Optothermal mathematical model and experimental studies for laser irradiation of arteries in the presence of blood flow

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We have developed an optothermal model for the interaction of laser light and the tissue of arterial walls and have checked its validity with animal experiments. The mathematical model consists of a laser diffusing tip positioned intraluminally in a cylindrical artery, in which the diffused laser light is incident on a blood–tissue interface at a distance from the tip. A temperature profile throughout the interface is obtained by considering the optical interaction and the thermal conduction and convection of the blood and tissue. The distribution of light in the media is determined using both Beer's law and the Kubelka-Munk two-flux theory in cylindrical coordinates. For experimental in vivo verification, a diffusing tip was inserted in canine arteries and the temperature profile varied by restricting the volume of blood; this simulated degrees of occlusion to determine the influence of blood flow on heat transport. The measured temperature profiles compared favorably to the theoretical results. Temperature profiles are also predicted for a water-filled lumen. The theoretical model will be useful in predicting the depth of ablation and extent of normal tissue damage during laser angioplasty treatment of atherosclerosis.

1. Introduction

The leading cause of death in the United States is coronary artery disease, known as atherosclerosis. This consists of yellowish to white plaques of cholesterol, fibrous and smooth muscle cells, and calcium within the arterial walls. A new technique for clearing atherosclerotic deposits, called laser angioplasty, has recently been demonstrated using fiber optics and lasers to ablate the plaque. For this procedure, an optical fiber is inserted into an artery and guided to the coronary or other atherosclerotic arteries. Laser light for removing the plaque is coupled into the fiber and transmitted to a variety of tips, including cleaved, sapphire, cylindrical diffusing, metal, or lensed tips.

One version of the cylindrical diffusing tip from Laserguide Corp. scatters the laser light in an approximate Gaussian distribution along the longitudinal axis. The risk of perforating the arterial wall is lower with a diffused light source than with a direct light beam. Using incident light rather than heating a metal tip with the light also provides an opportunity for selective ablation. Studies in atherosclerotic swine and rabbits have shown photosensitive dyes [hematoporphyrin derivative (HPD) and tetraphenyl porphine sulfonate (TPPS₄)] injected in the blood stream preferentially accumulate in the plaque vs the tissue. HPD and TPPS₄ absorb visible wavelengths, and at 630 nm both photochemical and photothermal processes may be possible mechanisms for selectively ablating plaque with minimal normal tissue damage. Photodynamic therapy is the photochemical process by which a biological system is damaged by the interaction of light and photosensitizers, producing singlet oxygen.

To support studies on selective plaque ablation procedures with the laser diffusing tip and predict potential damage to normal tissue, we developed a mathematical model of the laser diffusing tip which determines temperature profiles during laser angioplasty. Previous models have dealt mainly with direct laser light irradiation of tissue–tissue and tissue–plaque interfaces. Our model considers a cylindrical geometry and a Gaussian axial distribution of diffused light and includes arterial blood flow and tissue perfusion.

Low continuous optical powers for several minutes are used in laser angioplasty procedures with photosensitive dyes. During this length of time, the
blood flowing in the lumen and perfusing the tissue may provide appreciable heat transport away from the area to be ablated. The blood is flowing along the cylindrical tip, so a realistic model must include both axial and radial dimensions (see Fig. 1). At 630 nm, the blood significantly reduces the amount of light that reaches the tissue, since the blood has relatively high absorption at this wavelength. As a result, we also modeled the case of a water flush, which allows more light to propagate into the tissue while transporting heat away from the tissue. The time dependent optothermal cylindrical model is presented in the next section with experimental verification following.

II. Theory

The time dependent heat equation is used to model the axial and radial temperature distribution of the diffusing tip. This equation follows from the conservation of energy, by which the amount of heat entering a region plus the heat generated in the region equals the amount of heat leaving the region plus the amount of heat stored. In this model, the heat generated in a region is due solely to absorption of light. Heat enters or leaves the region through the conduction from or to adjacent tissue, and heat is also transported by the blood flow in the lumen. Heat added or lost per unit volume Q in that region is related to temperature T change by

$$Q = \rho c \frac{dT}{dt},$$

where \( \rho \) = density of the medium in the region, and \( c \) = heat capacity of the medium in the region.

The time dependent heat equation states that the temperature change \([T(r,z,t + \Delta t)] - [T(r,z,t)]\) is equal to the contributions due to light absorption \(\Delta T_l(r,z,t)\) plus the blood or water flow and perfusion \(\Delta T_p(r,z,t)\) and conduction to and from adjacent tissue \(\Delta T_c(r,z,t)\). This equation can be expressed as

$$T(r,z,t + \Delta t) - T(r,z,t) = \Delta T_l(r,z,t) + \Delta T_p(r,z,t) + \Delta T_c(r,z,t). \tag{2}$$

Each of these contributing terms is described in detail.

When considering boundary conditions at the blood–tissue interface, blood flow, and perfusion, the resulting equation can be solved numerically using the finite difference methods or finite element methods. The preferred method for this large scale model of the entire diffusing tip is the finite difference method, since a matrix does not have to be stored as required by the finite element method. Each conductive and convective temperature term is then expressed as a difference equation. Angular symmetry about the axis is assumed.

When laser light is incident from the diffusing tip, it will be absorbed and scattered by the blood and tissue. The light distribution may be modeled by using either the Kubelka-Munk two-flux theory or Beer's law, with the incident intensity used in Beer's law modified to include the effect of scattering in the media.

A. Modified Beer's Law

According to Beer's law, absorption and scattering will cause an exponential decay of the diffused light as it propagates through the tissue. In addition, the intensity will decrease by a factor of \( r_0/r \) due to the conservation of power requirements in a cylindrical geometry, where \( r_0 \) is the tip radius. The incident diffused light is assumed to be Gaussian distributed \(\exp[-(z/d)^2]\) in the axial direction z. Therefore, assuming minimal reflection occurs at the blood–tissue interface, the light intensity distribution from the tip is

$$I(r,z) = \frac{I_0}{r} \exp[-\frac{(z/d)^2}{2}] \exp[-\mu_1(r - r_0)] \quad r_0 \leq r \leq r_1, \tag{3a}$$

$$I(r,z) = \frac{I_0}{r} \exp[-\frac{(z/d)^2}{2}] \exp[-\mu_2(r - r_1)] \quad r \geq r_1, \tag{3b}$$

where \( \mu_1 = \alpha_1 + \beta_1 \), \( \alpha_1 = \) absorption coefficient in the blood, \( \beta_1 = \) scattering coefficient in the blood, \( \mu_2 = \alpha_2 + \beta_2 \), \( \alpha_2 = \) absorption coefficient in the tissue, \( \beta_2 = \) scattering coefficient in the tissue, and \( I_0 \) = incident intensity at \( r = r_0 \) and \( z = 0 \). This intensity will be absorbed as heat according to

$$Q = \alpha I(r,z), \tag{4}$$

where \( \alpha = \alpha_1 \) in blood and \( \alpha_2 \) in tissue.

Combining Eqs. (1) and (4) for small \( \Delta t \),

$$\Delta T_l(r,z,t) = \frac{\Delta t}{\rho c} \alpha I(r,z)s(t), \tag{5}$$

where

$$s(t) = 1 \text{ if the laser is on,} \quad 0 \text{ if the laser is off.}$$

In nonscattering media, the incident intensity \( I_0 \) in Eqs. (3a) and (3b) is just equal to the total delivered power \( P \) divided by the cylindrical surface area of the

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**Fig. 1.** Geometrical model of a laser diffusing tip positioned intraluminally to an arterial lumen with blood flow.
ential elements located at \( r + \Delta r, r - \Delta r, z + \Delta z, \) and \( z - \Delta z \) is
\[
\Delta T_c(r,z,t) = \frac{\Delta t}{\rho c} \eta \nabla^2 T(r,z,t), \tag{7}
\]
where \( \eta \) = conductivity and \( \nabla^2 \) is written as finite difference equations in cylindrical coordinates.

Blood flow through the lumen and tissue perfusion also contribute to heating and cooling of the blood and tissue. The blood flow is modeled by moving finite elements of blood at the previous spatial temperature along the \( z \) axis during the computation. The maximum velocity occurs halfway between the tip wall and the blood–tissue wall and then parabolically tapers to zero velocity at the inner and outer walls. The velocity as a function of the radius is expressed as
\[
V(r) = V_{\text{max}} \left( 1 - \left( \frac{r - \left( r_1 + r_0 \right)}{\left( r_1 - r_0 \right)} \right)^2 \right),
\]
where the maximum velocity \( V_{\text{max}} \) is related to the measurable average velocity \( V_{\text{ave}} \) according to
\[
V_{\text{ave}} = \frac{1}{\pi (r_1^2 - r_0^2)} \int_0^{2\pi} \int_0^{r_1} V(r) r dr d\theta.
\]
Solving this integral yields the following expression for \( V_{\text{max}} \) in terms of all measurable quantities \( r_1, r_0, \) and \( V_{\text{ave}} \):
\[
V_{\text{max}} = \frac{\pi (r_1^2 - r_0^2) V_{\text{ave}}}{2 \left( \frac{1}{2} r_1^2 + \frac{1}{2} r_0^2 + 2 \gamma (r_1^2 - r_0^2) + \frac{2 \gamma (r_1^2 - r_0^2)}{3 \chi} \right)},
\]
where \( \gamma = \left( r_1 + r_0 \right)/2 \) and \( \chi = \left( r_1 - r_0 \right)/2 \).

The blood flowing through the arterial lumen convectively modifies the arterial wall temperature proportional to the rate of flow. This temperature change also occurs with the perfusion of blood into the tissue. The blood perfuses into the tissue capillaries at a given temperature \( T_{\text{cap}} \), then increases or decreases the tissue temperature \( T_{\text{tissue}} \), proportional to the difference in these temperatures. The blood and tissue reach an equilibrium temperature as the blood perfuses radially outward. The equation governing this is:
\[
\Delta T_s(r,z,t) = w_b (T_{\text{cap}} - T_{\text{tissue}}) \Delta t, \tag{8}
\]
where \( w_b \) indicates the ratio of the volume of perfusing blood to the volume of tissue at a given flow rate. Values of \( w_b \) in typical tissue are much less than unity, and computational results for our model showed that this perfusion had a negligible effect on temperature for the given experimental conditions.

The final time dependent heat equation is the sum of Eqs. (7) and (8) and either (5) or (6) with parabolic blood flow. Theoretical temperature distributions are presented in the next section with the experimental verification. Finite difference methods introduce errors which may be reduced by using small spatial elements and short time increments, approaching a mathematically continuous evaluation. We found that the ratio of the time increment \( \Delta t \) and the radial increment \( \Delta r \) had to be less than or equal to 0.01 s/mm for proper convergence of our model.

### III. Experimental Procedure

A canine model was used for experimental verification of the theoretical model. One welded 36-gauge T-type thermocouple probe was sutured to the Laserguide fiber optic diffusing tip to measure accurately the temperature rise of the tip. Then the optical fiber was inserted through an incision in one of the main femoral arteries of the animal and into arterial branches varying in diameter from 1 to 5 mm. The final position of the diffusing tip was distal to the initial incision by a distance of \( \sim 13 \) cm.

At the location of the tip, a second incision was made so three to four more thermocouples could be positioned in the tissue next to the arterial wall. These thermocouples spaced from each other by \( \sim 1 \) mm were in a linear array normal to the tip. In addition, a blood flow probe (interfaced with a Transonic model T201 ultrasonic blood flowmeter) was positioned around the artery to measure the average blood flow rate. The thermocouples were sutured into place so that the incision could be closed. The thermocouples moved slightly on closing the incision, but the relative spacing remained the same. Closing the incision was necessary to model accurately an in vivo situation; leaving the artery exposed to air could result in a large artificial heat loss.

The thermocouples were interfaced to a µMAC A/D converter, and the measurements were recorded with an Apple Macintosh computer. The temperatures were recorded prior to, during, and after lasing while maintaining various blood flow rates. The normal average blood flow rate was unavoidably reduced to a value of 131 miler/min when the diffusing tip was positioned in the artery. The blood flow was further reduced to rates of 22, 9, and 0 miler/min by constriciting the artery. The light power level was maintained at 0.43 or 0.5 W at a wavelength of 632 nm (dye laser). The light distribution was approximately Gaussian (exp\([- [(x/a)^2]]) with measured \( a \) of 1.54 mm.

### IV. Results and Discussion

#### A. No Blood Flow

The experimental results for no blood flow in a small arterial lumen (radius = 0.75 mm) of one canine are given in Fig. 2. The radial temperature distribution is presented at times of 5, 10, and (steady state) 60 s. The temperatures were measured at the axial position of peak intensity to determine the maximum temperature of the normal tissue.

For comparison, predictions of the temperature profiles using the theories of Sec. II are also presented in Fig. 2. The Kubelka-Munk values for the absorption \( A_K \) and scattering \( S_K \) are taken from the literature. Values used are \( A_K = 4.5 \) cm\(^{-1} \), \( S_K = 9 \) cm\(^{-1} \) for blood, and \( A_K = 1.4 \) cm\(^{-1} \), \( S_K = 7.0 \) cm\(^{-1} \) for tissue. The absorption and scattering coefficients for Beer's law is related to the Kubelka-Munk coefficients by \( \alpha = \eta A_K \) and \( \beta = \chi S_K \), where \( \eta \) and \( \chi \) are given.
diffusing tip weighted by the Gaussian distribution. When scattering is present in the media, however, as it is in blood and tissue, the net effective incident intensity must be modified to account for the inwardly scattered light. To calculate the modified $I_0$ in Eqs. (3a) and (3b) we rely on the fact that the net power delivered $P$ must equal the total power absorbed in the blood and tissue:

\[
P = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \alpha_1 I(r,z) rdrdz + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \alpha_2 I(r,z) rdrdz,
\]

where $I(r,z)$ is from Eqs. (3a) and (3b). After performing the integration and solving for $I_0$, the complete expression for the incident intensity is given as

\[
I_0 = \frac{P}{2\pi\sigma_0 \left(\frac{a_1}{\mu_1} [1 - \exp(\mu_1 (r_0 - r))] + \frac{a_2}{\mu_2} \exp(\mu_2 (r_0 - r))\right)}.
\]

Note that in nonscattering media $\beta_1 = 0$ and $\beta_2 = 0$, so $\mu_1 = a_1$ and $\mu_2 = a_2$, and the above expression for incident intensity reduces to the expected $I_0 = P/\text{area}$, where area = $2\pi\sqrt{\pi\sigma_0}$, the surface area weighted by the Gaussian distributed light.

B. Kubelka-Munk Two-Flux Theory

In contrast to Beer’s law, the Kubelka-Munk model accounts for the fact that a fraction of the outward propagating light flux $F_+$ is scattered and immediately results in an inward propagating light flux $F_-$. This inward propagating light flux $F_-$ is further absorbed and scattered back into the outward flowing light flux $F_+$ (see Fig. 8). This results in a set of coupled equations.

The development of the Kubelka-Munk two-flux theory model for rectangular coordinates is presented by Ishimaru.20 The derivation for the same model in cylindrical coordinates is presented in the Appendix. The two-flux theory is considered adequate for describing the light distribution of a diffuse light source incident on a dull medium where light is diffusely scattered. Therefore, this model is directly applicable to the diffusing tip with a blood–tissue interface presented in Fig. 1. The Appendix yields two sets of outward and inward propagating flux equations for the two media shown in Fig. 9. These equations are:

\[
F_{1+}(r) = B \frac{\exp(\mu_0 r)}{\tau} + C \frac{\exp(-\mu_0 r)}{\tau},
\]

\[
F_{1-}(r) = B(A_{1+}^{[1]} \exp(\mu_0 r)/\tau + C(A_{1+}^{[1]} \exp(-\mu_0 r)/\tau)
\]

for medium 1 (blood), and

\[
F_{2+}(r) = D \frac{\exp(\beta_0 r)}{\tau} + E \frac{\exp(-\beta_0 r)}{\tau},
\]

\[
F_{2-}(r) = D(A_{2+}^{[2]} \exp(\beta_0 r)/\tau + E(A_{2+}^{[2]} \exp(-\beta_0 r)/\tau)
\]

for medium 2 (tissue).

The weighting coefficients $A_{1+}^{[1]}$ and $A_{2+}^{[2]}$, the absorption and scattering coefficients $\pm \mu_0$ and $\pm \beta_0$, and the optical distance $\tau = \rho_0 r$ are defined in the Appendix. The measured absorption and scattering coefficients of blood and tissue are available in the literature.15,18,20 They are obtained by first measuring the reflection and transmission of a finite thickness of the medium. Then the absorption and scattering coefficients are calculated using the Kubelka-Munk theory and are denoted $A_K$ and $S_K$, respectively.18 The coefficients $B, C, D,$ and $E$ are determined by the boundary conditions at the interfaces. The outer tissue radius extends to infinity relative to the depth of light penetration. This means the total flux in medium 2 must decay to zero as $r$ goes to infinity, so $D = 0$. At the tip–blood interface $r = r_0$ the experimentally measured input flux from the diffusing tip is $F_0$. With these equations and the known boundary conditions, the temperature contribution from the light source can be expressed as (for small $\Delta t$)

\[
\Delta T(r,z,t) = \frac{\Delta t}{\rho c} \exp[-(c/\sigma_0)\tau] [F_{1+}^{[1]}(r) + F_{1-}^{[1]}(r)]s(t) \quad r_0 \leq r \leq r_1,
\]

and

\[
\Delta T(r,z,t) = \frac{\Delta t}{\rho c} \exp[-(c/\sigma_0)\tau] [F_{2+}^{[2]}(r) + F_{2-}^{[2]}(r)]s(t) \quad r > r_1,
\]

where

\[
s(t) = \begin{cases} 1 & \text{if the laser is on,} \\ 0 & \text{if the laser is off,} \end{cases}
\]

and

\[
F_{1+}^{[1]}(r) = (\gamma A_{1+}^{[1]}) \text{ in medium 1 (blood), and} \\
F_{2+}^{[2]}(r) = (\gamma A_{2+}^{[2]}) \text{ in medium 2 (tissue),}
\]

where the values for $\gamma$ (in media 1 and 2) approach 1.0 for dominant absorption and 2.0 for dominant scattering.17 We have assumed that the Kubelka-Munk model does not alter the Gaussian distribution of the incident diffuse light.14

C. Time Dependent Heat Equation

Now that the light distribution has been described, the entire time dependent heat equation can be developed using Eq. (2). While lasing, some light is absorbed internal to the diffusing tip, so the tip temperature increases and conducts into the blood. The increase in temperature was determined experimentally and was found to increase exponentially to a final steady state temperature whose value depends on the presence of blood flow. This temperature is treated as a boundary condition at $r = r_0$ and enters into the conduction equations.

Once the light energy has propagated into the blood and tissue, transport mechanisms add or remove heat. Conduction is the rate of change in temperature attributed to adjacent blood and tissue. This results in a second-order differential equation in cylindrical coordinates for this model. The temperature of an element located at $r,z$ due to conduction from the adja-
by Klier as a function of the ratio of $A_K$ and $S_K$. The calculated values are $a_1 = 2.48 \text{ cm}^{-1}, \beta_1 = 13.5 \text{ cm}^{-1}$ for blood, and $a_2 = 0.94 \text{ cm}^{-1}, \beta_2 = 8.82 \text{ cm}^{-1}$ for tissue. The coefficient $\gamma$ in Eqs. (6a) and (6b) is given by Van Gemert and Star as a function of the ratio of $A_K$ and $S_K$ to be $\gamma = 1.82$ for blood and $\gamma = 1.9$ for tissue. The thermal properties of blood and tissue are assumed to be equivalent: conductivity of $\eta = 0.62 \text{ W m}^{-1} \text{ K}^{-1}$, density $\rho = 10^3 \text{ kg m}^{-3}$, and heat capacity $c = 3.5 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$.

Note in Fig. 2 that the peak predicted temperature occurs at the blood–tissue interface for this small arterial lumen. The estimated uncertainty in the location of the temperature probe is approximately ±0.25 mm, since the tip may have been eccentrically positioned in the lumen and actually closer to or farther from the tissue wall.

Irradiation profiles at $z = 0$ using the Kubelka-Munk model and the Beer’s law model with the modified incident intensity are presented in Fig. 3 for a power level of 0.43 W. The irradiation using Beer’s law without modifying the incident intensity to account for scattering is also plotted for comparison. In agreement with previously reported results, the unmodified Beer’s law theory is significantly lower than the Kubelka-Munk prediction. However, when the incident intensity in Beer’s law is modified to account for scattering as described earlier, the irradiation is higher than Kubelka-Munk as shown in Fig. 3.

The experimental temperature rise was slower than predicted theoretically. This could be the result of not accurately modeling heat sinks present in the tissue. For example, small capillaries branching from the artery carry blood from the lumen out into the tissue, dissipating heat from the blood into surrounding tissue. This is modeled in the perfusion equations; however, the values for the ratio of the volume of perfusing blood to the volume of tissue at a given rate $w_b$ available in the literature did not make a significant contribution to the theoretical temperature change. The actual experimental value of $w_b$ may have been higher than assumed in the model. Also, the literature assumed a uniform microstructure of capillaries. The canine capillaries in question were nonuniformly distributed and larger than indicated in the literature, suggesting a greater volume of blood flow perfusing into the tissue.
B. Varying Blood Flow

To determine the effect of varying blood velocity in the lumen, the average blood flow was maintained at 131, 22, 9, and 0 mliter/min in a larger artery (radius = 2.25 mm) of another canine. The experimental and theoretical Kubelka-Munk temperature distributions at a lasing time of 10 s and at a power of 0.5 W are presented in Fig. 4. The Kubelka-Munk model is used since it provides a more complete analysis of the irradiation of the highly scattering tissue than the modified Beer’s law model. The theoretical model agrees reasonably well with the measured values in the tissue. The temperatures in the tissue are lower than for the smaller artery [see Fig. 2(b)], since a greater percentage of the light is absorbed by the blood. Also, heated blood in the lumen flows away from the region. This limits the heating of the tissue via the conduction of the blood. Note in Fig. 4 that as the blood flow increases, the temperature across the lumen approaches a constant distribution equal to the initial body temperature of 40°C.

Fig. 4. Theoretical and experimental results for lasing 10 s intraluminally with blood flow of (a) 0, (b) 9, (c) 22, and (d) 131 mliter/min.

The peak temperature in the tissue from the data of Fig. 4 is replotted in Fig. 5 to illustrate the effect of blood flow on tissue cooling. A dramatic cooling effect is evident when only one-fourteenth (9 mliter/min) of the normal blood flow is present. As the blood flow is increased, minimal additional cooling occurs.

Fig. 5. Peak tissue temperature vs blood flow.
To explore the possibility of increasing the tissue temperature by reducing the absorption properties of the intervening medium, the theoretical temperature distribution for a water-filled lumen (negligible absorption and scattering at 632 nm) was calculated. Figure 6 presents the temperature distribution for the large artery \( r = 2.25 \text{ mm} \). When the blood is replaced with water, the peak tissue temperatures are 3°C higher on the average during no flow. A higher percentage of light is incident on the tissue wall, increasing its temperature. Note, however, that as the water is allowed to flow past the tissue surface, heat is carried away, dramatically cooling the tissue.

Figure 7 indicates that the cooling of the tissue as a result of flow in the lumen is more significant with the presence of water than with blood. In a blood-filled lumen, the heated blood is replaced by blood at the lower body temperature, which convectively cools the tissue wall. However, with parabolic flow a thin layer of blood moves slowly past the tissue surface increasing in temperature due to light absorption, thereby conductively heating the tissue. The cooling as a result of water flow is significant since the thin layer moving slowly past the tissue surface remains at approximately body temperature, convectively cooling the tissue.

The increase in tissue temperature in a water-filled lumen during no flow indicates an increase in light energy which may be desirable for enhancing photodynamic or photothermal therapy. Since photosensitizers (HPD and TPPS\(_4\)) have been shown to accumulate preferentially in atherosclerotic plaque vs normal tissue, photodynamic therapy may be useful in eliminating plaque with minimal normal tissue damage. With the increased light distribution in normal tissue for a water-filled lumen, the peak temperatures after 10 s are still within a safe range for 0.5-W input power (see Fig. 6), so normal tissue damage would be minimal during the therapy.

V. Conclusions

The mathematical model developed for predicting the temperature profile in arteries compares favorably to our in vivo experimental results. The temperature distributions predicted by the Kubelka-Munk two-flux theory and the modified Beer's law agree with the experimental data. This is a result of modeling the backscattering present in blood and tissue.

The blood has high absorption at 632 nm, so the peak temperatures occur in the blood-filled lumen for both the 1.5- and 4.5-mm diam arteries. The temperature rise time was faster theoretically than experimentally. This discrepancy in rise time may be a result of the perfusion, as discussed in Sec. IV, and the fact that the model parameters cited from the literature were measured in vitro and not in vivo.

The blood flow and light absorption by the blood both significantly reduce the peak temperature in the tissue. The water-filled lumen studies indicate that the temperature can be increased in the tissue during no flow by employing a water flush. This is equivalent to increasing the light incident on the tissue, which is beneficial in photodynamic and photothermal therapy. However, replacing the blood in the lumen with water and maintaining no flow are surgically difficult. Ideally, one would like to maintain the normal blood flow to minimize risk to the patient during surgery. If higher temperatures are desirable, they can be achieved by using a diffusing tip matched to the size of the lumen, and increasing power levels.

The mathematical model developed in this paper should be useful in predicting normal tissue damage during laser angioplasty. The model includes variable blood or water flow rates, tissue perfusion, lasing time, radial distances, Gaussian weighting, and absorption and scattering.

VI. Appendix

The following derivation of the Kubelka-Munk equations for cylindrical coordinates is based on the two-flux theory for rectangular coordinates presented by Ishimaru. The two-flux theory is considered adequate for describing the light distribution of a diffuse light source incident on a dull medium so that the light is diffusely scattered.
Consider diffuse fluxes \( F_+(r) \) and \( F_-(r) \) propagating in the outward and inward radial directions, respectively. The outward flowing flux \( F_+ \) decreases by the amount absorbed in a differential distance \( dr \) and by the scattering in the inward radial direction (see Fig. 8). This can be expressed by
\[
dF_+ = -(K + S)F_+dr = -(K + S)F_+dr, \tag{A1}
\]
where \( \rho_K \) = number of particles per unit volume,
\( \sigma_t \) = total cross section of a single particle,
\( K \sigma_t \) = equivalent absorption cross section for \( F_+ \),
\( K = \) nondimensional absorption coefficient, and
\( \tau = \rho_K \sigma_t r = \) the optical distance.

The values of \( K \) and \( S \) are not clearly related to the parameters of the particle, so they are difficult to calculate directly. As a result, these values are determined experimentally.

The outward flowing flux \( F_+ \) decreases according to Eq. (A1) and increases as a result of outward scattering of the inward scattered flux \( F_- \). This increase in the outward flux can be expressed as
\[
dF_+ = SF_-dr. \tag{A2}
\]

Accordingly, the inward flowing flux decreases by the amount that is absorbed and scattered by
\[
dF_- = -(K + S)F_-dr = (K + S)F_-dr \tag{A3}
\]
and increases by the amount of outward flux \( F_+ \) that is scattered inward by
\[
dF_- = SF_+dr. \tag{A4}
\]

Summing the respective increases and decreases in flux yields the two coupled differential equations
\[
\frac{dF_+}{d\tau} = -(K + S)F_+ + SF_-, \quad \frac{dF_-}{d\tau} = (K + S)F_- - SF_+. \tag{A5}
\]
To solve these coupled equations, the solution of the form \( \exp(\mu \tau) \) is assumed. Noting that
\[
\frac{d}{d\tau} \exp(\mu \tau) = \frac{d}{d\tau} \left( \mu - \frac{1}{\tau} \right),
\]
then
\[
F_+(\mu_+ - 1/\tau) = -(K + S)F_+ + SF_-, \\
F_-(\mu_- - 1/\tau) = (K + S)F_- - SF_. \tag{A6}
\]
These can be rewritten as
\[
F_+[\mu_+ - 1/\tau + (K + S)] - SF_+ = 0, \\
F_-[\mu_- - 1/\tau - (K + S)] + SF_- = 0.
\]
For a nontrivial solution of the fluxes, the determinant of the coefficients of \( F_+ \) and \( F_- \) must be zero. So,
\[
\begin{vmatrix}
\mu_+ - 1/\tau + (K + S) & -S \\
S & \mu_- - 1/\tau - (K + S)
\end{vmatrix} = 0.
\]
Solving for the values of \( \mu \),
\[
\mu_+ = 1/\tau \pm \sqrt{(K + 2S)} = \pm \mu_0.
\tag{A7}
\]
The ratios of \( F_+ \) and \( F_- \) can be expressed as
where \( \pm \mu_0 \) and \( [1] \) denote the inner interfacial media, and \( \pm \beta_0 \) and \( [2] \) denote the outer interfacial media.

Assuming that negligible reflection occurs at the boundaries (a good approximation for a blood–tissue interface), the following boundary conditions result:

1. \( F^0_{[r]}(r) = F_0^0 \)
2. \( F^0_{[r]}(r) = F_{[r]}^0 \)
3. \( F^0_{[r]}(r) = F^0_{[r]} \)
4. \( \frac{\gamma^0_{[r]}}{r} = 0 \).

Since the fluxes and optical distance \( \tau \) is a function of the medium, superscripts are used to indicate the respective media. The diffuse light input is experimentally known, with the power density equal to \( F_0 \). Applying these boundary conditions to the flux equations will yield expressions for the unknown coefficients. Assuming that the boundary \( r_0 \) goes to infinity, immediately it is known that \( D \) equals zero since the light has exponentially decay to zero. The intermediate equations are:

1. \( F_{[r]}^0 = B \exp(\mu_0 r_0^2) + C \exp(-\mu_0 r_0^2), \)
2. \( E \exp(-\beta_0 r_0^2) r_0^2 = r_0^2[B \exp(\mu_0 r_0^2)] \)
   \[ + C \exp(-\mu_0 r_0^2) \],
3. \( E \exp(-\beta_0 r_0^2) r_0^2 = r_0^2[B \exp(\mu_0 r_0^2)] \)
   \[ + C \exp(-\mu_0 r_0^2) \].

The simplest way to solve for the coefficients is to leave these equations in this form and evaluate them with the experimental values. The measured values of \( K \) and \( S \) automatically account for the factor \( \rho K \sigma_0 \), which is the particle volume density times the total cross section of a single particle, also referred to as the transport coefficient. Therefore, the measured values have units of inverse distance and are usually denoted as \( AK \) and \( SK \). Knowing this, the exponents in Eqs. (A10) can be written as

\[
\pm \mu_0 r = \pm \sqrt{AK} + \frac{2S}{3}
\]

with the same form for \( \pm \beta_0 r \). The coefficients can be solved readily from the above relations once the input power density \( AK \), \( SK \), and the transport coefficients \( \rho K \sigma_0 \) have been provided. For example, the \( \sqrt{AK} + \frac{2S}{3} \) = 1.006 l/mm in blood and 0.509 l/mm in tissue, so \( A^1 = 2.618, A^2 = 0.382, A^2 = 2.09 \), and \( A^2 = 0.478, 18, 23 \). The transport coefficient for oxygenated blood is equal to 0.747 l/mm at 660 nm. The transport coefficient for tissue is calculated using the relationships given by Van Gemert and Star. The transport coefficient is expressed as \( [\sigma_a + \sigma_t(1 - g)] \), where

\[
A_K = 2\sigma_a - \frac{\sigma_t^2}{\sigma_t(1 - g)}.
\]

Using these equations and noting that the anisotropy \( g \) of tissue is \( \sim 0.9 \), the transport coefficient of tissue is \( \sim 1.096 \) l/mm. Assuming a geometry with \( r_0 = 0.345 \) mm and \( r_1 = 0.75 \) mm, the coefficients are readily calculated using Eqs. (A10), yielding \( B = 0.0012F_0, C = 0.132F_0, \) and \( E = 0.139F_0 \). The complete set of equations describing the light distribution for the cylindrical geometry described with blood and tissue present is:

\[
F_{[r]}^0 = \frac{F_0}{r} \left[ 0.00161 \exp(1 + 1.006r) + 0.177 \exp(1 - 1.006r) \right],
\]

\[
F_{[r]}^0 = \frac{F_0}{r} \left[ 0.0042 \exp(1 + 1.006r) + 0.0675 \exp(1 - 1.006r) \right],
\]

\[
F_{[r]}^0 = \frac{F_0}{r} \left[ 0.126 \exp(1 - 0.509r) \right],
\]

\[
F_{[r]}^0 = \frac{F_0}{r} \left[ 0.0606 \exp(1 - 0.509r) \right].
\]

where the coefficients given have units of millimeters, so the units of flux are W/mm².

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References


NIST Report Summarizes Inventions Program

A portable pothole patcher, a new composite material made of high-strength fibers, a new process for continuous casting of steel cylinders, and a lightweight aluminum cylinder which makes it practical to use natural gas as a vehicle fuel are among the 400-plus inventions which have received support from the federal Energy-Related Inventions Program. The program, which began in 1975, is conducted jointly by NIST and the U.S. Department of Energy and aims at helping inventors get their ideas from the workshop to the marketplace. NIST provides, at no cost to the inventor, evaluations of energy-related inventions and recommends those it considers promising to DoE. In turn, DoE can provide financial support or help in marketing an inventor’s idea. A new report is available which describes the program as well as the inventions which have been recommended for DoE support. Energy Related Inventions Program: A Joint Program of the Department of Energy and the National Institute of Standards and Technology–Status Report (NISTIR 88-4005) can be ordered from the National Technical Information Service, Springfield, Va. 22161, for $36.95 prepaid. Order by PB #89-141154.