Characterization and Sensitivity Analysis of Hyperelastic Materials in Biaxial Tension

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Overview

• Motivation for Research
• Specific Problem & Goals
  • Test Equipment Improvements
  • Internal Stress Characterization Method
  • Test Specimen Design Improvements
  • Specimen and Material Model Sensitivity
  • Material Characterization and Validation
Motivation for Research

• The **human lung** – particle motion and distribution unknown
• Helpful for smoking and medicine dosing studies

• Alveoli too small! (200-300 microns[^1])

• Scaled models are **linear** elastic

• Lung mechanical properties are **nonlinear**...

[^1]: Berend, et. al. 2000

http://hyperphysics.phy-astr.gsu.edu/HBASE/ptens2.html
Motivation for Research

Long-term Project Goals

• Material that replicates the human lung mechanical response.

• Accurate biaxial properties for use in computer simulations.

Estimated PV curves for non-excised healthy and emphysematous human lungs
Specific Problem & Goals

Surrogate Materials: Testing and Characterization

- **Test Fixture:** Original test fixture at RIT was **not acceptable**
  - Goal 1: Improve test fixture and collect acceptable equibiaxial biaxial data.

- **Geometry:** Original specimen geometry was **not optimal**
  - Goal 2: Develop criteria and conduct a numerical analysis to determine a planar biaxial specimen geometry that best satisfies the criteria.

- **Method:** Biaxial material characterization methods are **not standardized**
  - Goal 3: Develop a material characterization process that accurately estimates equibiaxial stress-strain response.

- **Sensitivity:** No knowledge of how test parameters affect **material properties**
  - Goal 4: Numerically analyze common test parameter perturbations and their potential effect on stress-strain characterization.
Background – Test Equipment at RIT

Planar Biaxial Test Fixture
Background – Test Equipment at RIT

Boiling Flask Test Fixture

Experimental stereoPIV test setup from Berg (2010).

Boiling flask specimen and model from Ferrara (2009).

Pressure difference at boiling flask wall:

\[ P_{in} - P_{out} = P_{ch}(0) - \rho g H g(t) - P_{ch}(t) \]
Test Fixture Misalignment - Calculations

- Test fixture experienced **clamp misalignment**

- Causes:
  - Non-equibiaxial displacement
  - Binding
  - Uneven specimen deformation

- Quantifying clamp location, pt. R

- **2-D vector analysis** of the linkages revealed the cause for misalignment

- Images of the system in motion verified the vector analysis
Test Fixture Misalignment - Results

- Majority of misalignment from **top crossbar** offset
- Except for **pin hole diameters**, components machined to acceptable tolerances.
- Pin play creates **backlash** in joints.
- Result is different **extension & contraction** paths.

![Graph showing X-clamp Height Comparison (Images & Calculations)]

\[
y = 0.8611x - 0.0222 \\
R^2 = 0.9994
\]
## Test Fixture Misalignment - Results

### X-axis Clamp Height Bounding Paths

<table>
<thead>
<tr>
<th>Crossbar Height (in)</th>
<th>-X Clamp Min</th>
<th>-X Clamp Max</th>
<th>+X Clamp Min</th>
<th>+X Clamp Max</th>
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<tbody>
<tr>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>7.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Clamp Height (in):**
- Old Operating Range: 3.3 - 3.5
- New Operating Range: (Assumed to be within the same range as Old Operating Range for simplicity)
Test Fixture Design – MCD Analysis

- Minimum Constraint Design (MCD) - A design method to develop a device that operates with a particular number of Degrees of Freedom (DOF).

The original design is over-constrained, which seems to create the binding.

Original System MCD Analysis

<table>
<thead>
<tr>
<th>Joint Type</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>6</td>
</tr>
<tr>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>Slider</td>
<td>4</td>
</tr>
<tr>
<td>Ball</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Introduced Component</th>
<th>Connecting Component</th>
<th>Connecting Joint Type</th>
<th>No. of Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ground</td>
<td>Fixed</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Slide (Rot)</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>Pin</td>
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<td>6</td>
<td>5</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>Ground</td>
<td>Fixed</td>
<td>6</td>
</tr>
</tbody>
</table>

Total DOF = 36
Total Constraints = 41
Remaining DOF = -5
## Test Fixture Design – MCD Analysis

### Redesigned System MCD Analysis

<table>
<thead>
<tr>
<th>Introduced Component</th>
<th>Connecting Component</th>
<th>Connecting Joint Type</th>
<th>No. of Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ground</td>
<td>Fixed</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Ground</td>
<td>Slide (Rot)</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>Pin</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>Ball</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>6</td>
<td>Slide</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>9</td>
<td>Slide</td>
<td>4</td>
</tr>
</tbody>
</table>

**Original MCD**

- **Total DOF** = 78
- **Total Constraints** = 73
- **Remaining DOF** = 5
- **Insignificant** = #4, #11, #13 (x2)
- **Significant DOF** = 1

**MCD**

- + 2 DOF (2x)
- Tolerance (2x)
- + 3 DOF
Test Fixture Design - Implementation

New Components

1. Inlaid mounts (4x)
2. Motion links (4x)
3. Platform
4. Radial bearings (4x)
5. Spherical bearings (6x)
6. Precision shoulder bolts (4x)
Test Fixture Design – MCD Results

X-axis Clamp Height

Original

MCD

<table>
<thead>
<tr>
<th>Crossbar Height (in)</th>
<th>Clamp Location (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+X clamp (Actual)</td>
</tr>
<tr>
<td></td>
<td>-X clamp (Actual)</td>
</tr>
</tbody>
</table>

- X clamp height
- +X clamp height
Test Fixture Design – MCD Results

Load Ratio

**Original**

\[ y = 0.8611x - 0.0222 \]
\[ R^2 = 0.9994 \]

**MCD**

\[ y = 1.0135x - 0.0166 \]
\[ R^2 = 0.9999 \]
Background – Stress and Strain

Planar Biaxial Geometry

\[ \sigma = \frac{F_{\text{load cell}}}{A_{\text{clamp edge}}} \]

\[ A_{\text{clamp edge}} = \text{Clamp width} \times \text{Specimen thickness} \]

\[ \varepsilon = \frac{\Delta l}{l_0} = \frac{y_a - y_b - l_0}{l_0} \]
Internal Stress Estimation

- **Perpendicular loads** work against each other.

- **Non-uniform stress** across specimen

- Relates edge stress to internal stress level

\[
SDF = \frac{\text{Central Stress (Estimated)}}{\text{Edge Stress (Applied)}}
\]

Stress Decay Across Path A-A, \(SDF = 0.808\)
Test Method - Stress Decay Factor (SDF)

1. Experimental data collected (Figure 2.21):
   a. Clamp edge stress
   b. Central diamond strain

2. First simulation with data from step 1:
   a. Best available data
   b. Modeled as equibiaxial

3. First numerical results (Figure 2.22):
   a. ANSYS model verification

4. Calculate SDF from step 3:
   a. Stress: $\frac{\text{Central Diamond}}{\text{Clamp Edge}}$

Figure 2.21: Raw experimental stress-strain plot for the R04 CCS no leg specimen.

Figure 2.22: Raw edge stress data modeled and verified as equibiaxial central diamond stress.
Test Method - Stress Decay Factor (SDF)

4. Calculate predicted central diamond stress (Figure 2.23):
   a. \( \sigma_{\text{diamond}} = \text{SDF} \times \sigma_{\text{edge}} \)

5. Final simulation (Figure 2.23):
   a. Using SDF-modified data from step 5

6. Additional verification (Figure 2.24):
   a. Experimental clamp edge data (original data) and
   b. ANSYS clamp edge results (from simulation in step 5)
Background – Specimen Design

Sacks et al. (2005)
Background – Specimen Design

Small radius
- Partially thinned center region
  - Breiu et al. (2007)

Large radius
- Completely thinned center region
  - Yu et al. (2002)
  - Bhatnagar et al. (2007)
Specimen Candidates & Criteria

(1) High SDF
    Minimize stress decay

(2) Large deformation
    Accurate characterization

(3) Large equibiaxial stress region
    Accurate SDF calculation

(4) Testable in-house

Cruciform clamped at end of leg
Dimensions:
25.4mm end to end, 25.4mm leg width
Dimensions varied:
Fillet radius (1, 5 & 10mm)

Cruciform clamped at fillet foot
Dimensions:
33.4mm end to end, 25.4mm leg width
Dimensions varied:
Fillet radius (1, 4, 5, 6 & 10mm)

Square with interfering clamps
Dimensions:
50.8mm sides, 25.4mm clamp width,
5mm clamp inset
Dimensions varied:
Side length (50.8 & 76.2mm), clamp inset (5 & 7.5mm)

CCS R01 leg (current geometry)
CCS R04 no leg
CSS 2x2*
* certain regions free meshed
Finite Element - Overview

- **Element Type:** Shell281 (8-node)
- **Stiffness:** Membrane only (no bending)
- **Mesh:** Mapped (Global Size = 1)
- **Material Model:** Mooney-Rivlin 2-parameter

\[
\bar{U} = \frac{\mu_1}{2} (\bar{I}_1 - 3) + \frac{\mu_2}{2} (\bar{I}_2 - 3) + \frac{K_1}{2} (J - 1)^2
\]

- ANSYS hyperelastic curve-fitting tool estimates Mooney-Rivlin 2-parameter coefficients, \( C_{10} \) and \( C_{01} \). For material incompressibility, \( d = 0 \).

- Candidate models use **identical Mooney-Rivlin coefficients and displacement loads** for consistency.
New Specimen Selection - Results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>SDF (%)</th>
<th>Range (mm)</th>
<th>Range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCS R01 leg*</td>
<td>80.74%</td>
<td>3.6</td>
<td>7.0%</td>
</tr>
<tr>
<td>CCS R04 no leg</td>
<td>83.15%</td>
<td>3.6</td>
<td>10.8%</td>
</tr>
<tr>
<td>CSS 2x2</td>
<td>83.10%</td>
<td>3.6</td>
<td>8.8%</td>
</tr>
</tbody>
</table>

*Specimen design from Ferrara 2009
Specimen Sensitivity Analysis

- How does a specimen respond to:
  1) **Offset clamp** deformation
  2) Non-equibiaxial **load ratio**

- By how much are stress-strain results effected?

- How are metrics such as SDF and equibiaxial range effected?

Von Mises stress contour of CCS with offset ½” clamps.
Specimen Sensitivity Analysis - Results

Offset Clamp Graph

- Nominal
- 0.1 in
- 0.125 in
- 0.25 in
- 0.375 in
- 0.5 in

Stress (Mpa) vs Strain

Nominal
0.1 in
0.125 in
0.25 in
0.375 in
0.5 in
Specimen Sensitivity Analysis - Results

Non-equibiaxial Load Ratios

<table>
<thead>
<tr>
<th>Load Ratio</th>
<th>1</th>
<th>0.95</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDF</td>
<td>0.658</td>
<td>0.641</td>
<td>0.613</td>
</tr>
<tr>
<td>Strain</td>
<td>Stress</td>
<td>Stress</td>
<td>% Difference</td>
</tr>
<tr>
<td>0.04</td>
<td>0.0015</td>
<td>0.0014</td>
<td>5.4%</td>
</tr>
<tr>
<td>0.08</td>
<td>0.0027</td>
<td>0.0025</td>
<td>5.4%</td>
</tr>
<tr>
<td>0.13</td>
<td>0.0037</td>
<td>0.0035</td>
<td>5.3%</td>
</tr>
<tr>
<td>1.16</td>
<td>0.0137</td>
<td>0.0129</td>
<td>6.1%</td>
</tr>
<tr>
<td>1.20</td>
<td>0.0139</td>
<td>0.0131</td>
<td>6.1%</td>
</tr>
<tr>
<td>1.24</td>
<td>0.0142</td>
<td>0.0134</td>
<td>6.2%</td>
</tr>
</tbody>
</table>

Load Ratio: 1.05 1.15

<table>
<thead>
<tr>
<th>SDF</th>
<th>0.658</th>
<th>0.678</th>
<th>0.723</th>
</tr>
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<tbody>
<tr>
<td>Strain</td>
<td>Stress</td>
<td>Stress</td>
<td>% Difference</td>
</tr>
<tr>
<td>0.04</td>
<td>0.0015</td>
<td>0.0016</td>
<td>5.6%</td>
</tr>
<tr>
<td>0.08</td>
<td>0.0027</td>
<td>0.0028</td>
<td>5.6%</td>
</tr>
<tr>
<td>0.13</td>
<td>0.0037</td>
<td>0.0039</td>
<td>5.5%</td>
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<td>1.16</td>
<td>0.0137</td>
<td>0.0146</td>
<td>6.3%</td>
</tr>
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<td>0.0139</td>
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<td>1.24</td>
<td>0.0142</td>
<td>0.0151</td>
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</table>
Mooney-Rivlin Coefficient Sensitivity

Material properties:

<table>
<thead>
<tr>
<th></th>
<th>Ferrara 2009</th>
<th>Current</th>
</tr>
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<tbody>
<tr>
<td>C10</td>
<td>3.46E-02</td>
<td>3.33E-03</td>
</tr>
<tr>
<td>C01</td>
<td>-1.085E-02</td>
<td>-2.43E-05</td>
</tr>
</tbody>
</table>

Material Stability:

\[ \frac{d\sigma_{ij}}{d\varepsilon_{ij}} > 0 \]

Mooney-Rivlin model for equibiaxial tension:

\[ S_1 = S_2 = \mu_1(\lambda - \lambda^{-5}) + \mu_2(\lambda^3 - \lambda^{-3}) \]

(Applied Mechanics of Solids, A. F. Bowers)
Planar Biaxial Characterization Results

New Material Properties:

**Mooney-Rivlin 2-parameter**
- **C10**: 3.33E-03
- **C01**: -2.43E-05
Validation: Inflated Boiling Flask

ANSYS predicts a more compliant boiling flask than experimentally measured.

Calculation for pressure on boiling flask wall:

\[ P_{\text{in}} - P_{\text{out}} = P_{\text{ch}}(V) - \rho g \Delta h_{\text{ch}}(t) - P_{\text{ch}}(t) \]

Calculation used for pressure on boiling flask wall (blue dots)

\[ P_{\text{in}} - P_{\text{out}} = P_{\text{ch}}(V) - P_{\text{ch}}(t) \]
Future Work

- Validation
  - Boiling Flask
- More strain
  - Material stability at higher strain?
- Material search:
  - $\sigma-\varepsilon$ to P-V correlation
- Simulation convergence
  - Boiling flask and more complex geometries
Summary

- Test Fixture Improvement
  - 1:1.01 ratio
  - Symmetric clamps
  - No binding
- Specimen Design Improvement
  - Increased strain
  - Higher SDF
  - Increased % equibiaxial region
- Sensitivity Analysis
  - Low sensitivity to clamp alignment
  - High sensitivity to load ratio
- SDF Test Method
  - Estimate equibiaxial stress-strain
  - More stable coefficients
  - Estimates a more compliant material
- Boiling Flask prediction
  - Experimental pressure is actually less than currently measured.
References

References

References

QUESTIONS? COMMENT? ANSWERS?

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Department of Mechanical Engineering

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