One of the best ways to validate a Kalman filter is to compare the expected standard deviations of the state error components with the actual state error's deviations. (i.e. compare $\sqrt{\text{diag}(P)}$ with $\sqrt{\text{Avg}[(x-x')^2]}$)

Where you compute the actual deviation with a Monte Carlo simulation ($20 < \# \text{ of runs} < 100$).

The most common error committed with this technique is in the initialization of the individual Monte Carlo runs. The filter is seeded with an initial state estimate, $\hat{x}(0)$, and an expected error in that state, $\hat{P}(0)$. These could be guesses that never change run-to-run, or they could depend on the first measurement. Either way, to get the expected error to match the actual error, the Monte Carlo runs must obey the statistics in $\hat{P}(0)$. 

This means that if \( \hat{P}(\epsilon) \) indicates that an initial state component has a certain variance, then each Monte Carlo run should be initialized with a state component random variable that obeys that variance. For example, if we tell the filter through \( \hat{P}(\epsilon); \hat{x}_0 \) that the initial range is \( R_0 \) with an error of \( \sigma_R \), then we should initialize the filter each time with \( -R_0 \), but each Monte Carlo initial range value should be a different value of a random variable whose standard deviation is \( \sigma_R \).

Note: Also make sure that the noise seed for each individual run is different to guarantee a different sample for every element in the Monte Carlo ensemble.

Another check on the filter is the innovation sequence, \( \hat{z} = f(\hat{x}) \). If you plot it, it should be white with zero-mean.

Single runs only validate filter code in the absence of noise.
The intent is to isolate as many terms and components as possible.

1. Ownship Motion / Az-Plane / No-Crossover

   a) Choose $V_i R$ to give $\Delta \delta_{az}$

   b) North-only motion

   c) Do Not Cross over to avoid discontinuities

   Look for: Zero out-of-plane terms, No ownship.

2. Ownship Motion / El-Plane / No-Crossover

   Repeat 1 in N-D plane to exercise el.

3 & 4. Reverse directions in 1 & 2 to change signs and check for errors.

   Try in different quadrants to get different angles & signs.
6) At Cross-Over

Look for Discontinuity here.

6) Repeat Variations of 1-5 with coupled quadrant motion and with target motion.