Optimization of Reactive Mufflers

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Abstract—A new approach to optimization of reactive mufflers, which is based on use of muffler prototype with nondimensional geometrical parameters and integral criterion of acoustic performance of mufflers, is proposed. Implementation of the approach using the example of chamber mufflers is considered.

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Reactive mufflers are used extensively for reducing the gas-dynamic noise of machines. Optimization of its configuration is important for the performance of a muffler. This permits, on the one hand, determining the configuration of a muffler with maximum acoustic performance under given dimensional restrictions and, on the other hand, estimating the necessary minimum volume of a muffler to ensure required acoustic performance. Meanwhile, until recently, problems of muffler optimization were not given detailed consideration, and the works dealing with it [1–4] gave attention only to several aspects of the question.

To some extent, this is due to the fact that muffling of noise by reactive mufflers is frequency selective; while in one frequency range a muffler significantly reduces noise, in other ranges noise passes through a muffler almost without reduction. Such frequency selectivity is determined, not only by the muffler itself, but by the system parameters in which the muffler is set [5, 6]. From this point of view, it is reasonable to use so-called transmission loss $TL$ [5], definable by the logarithm of the ratio of the acoustic power of the incident wave at the muffler input to acoustic power of the wave after passing a muffler by matched output load as the acoustic performance rate. Transmission loss $TL$ is also a frequency function; they depend only on the parameters of the muffler itself and can be expressed in terms of coefficients of the muffler transfer matrix, which connects sound pressures and acoustic particle velocity at the input and output of the muffler [5].

In the above-mentioned works, optimization is performed with the use of the $TL$ rate for a certain frequency or limited band of frequencies. This factor significantly reduces the importance of the received results because muffling noise is usually broadband and the optimal muffler parameters for the specific frequency can take different values for another frequency.

In order to make the muffler optimization procedure effective, it is necessary, first, to proceed to use of the integral criterion of the acoustic performance of mufflers, which would depend on frequency and which would be evaluated only by numerical value. Second, as in the case of electrical filter calculation [7], the procedure of reactive muffler optimization should be performed with the use of a muffler prototype with nondimensional geometrical parameters that enables one later to proceed to the actual-size muffler, which meets specific requirements. The present article is concerned exactly with such an approach.

As the integral criterion of acoustic performance of mufflers, we suggest using overall transmission loss $OTL$ introduced for consideration in [8, 9] and determined by the formula

$$OTL = -10 \log \int_{f_1}^{f_2} W G_n^2 10^{-0.1 TL} df,$$  \hspace{1cm} (1)

where $f_1,f_2$ are the lower and upper limiting frequencies of the frequency range under consideration, respectively; $W$ is the weight function (when $W = 1$, we have overall linear loss; using $WA$-correction, which is applied for determining of sound levels, as the weight function, we will get overall loss measured on the $A$-scale); and $G_n$ is the normalized spectrum of the sound pressure of the incident wave at the muffler input that de facto determines the spectrum shape of inlet sound pressure.

As seen in (1), overall muffler loss depends not only on the characteristics of the muffler itself, which are determined by transmission loss $TL$, but on the spectrum shape of the inlet sound pressure and on the type of weight function used. When this spectrum is uniformly distributed with frequency in the frequency range under consideration with width $F = f_2 - f_1$, like the white noise spectrum, then $G_n^2 = 1/F$. Moreover, if weight function $W = 1$ and the transmission loss of a muffler are represented in this range $N$ by discrete val-
ues, then the formula (1) becomes simpler and takes the shape

\[
OTL = -10 \log \left( \sum_{i=1}^{N} 10^{-0.1TL_i} \right). \tag{2}
\]

In such an approach to optimization, the objective function is the extremum (maximum) of overall transmission loss \( OTL_{\text{max}} \). At the same time, the position of the extremum will depend not only on the parameters of the muffler itself, but on the frequency range under consideration as well. Consequently, one of the major problems of using overall loss for the optimization procedure is the necessity of reasonable selection of band limits, in which integration of the muffler transmission loss in accordance with (1) is performed.

Accordingly, it is reasonable to represent muffler transmission loss not as the frequency function, but as nondimensional parameter \( \mu = d_p/\lambda \) [10], which is connected with frequency through wavelength \( \lambda \). In this case, by computation of overall loss, the upper limit of the frequency range under consideration that corresponds to wavelength \( \lambda_{\text{gr}} \) will be determined by the parameter \( \mu_{\text{gr}} = d_p/\lambda_{\text{gr}} \) [10]. If we limit this frequency range by the condition that only plane sound waves can propagate in the muffler inlet and outlet, this condition will be written down as \( \mu_{\text{gr}} < \mu_p \), where \( \mu_p \) is a value determined by the cross-section shape of the inlet/outlet pipes that corresponds to the occurrence of the first transverse mode of acoustic oscillations in the inlet/outlet pipes. For example, for circular cross-section inlet/outlet pipes, wavelength \( \lambda_1 \), which corresponds to the occurrence of the first higher mode in the inlet/outlet pipe, is related to the zero of the Bessel function of the first order \( \alpha_1 \) by the ratio \( \mu_p = \frac{d_p}{\lambda_1} = \frac{\alpha_1}{\pi} = 1.22 \).

In addition, it is reasonable to express all geometrical dimensions of a muffler in terms of value \( d_p \), which enables one to turn from the original muffler to its prototype, wherein muffler configuration is determined by nondimensional parameters. Meanwhile, transmission loss of such a muffler prototype will be a universal characteristic that describes the acoustic features of the muffler family with the specific configuration and corresponds to the nondimensional parameters of the prototype.

This approach has the advantage that, as a rule, \( d_p \) ranges among original values that are set at the initial stage of muffler engineering and others that remain unchanged. Therefore, if we determine the muffler configuration, which meets the required acoustic performance, in terms of nondimensional parameters, then with knowledge of value \( d_p \) we can turn to the actual-size muffler, which also meets specific requirements.

Let us consider application of such approach to a simple chamber muffler. Acoustic characteristics of such a muffler are thoroughly studied [6, 11]. Aside from frequency they depend on its geometrical dimensions: chamber length \( l \) and cross dimensions of chamber \( D \) and inlet/outlet pipes \( d_p \). In the simplest model of an expansion chamber, which concerns only plane sound waves, transmission loss can be expressed in terms of ratio \( l/D \) and expansion ratio \( m = S/S_p \), where \( S \) and \( S_p \) are the cross-section areas of a chamber and inlet/outlet pipes. In the overall model, which also deals with higher modes of oscillations, there is another nondimensional parameter alongside the two above-mentioned ones, the ratio \( l/D \) usually being used as such [12]; additionally, the cross-section shape of the chamber here becomes important. Taking into account that, in accordance with the proposed approach, the fundamental value is cross dimension of a inlet/outlet pipe \( d_p \), let us take the values \( \mu, m \) and \( n = l/d_p \) as three nondimensional parameters that characterize an expansion chamber.

Figure 1 shows diagrams of transmission loss of the circular cross-section expansion chamber as functions of the \( \mu \) parameter. Transmission loss was calculated in terms of the coefficients of the expansion chamber transfer matrix, formulas for which are given in [13, 14]. The results conform to chambers with expansion ratio \( m = 8 \) and relative lengths \( n = 5 \) and \( n = 1 \). The presented diagrams can be divided into two parts. For the chamber where \( n = 5 \), by \( \mu < 0.4 \), the left half of the figure shows propagation in the expansion chamber of plane sound waves and the right half shows propagation of higher modes of acoustic oscillations. As in the case with a inlet/outlet pipe, wavelength \( \lambda_{\text{ik}} \), which corresponds to occurrence of the first higher mode in the chamber, is related to its cross dimension \( D \) by the ratio \( D/\lambda_{\text{ik}} = \mu_{\text{p}} \). Taking into account that chamber diameter \( D = d_p m^{1/2} \), the last ratio is transformed to the form \( d_p/\lambda_{\text{ik}} = \mu/m^{1/2} \). The left part of the last ratio defines the limit of solely plane sound waves propagation in a chamber on the presented figure. If we define this value as \( \mu_k \), we will get

\[
\mu_k = \mu_{\text{p}}/m^{1/2}. \tag{3}
\]

When \( m = 8 \), the value of \( \mu_k = 0.43 \); it is in full conformity with the diagram of transmission loss of a chamber with relative length \( n = 5 \), which is presented in Fig. 1. When \( n = 1 \), the diagram of expansion chamber transmission loss changes significantly. For such a chamber, the ratio \( l/D = d_p n/(d_p m^{1/2}) = n/m^{1/2} = 0.35 \); this chamber belongs to the class of so-called short chambers [12], when it starts working as a resonator in the low-frequency region [15].

The represented data show that expansion chambers’ transmission loss changes variously, depending on their geometry; therefore, they cannot intrinsically give an unambiguous answer as to which of the compared chambers preferable from the point of view of acoustic performance.

Now let us consider overall transmission loss of expansion chambers, a calculation of which we will
produce with the use of formula (2). Figure 2 shows the dependences of overall loss on expansion ratio $m$ of chambers with varying relative length $n$. Overall loss is represented for two frequency ranges corresponding to $\mu_{gr} = 1$ and $\mu_{gr} = 1/2$. Moreover, for comparison, overall loss is presented here that correspond to a plane model of an expansion chamber. At low values of $m$, overall expansion chamber loss, calculated with and without higher modes, is almost the same. For long chambers ($n = 5$), further increase in $m$ results in termination of overall loss and an increase in occurrence of higher modes of oscillations in the chamber. For short expansion chambers, increase in $m$ results in the occurrence of an extremum on the transmission loss diagrams when overall loss takes maximum values $OTL_{\text{max}}$. The values of $m = m_o$ and $n = n_o$, which correspond to this maximum, will depend on $\mu_{gr}$ value. When $m > m_o$ and $n > n_o$, overall loss decreases; meanwhile, the expansion chamber volume will increase. For this reason, the values of $m_o$ and $n_o$ can be considered optimal. If $\mu_{gr}$ parameter decreases from 1 to 1/2 (twofold narrowing of operating frequency range), the value of $OTL_{\text{max}}$ and optimal chamber length ($n_o$ parameter) increase approximately in the same way and optimal expansion ratio $m_o$ increases to an even greater degree. Let us note that the transmission loss that is
The nature of the dependence of optimal parameters of the expansion chamber on coefficient \( \mu_{gr} \) is reflected in Table 1. The depicted results show that decrease in frequency range under consideration is accompanied with a significant increase in the \( OTL_{\text{max}} \) value, as well as the chamber volume, which is expressed in nondimensional units with product \( n_o \cdot m_o \). At the same time, changes of the \( OTL_{\text{max}}/n_o \) value are not so essential, because this value hardly falls outside the limits of the range 6–8 dB. The presented data also show that, although a decrease in \( \mu_{gr} \) is accompanied by a significant increase in the optimal chamber’s dimensions, which are expressed in terms of the parameters \( m_o \) and \( n_o \), the dependence of the chamber length on its diameter, by which the global maximum of overall loss is provided, increases only slightly, from 0.38 to 0.42, i.e., less than 10%. Therefore, in any case, optimal expansion chambers will belong to the class of short chambers, the nature of sound propagation in which is closer to that in resonators.

It is possible to use a somewhat different approach to the procedure of expansion chamber optimization. For given expansion ratio of the chamber \( m \), we determine relative chamber length \( n_o \) and parameter \( \mu_{gr} \), which provide \( OTL_{\text{max}} \). The results of the appropriate calculations for circular cross-section chambers are depicted in Table 2. These results show that parameter \( \mu_{gr} \), which determines the optimal frequency range of chamber muffling, can be approximated with formula (3), in which constant \( \mu_{gr} \) for a circular cross-section muffler is taken to be equal 1.22. Therefore, integration of a frequency range by calculation of overall loss is always restricted to the range of plane sound wave propagation in the chamber of the given cross section. At the same time, however, in the chamber itself, plane waves per se do not propagate, because the chamber belongs to the class of short chambers, the nature of sound propagation in which is closer to that in resonators.

The presented data also show that overall transmission loss can be expressed as the expansion ratio with the approximation formula

\[
OTL_{\text{max}} \approx 10 \log m,
\]

which gives fairly good agreement with the results of numerical calculations.

Formally, the value under the logarithm sign in the formula (4) can be considered as the square of transmission coefficient \( \tau^2 \) that shows how much the sound wave amplitude decays in passing through a muffler. It appears that, in this case, transmission coefficient \( \tau = m^{1/2} \). Therefore, with regard to (3), we discover that the product of transmission coefficient \( \tau \) and normalized width of operating frequency range \( \mu_{gr} \) is a constant value:

\[
\tau \mu_{gr} = \mu_p = \text{const.}
\]

Formula (5) can be considered as a kind of uncertainty relation. This relation shows that the larger the required value of overall loss, the narrower the frequency range where it can be achieved by an expansion chamber; on the contrary, the broader the operating frequency range, the lower the values of the maximum achievable overall loss on it.

<table>
<thead>
<tr>
<th>Chamber parameters</th>
<th>( \mu_{gr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_o )</td>
<td>0.63 0.75 0.83 1.06 1.53 2.01 3.96</td>
</tr>
<tr>
<td>( m_o )</td>
<td>2.8 3.9 4.7 7.8 14.0 23.5 89.4</td>
</tr>
<tr>
<td>( l/D = n_o/m_o^{1/2} )</td>
<td>0.38 0.38 0.39 0.40 0.41 0.42 0.42</td>
</tr>
<tr>
<td>( OTL_{\text{max}} )</td>
<td>4.5 6.1 6.9 8.2 7.5 13.8 21.1</td>
</tr>
<tr>
<td>( OTL_{\text{max}}/n_o )</td>
<td>7.1 8.1 8.3 8.2 7.5 6.9 5.3</td>
</tr>
</tbody>
</table>

Table 1. Dependence of optimal chamber parameters on parameter \( \mu_{gr} \)

<table>
<thead>
<tr>
<th>Chamber parameters</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_o )</td>
<td>0.68 0.79 0.99 1.24 1.44 1.67 1.90 2.13</td>
</tr>
<tr>
<td>( \mu_{gr} ) (calculation)</td>
<td>0.85 0.69 0.53 0.41 0.35 0.30 0.26 0.23</td>
</tr>
<tr>
<td>( \mu_{gr} = 1.22/m^{1/2} )</td>
<td>0.70 0.61 0.50 0.41 0.35 0.30 0.27 0.24</td>
</tr>
<tr>
<td>( OTL_{\text{max}} ) (calculation)</td>
<td>5.2 6.4 8.1 9.7 11.0 12.2 13.2 14.2</td>
</tr>
<tr>
<td>( OTL_{\text{max}} = 10\log m )</td>
<td>4.8 6.0 7.8 9.5 10.8 12.0 13.0 13.9</td>
</tr>
</tbody>
</table>

Table 2. Dependence of optimal chamber parameters on its expansion ratio \( m \)
Let us note that the optimal chambers’ characteristics depicted in the tables are expressed in terms of nondimensional parameters. Therefore, such chamber mufflers are, in essence, muffler prototypes. Having determined the actual cross dimensions of the inlet and outlet inlet/outlet pipes of muffler, we can define the dimensions of an optimal muffler. For example, when inlet/outlet pipe diameter \( d_p \) is 50 mm and the muffling diapason corresponds to \( \mu_{gr} = 1/2 \), optimal expansion chamber, as it appears from Table 1, should have a length \( l = n_o \cdot d_p = 53 \) mm and diameter \( D = m_o^{1/2} d_p = 134 \) mm. At the same time, the upper limiting frequency of muffling diapason \( f_{gr} \) is equal to \( \mu_{gr} c/d_p \) and approximately equal to 3400 Hz.

It should also be stated that, if we are restricted by the cross dimensions of the expansion chamber and cannot prescribe large values of \( m \), overall chamber loss will be low as well, even in a narrow frequency range, because large overall loss of an expansion chamber is provided only with large expansion ratio \( m \). In this case, the only way to increase chamber muffler acoustic performance is to make it multisectional.

Analysis of the acoustic characteristics of a two-section chamber muffler generated from the abovementioned simple chamber by means of placement of a diaphragm with an axisymmetric hole with a diameter equal to the chamber inlet/outlet pipes’ diameter inside it was carried out. At the beginning, under a fixed overall length of such a muffler \( n \), the relations between sections \( n_1 \) and \( n_2 \) were varied in order to find an optimal alternative that can provide a maximum of overall loss. As a result, it was discovered that, regardless of overall length \( n \) of a two-section chamber muffler, its optimal configuration is when the length of one of the sections is equal to the optimal length of a one-section chamber muffler \( n_{opt} \) under given expansion ratio \( m \). At the second stage, having assumed the length of the first muffler section \( n_1 \) to be equal \( n_{opt} \), the length of the second section \( n_2 \) varied in order to find the global maximum of overall transmission loss. The appropriate calculations were carried out and their results for \( \mu_{gr} = 1 \) are represented in Fig. 3, where overall transmission loss curves for mufflers with one and two sections are given for comparison. The calculations according to the data of Table 1 were done under the condition that the expansion ratio of chambers \( m \) is equal 2.8. The presented data show that the presence of the second section significantly increases the overall loss of a chamber muffler. These loss reach their peak value when the length of the two-section muffler \( n \) is equal 1.8.

Let us note that overall loss of a two-section muffler will change significantly under the conditions of change of the second chamber length if this chamber remains in the class of short chambers or is near to it. Further increase of chamber length is accompanied with termination of overall loss growth; they oscillate about a certain average level. As appears from Fig. 3, this average level for one-chamber muffler is less than 3 dB. For the two-chamber muffler, this level increases significantly and reaches 5 dB.

Fig. 3. Overall transmission loss of one-section (dashed line) and two-section (solid line) expansion chambers.
and, consequently, the higher the overall muffler loss in this range, the larger the muffler volume needed for it.

In conclusion, it should be noted that this work suggests a new approach to designing and engineering mufflers. It is based, first, on the use of such integral values of acoustic performance as overall transmission loss and, second, on the use of the cross dimension of the muffler inlet/outlet pipes as the base dimension of the muffler itself; the rest of the geometrical muffler dimensions and muffling frequency range are expressed in terms of this dimension. It enables one to start solving the problem of finding a muffler prototype that provides the required muffling level in a certain frequency range. Then, with the actual dimensions of the inlet/outlet pipe being known, we can turn to the actual muffler configuration. The proposed approach was tested on chamber mufflers. It was discovered that there is a kind of uncertainty relation for an expansion chamber; it links the width of muffling frequency range and muffling rate in this range. Moreover, the proposed approach enables one to resolve the problem of optimization of muffler geometry, this being shown using the example of a chamber muffler with one and two sections.

**Fig. 4.** Optimal configurations of two-section chamber mufflers. (a) $\mu_{gr} = 1$; (b) $\mu_{gr} = 1/2$.

**REFERENCES**


*Translated by Bataeva*