Successive Order Scattering Transport Approximation for Laser Light Propagation in Whole Blood Medium

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***Abstract*—An analytical solution method of the radiative transport equation, describing light scattering distribution in whole blood, is derived by applying successive order scattering approximation and transport approximation. By separating coherent components of scattered fluxes, the transport equation can be represented in terms of each order scattering flux, and the equations for each order scattering flux have a simplified integration term of scattering contribution that usually makes the solution complicated or even impossible. Also, actual phase function can be used for calculation of angular dependent scattering distribution that is approximated by the sum of the zeroth- and first-order Legendre polynomial in diffusion theory, or the sum of isotropic and coherent components in transport approximation. The method is then used to calculate reflectance from a half-space blood medium. It is found that first-order scattering flux alone produces a good agreement with experimental data and higher-order scattering fluxes are negligible in whole blood.**

***Index Terms*—Blood, light propagation and scattering, multiple scattering, radiative transport equation.**

I. Introduction

Light reflection has been a useful tool for *in vivo* or *in vitro* measurements of hemoglobin oxygen saturation or pH in whole blood [1]–[4]. The changes in absorption of light with changes in oxygen saturation or pH have been used for these measurements. More recently, light scattering distribution was utilized to detect the size of microemboli generated by direct blood contact with biomaterials [5] or in stored blood [6]. This method has been used for *ex vivo* or *in vitro* biomaterial testings.

Diffusion theory, as an approach to the solution of light propagation in whole blood, has been suggested and applied because three-dimensional (3-D) reflectance and transmittance calculations are readily obtained [7], [8]. Also, transport approximation has been applied for microemboli detection in whole blood, since it is particularly effective in a scattering medium whose angular distribution of scattered flux is strongly peaked in the forward direction as in whole blood [5], [9], [10]. For both theories, the phase function is too simplified even though they produce a good agreement with the experiment, especially for back scattering. Diffusion theory approximates intensity distribution and phase function as the sum of the zeroth- and first-order Legendre polynomial, while transport approximation considers the phase function as the sum of isotropic scattering and high forward scattering terms. These simplifications show good agreement with experimental data of backscattering of blood cells [7], [9] because the backward scattering distribution $(-1 < \mu < 0)$ is relatively simple. However, for forward scattering $(0 < \mu < 1)$, the high anisotropy of scattering distribution of erythrocyte cannot be easily modeled by these simple approximations.

There is a need for an analytical tool that can produce angular-dependent distribution of light in whole blood for the design of optical methods of microemboli characterization (size and nature of microemboli). In this paper, an analytical method, namely, the successive order scattering transport approximation that uses actual phase function of scatterers and is still easy to calculate, is derived and compared with experimental data in a half-space whole blood medium.

In a radiative transport equation that describes flux distribution in whole blood, the flux is decomposed into coherent and incoherent fluxes. The coherent flux is the sum of reduced incident flux and forwardly scattered flux, and the incoherent flux is the scattered flux into all space except the forward direction. This decomposition of fluxes enables the equation to be separated in terms of each order scattering and the separated equations become readily solvable.

II. Successive Order of Scattering Approximation

The source-free one-speed radiative transport equation for a plane geometry is given by [11]

$$\mu\frac{d\varphi(z,\mu)}{dz}+\Sigma_t\varphi(z,\mu) = \Sigma_s\int_0^{2\pi}d\phi'\int_{-1}^{+1}d\mu' f\left(\hat{\Omega}'\cdot\hat{\Omega}\right)\varphi(z,\mu') \qquad (1)$$

where the angular flux (radiation intensity) $\varphi(z,\mu)$ is a function of the coordinate (z) and the cosine of polar angle (μ), Σ_t is the total cross section, Σ_s is the scattering cross section, and $f(\hat{\Omega}'\cdot\hat{\Omega})$ is the scattering phase function where $\hat{\Omega}'$ is the incident direction and $\hat{\Omega}$ is the scattering direction represented in polar coordinate. Since the scatterers are assumed to take

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the shape of spheres, the azimuthal angle (ϕ) disappears from this equation on account of symmetry.

For a semi-infinite half space, the boundary condition for the flux incident at $z = 0$ is

$$\varphi(0,\mu) = F(2\pi\mu)^{-1}\delta(\mu - \mu_0), \quad \mu > 0 \tag{2}$$

where F is the flux magnitude of an incoming beam. A second condition is that the flux vanishes for large distance in the positive z direction.

The angular flux $\varphi(z,\mu)$ can be decomposed into two components: a reduced incident flux $\varphi_{\rm ri}$ that decreases due to scattering and absorption, and a diffused flux that is created within the medium due to scattering. If we decompose the diffused flux into a sum of successive orders of scattering fluxes, the angular flux can be written as

$$\begin{aligned}\varphi &= \varphi_{\rm ri} + \varphi_d \\ &= \varphi_{\rm ri} + (\varphi_1 + \varphi_2 + \varphi_3 + \cdots) \\ &= \varphi_{\rm ri} + \sum_{n=1}^{\infty} \varphi_n \end{aligned} \tag{3}$$

where the $\varphi_{\rm ri}$ is the reduced incident flux and $\varphi_1, \varphi_2, \cdots, \varphi_n$ are the first-, second-, and the nth-order scattering angular fluxes, respectively. After substituting (3) into (1), we have

$$\begin{aligned}&\mu\frac{d\left[\varphi_{\rm ri}(z,\mu) + \sum_{n=1}^{\infty}\varphi_n(z,\mu)\right]}{dz} \\ &\quad + \Sigma_t\left[\varphi_{\rm ri}(z,\mu) + \sum_{n=1}^{\infty}\varphi_n(z,\mu)\right] \\ &\quad = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' f(\hat{\Omega}'\cdot\hat{\Omega}) \\ &\quad\quad \times \left[\varphi_{\rm ri}(z,\mu') + \sum_{n=1}^{\infty}\varphi_n(z,\mu')\right].\end{aligned} \tag{4}$$

The expanded radiative transport equation can be separated according to the scattering order. In principle, the number of equations are infinite because of the infinite number of multiple scattering. But, in practice, higher-order scattering effects become increasingly small and can be truncated. This depends on the density of medium, wavelength of incident beam, and scattering characteristics of scatterers. Accordingly, the reduced incident flux that decreases due to scattering and absorption can be separated as

$$\mu\frac{d\varphi_{\rm ri}(z,\mu)}{dz} + \Sigma_t\varphi_{\rm ri}(z,\mu) = 0 \tag{5}$$

and the diffused flux created within the medium due to multiple scattering is satisfied by the equation

$$\begin{aligned}&\mu\frac{d\left[\sum_{n=1}^{\infty}\varphi_n(z,\mu)\right]}{dz} + \Sigma_t\sum_{n=1}^{\infty}\varphi_n(z,\mu) \\ &\quad = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' f\left(\hat{\Omega}'\cdot\hat{\Omega}\right) \\ &\quad\quad \times \left[\varphi_{\rm ri}(z,\mu') + \sum_{n=1}^{\infty}\varphi_n(z,\mu')\right].\end{aligned} \tag{6}$$

The first term on the right-hand side is considered as the first-order scattering and denotes the diffused flux created by scattering of the reduced incident flux. The equations for each successive order of scattering can be written as

$$\begin{aligned}&\mu\frac{d\varphi_1(z,\mu)}{dz} + \Sigma_t\varphi_1(z,\mu) \\ &\quad = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' f\left(\hat{\Omega}'\cdot\hat{\Omega}\right)\varphi_{\rm ri}(z,\mu')\end{aligned} \tag{7}$$

$$\begin{aligned}&\mu\frac{d\varphi_2(z,\mu)}{dz} + \Sigma_t\varphi_2(z,\mu) \\ &\quad = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' f\left(\hat{\Omega}'\cdot\hat{\Omega}\right)\varphi_1(z,\mu')\end{aligned} \tag{8}$$

$$\vdots$$

$$\begin{aligned}&\mu\frac{d\varphi_n(z,\mu)}{dz} + \Sigma_t\varphi_n(z,\mu) \\ &\quad = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' f\left(\hat{\Omega}'\cdot\hat{\Omega}\right)\varphi_{n-1}(z,\mu').\end{aligned} \tag{9}$$

Equation (5) is a simple homogeneous differential equation governing the reduced incident flux, and can be solved easily for known boundary conditions. This solution is then used as the source function for (7), governing first-order scattering. The solution of (7) is used for (8), governing second-order scattering, and so on.

For a scattering medium of low albedo, the above equations can be solved easily [12]. But, for a medium of high albedo, such as whole blood ($\sim$1), those equations cannot be solved easily and we have to apply other solution methods.

III. Successive Order Scattering Transport Approximation

For scatterers whose angular scattering distributions are strongly peaked in the forward direction, transport approximation is particularly effective and gives quite accurate results for a wide range of problems [12], [13]. In this section, by applying the transport approximation concept and the successive order scattering consideration, we will derive an analytical solution method, namely, the successive order scattering transport approximation, for erythrocytes in whole blood.

In (3) we decomposed the angular flux into a reduced incident flux and a diffused flux. The same angular flux, also, can be written in terms of a coherent flux and an incoherent flux as

$$\varphi = \varphi_{\rm ri} + \varphi_d = \varphi_c + \varphi_{\rm inc} \tag{10}$$

where the coherent flux φ_c is the sum of reduced incident flux and forwardly scattered flux, and the incoherent flux $\varphi_{\rm inc}$ is the scattered flux into the whole 4π space except the forward direction.

After substituting (10) into the radiative transport equation, (1), we have

$$\begin{aligned}&\mu\frac{d[\varphi_c(z,\mu) + \varphi_{\rm inc}(z,\mu)]}{dz} + \Sigma_t[\varphi_c(z,\mu) + \varphi_{\rm inc}(z,\mu)] \\ &\quad = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' f(\hat{\Omega}'\cdot\hat{\Omega})[\varphi_c(z,\mu') + \varphi_{\rm inc}(z,\mu')].\end{aligned} \tag{11}$$

The transport equation for the coherent flux can be separated as

$$\mu_0 \frac{d\varphi_c(z,\mu_0)}{dz} + \Sigma_t \varphi_c(z,\mu_0) = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' f\left(\hat{\Omega}' \cdot \hat{\Omega}_0\right) \varphi_c(z,\mu')\delta(\mu'-\mu_0) \tag{12}$$

where μ_0 is the cosine of the angle between an incident flux direction and the z-axis. The integrals on the right side represent the forwardly scattered flux in the incident direction by the coherent incident flux. The transport approximation for the phase function is given by [15]

$$f(\hat{\Omega}' \cdot \hat{\Omega}) = \bar{\mu}\delta\left(\hat{\Omega}' \cdot \hat{\Omega} - 1\right) + \frac{1}{4\pi}(1-\bar{\mu}) \tag{13}$$

where $\bar{\mu}$ is the average scattering angle cosine whose value is very close to one for erythrocytes. Inserting (13) into (12), we have

$$\begin{aligned} \mu_0 \frac{d\varphi_c(z,\mu_0)}{dz} &+ \Sigma_t \varphi_c(z,\mu_0) \\ &= \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' \left[\bar{\mu} + \frac{1}{4\pi}(1-\bar{\mu})\right] \\ &\quad \times \varphi_c(z,\mu')\delta(\mu'-\mu_0) \\ &= \bar{\mu}\Sigma_s \varphi_c(z,\mu_0) + \frac{\Sigma_s}{4\pi}(1-\bar{\mu})\varphi_c(z,\mu_0) \\ &\approx \bar{\mu}\Sigma_s \varphi_c(z,\mu_0). \end{aligned} \tag{14}$$

Since $\mu_0 = 1$ for the coherent forward direction, the above equation can be written as

$$\frac{d\varphi_c(z,\mu_0)}{dz} + \Sigma_t \varphi_c(z,\mu_0) = \bar{\mu}\Sigma_s \varphi_c(z,\mu_0). \tag{15}$$

Using transport cross section $\Sigma_{\rm tr} = \Sigma_t - \bar{\mu}\Sigma_s$, the above equation becomes

$$\frac{d\varphi_c(z,\mu_0)}{dz} + \Sigma_{\rm tr}\varphi_c(z,\mu_0) = 0 \tag{16}$$

and solution of this equation is obtained as

$$\varphi_c(z,\mu_0) = F_0 e^{-\Sigma_{\rm tr} z} \tag{17}$$

where F_0 is the incident angular flux at $z = 0$ in the forward direction.

The transport equation for the incoherent flux can be obtained from (11) as

$$\begin{aligned} \mu \frac{d\varphi_{\rm inc}(z,\mu)}{dz} &+ \Sigma_t \varphi_{\rm inc}(z,\mu) \\ &= \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' [f\left(\hat{\Omega}' \cdot \hat{\Omega}\right)\varphi_{\rm inc}(z,\mu') \\ &\quad + f\left(\hat{\Omega}_0 \cdot \hat{\Omega}\right)\varphi_c(z,\mu')\delta(\mu'-\mu_0)] \end{aligned} \tag{18}$$

for $\hat{\Omega} \neq \hat{\Omega}_0$. Even though the second term within integral concerns the coherent flux, it represents the incoherent flux generated by scattering of the coherent incident flux. Note the forwardly scattered flux by the coherent incident flux is already included in (12).

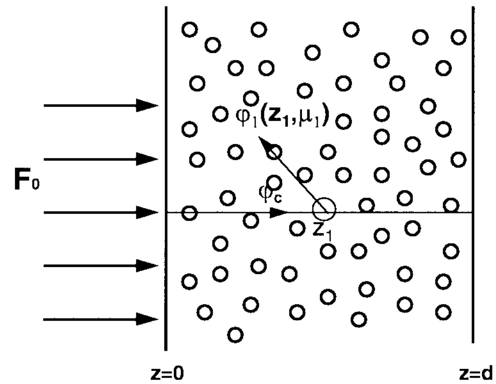

Fig. 1. Definition of first-order scattering in successive order scattering transport approximation.

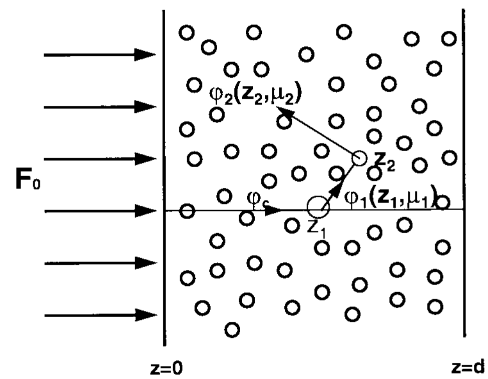

Fig. 2. Definition of second-order scattering in successive order scattering transport approximation.

For separation of the incoherent flux associated with each order of scattering, let us consider the scattering geometry shown in Fig. 1. The coherent component, the sum of reduced incident flux and forwardly scattered flux, arriving at $z1$, is scattered by the differential volume of particles located at $z1$. We define this type of scattering in any direction except the forward direction by a coherent incident flux as the first-order scattering in the successive order scattering transport approximation. Even though the coherent component arriving at $z1$ has experienced scatterings along its path, we do not define them as scattering in this approximation. We define the second-order scattering as shown in Fig. 2. The first-order scattering flux at $z1$, scattered portion of the coherent flux, is now propagating in an arbitrary direction $\mu 1$ and finally arrives at $z2$. This arriving first-order scattering flux at $z2$, attenuated by the particles located along its path, is scattered again by the differential volume of particles at $z2$. This is defined as the second-order scattering, and so on for higher-order scatterings. By applying these definitions of scattering, the incoherent flux can be written as the sum of each order scattering such that

$$\varphi_{\rm inc} = \varphi_{\rm inc1} + \varphi_{\rm inc2} + \varphi_{\rm inc3} + \cdots = \sum_{m=1}^{\infty} \varphi_{\rm incm}. \tag{19}$$

By substituting (19) into (18), we have

$$\mu \frac{d \sum_{m=1}^{\infty} \varphi_{\text{incm}}(z,\mu)}{dz} + \Sigma_t \sum_{m=1}^{\infty} \varphi_{\text{incm}}(z,\mu) = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' \left[f\left(\hat{\Omega}' \cdot \hat{\Omega}\right) \sum_{m=1}^{\infty} \varphi_{\text{incm}}(z,\mu') + f\left(\hat{\Omega}_0 \cdot \hat{\Omega}\right) \varphi_c(z,\mu') \delta(\mu' - \mu_0) \right]. \tag{20}$$

In order to represent this equation in terms of transport cross section, we can subtract from both sides the component $\bar{\mu}\Sigma_s \sum_{m=1}^{\infty} \varphi_{\text{incm}}(z,\mu)$. Then, we have

$$\mu \frac{d \sum_{m=1}^{\infty} \varphi_{\text{incm}}(z,\mu)}{dz} + (\Sigma_t - \bar{\mu}\Sigma_s) \sum_{m=1}^{\infty} \varphi_{\text{incm}}(z,\mu) = \Sigma_s \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu' \left\{ \left[f\left(\hat{\Omega}' \cdot \hat{\Omega}\right) - \bar{\mu}\delta\left(\hat{\Omega}' \cdot \hat{\Omega}\right) \right] \times \sum_{m=1}^{\infty} \varphi_{\text{incm}}(z,\mu') + f\left(\hat{\Omega}_0 \cdot \hat{\Omega}\right) \right\} \times \varphi_c(z,\mu') \delta(\mu' - \mu_0). \tag{21}$$

By substituting the transport cross section $\Sigma_{\text{tr}} = \Sigma_t - \bar{\mu}\Sigma_s$ and separating fluxes associated with each scattering order, the equations of each successive order scattering can be written as

$$\mu_1 \frac{d\varphi_1(z,\mu_1)}{dz} + \Sigma_{\text{tr}} \varphi_1(z,\mu_1) = \Sigma_s f\left(\hat{\Omega}_0 \cdot \hat{\Omega}_1\right) \varphi_c(z,\mu_0) \tag{22}$$

$$\mu_2 \frac{d\varphi_2(z,\mu_2)}{dz} + \Sigma_{\text{tr}} \varphi_2(z,\mu_2) = \Sigma_s \int_0^{2\pi} d\phi_1 \int_{-1}^{+1} d\mu_1 f\left(\hat{\Omega}_1 \cdot \hat{\Omega}_2\right) \varphi_1(z,\mu_1) \tag{23}$$

$$\vdots$$

$$\mu_n \frac{d\varphi_n(z,\mu_n)}{dz} + \Sigma_{\text{tr}} \varphi_n(z,\mu_n) = \Sigma_s \int_0^{2\pi} d\phi_{n-1} \int_{-1}^{+1} d\mu_{n-1} \times f\left(\hat{\Omega}_{n-1} \cdot \hat{\Omega}_n\right) \varphi_{n-1}(z,\mu_{n-1}) \tag{24}$$

where $\mu1$, $\mu2$, and μn represent arbitrary directions of the first, second, and the nth-order scattering, respectively. The delta function in the bracket of (21) has disappeared since there is no scattering in the forward direction in the successive order scattering transport approximation (the coherent component is already considered in the previous scattering order). Note that the right-hand sides of the higher-order equations [(23) and (24)] require integration about the entire space because lower-order scattered flux arrives from every direction. The above equations can be solved successively. The solution of a lower-order scattering equation is applied to solve the next order scattering equation. The actual integrals for second- or higher-order scatterings depend on the geometry of scattering medium and the diameter of incident flux on the surface of the slab.

For the first-order scattered flux in the direction between $-1 < \mu 1 < 0$ at the boundary $(z = 0)$ of the slab medium (Fig. 1), (22) is solved as

$$\varphi_1(0,\mu_1) = \int_0^d \Sigma_s f\left(\hat{\Omega}_0 \cdot \hat{\Omega}_1\right) \varphi_c(z,\mu_0) e^{-\Sigma_{\text{tr}} \frac{z}{\mu_1}} \frac{dz}{\mu_1} = \frac{\Sigma_s}{\Sigma_{\text{tr}}} F_0 f\left(\hat{\Omega}_0 \cdot \hat{\Omega}_1\right) \frac{1}{1 + |\mu_1|} \left[1 - e^{-\Sigma_{\text{tr}} \left(1 + \frac{1}{|\mu_1|}\right) d} \right]. \tag{25}$$

For a semi-infinite space, the d should approach infinity. Solution of the second-order scattering equation given in (23) is obtained as

$$\varphi_2(0,\mu_2) = \int_0^d \left[\int_0^{2\pi} d\phi_1 \int_{-1}^{+1} d\mu_1 \Sigma_s f\left(\hat{\Omega}_1 \cdot \hat{\Omega}_2\right) \varphi_1(z,\mu_1) \right] \times e^{-\Sigma_{\text{tr}} \frac{z}{\mu_2}} \frac{dz}{\mu_2}. \tag{26}$$

Likewise, the n-th-order scattering is obtained as

$$\varphi_n(0,\mu_n) = \int_0^d \left[\int_0^{2\pi} d\phi_{n-1} \int_{-1}^{+1} d\mu_{n-1} \Sigma_s \times f\left(\hat{\Omega}_{n-1} \cdot \hat{\Omega}_n\right) \varphi_{n-1}(z,\mu_{n-1}) \right] e^{-\Sigma_{\text{tr}} \frac{z}{\mu_n}} \frac{dz}{\mu_n}. \tag{27}$$

The total angular flux at z in the direction μ can be written as

$$\varphi(z,\mu) = \varphi_c(z,\mu)\delta(\mu \cdot \mu_0) + \varphi_1(z,\mu) + \varphi_2(z,\mu) + \varphi_3(z,\mu) + \cdots + \varphi_n(z,\mu). \tag{28}$$

The accuracy of this method will depend upon how many higher-order scattering fluxes are considered. However, we will see that the first-order scattering approximation is sufficient for a medium of highly anisotropic scatterers such as whole blood in Section IV.

IV. First-Order Scattering Transport Approximation in Light Propagation in Whole Blood

In order to validate the successive order scattering transport approximation derived in Section III, we apply the approximation method to the half space of whole blood medium shown in Fig. 3. Two fibers are positioned perpendicularly on the surface of the half-space medium with a distance of separation r. The diameters of source and detector fibers are 0.508 mm and 0.127 mm, respectively, and the numerical aperture of the detector fiber is 0.689. The first- and second-order scattering transport approximations were calculated by using the phase function obtained from Mie's scattering theory for erythrocytes. The parameters used were $\bar{\mu} = 0.9954$, $\Sigma_t = 146.25$, and $\Sigma_s = 145.99$ by assuming 100% oxygen saturated blood, 41% hematocrit, and a wavelength $\lambda = 0.685$. These parameters were obtained from Mie scattering theory by considering erythrocyte as a sphere whose equivalent diameter is 5.58 μm and volume 90.97 μm^3. The index

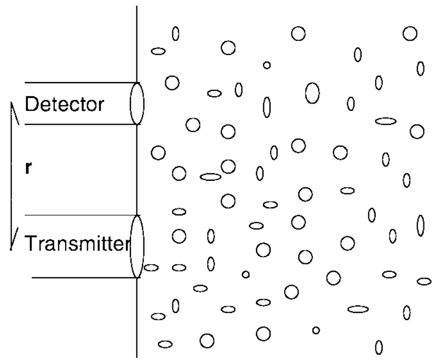

Fig. 3. Geometry of two fibers facing a half-space blood medium perpendicularly.

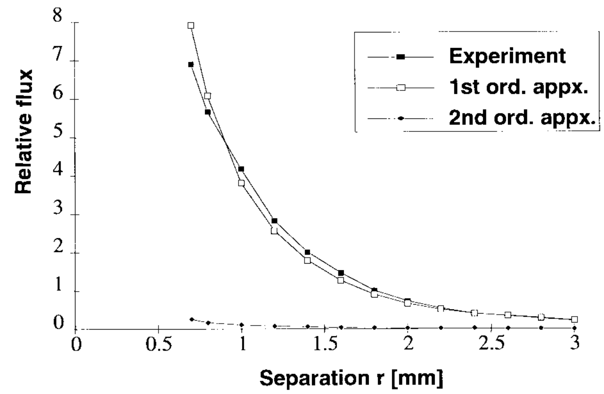

Fig. 4. Comparison of experimental data, first-order scattering transport approximation, and second-order scattering transport approximation for reflected flux from whole blood versus radial separation distance.

of refraction for erythrocyte is $1.034 + i5(10^{-5})$ and the surrounding medium of erythrocytes is assumed to be water (1.33). For the calculation, we selected 3-D sampling points whose interdistances along each axis are 0.0635 mm in the medium shown in Fig. 3 and changed the separation distance r from 0.7 mm to 3 mm. The sampling point distance has been chosen because the application of diffusion theory [7] to the geometry of the sampling points produced a good agreement with experimental data. For the first- and second-order scattering fluxes, (22) and (23) were solved for each sampling points, respectively. Each sampling point can be denoted by a delta function and the delta function simplifies the calculations. Contribution of scattered fluxes from every sampling point was summed, and the total fluxes for each separation distance were normalized and compared to the measured data as shown in Fig. 4. The result shows that the second-order scattering contribution is negligible (less than 3%) compared to the first-order scattering. Even though only first-order transport scattering is considered, thereby neglecting all higher-order transport scatterings, the result is very acceptable. If we recall the fact that erythrocyte scatters energy mostly in the forward direction, it is understandable that only first-order scattering transport approximation is sufficient for the whole blood medium. It is also possible to calculate the higher-order transport scattering fluxes if scattering geometry is well defined. If the anisotropy of scatterers decreases, the contribution of higher-order scattering becomes significant and must be considered. Also, scattering geometry, diameter of incident beam, reception characteristic of detectors, and scattering particle density should be considered for determination of proper scattering orders.

The main advantage of this successive order scattering transport approximation compared to diffusion approximation or transport approximation is that the actual phase function of scatterers can be used and, therefore, angular-dependent scattered flux distribution can be obtained. For transport approximation, the phase function of scatterers is approximated as the sum of a small isotropic component and a large, forwardly directed, component. That means the scattered flux in all directions except the forward direction is considered as constant. The diffusion approximation uses only the first and second terms of Legendre expansion of fluxes and cannot represent the flux distribution at small angles in detail. Nevertheless, the two approximation methods are in good agreement with experimental data because the detector has a finite reception area. The fluxes within the reception angle will be averaged and considered as constant.

On the other hand, the successive order scattering transport approximation allows us to calculate the angular-dependent flux distribution in any direction in any scattering geometries, easily and accurately. The scattering flux can be obtained simply by considering the scattering characteristic of scatterers, located at a sampling point in a scattering medium, in the direction of a detector and the distance between them.

V. Conclusion

A successive order scattering transport approximation is derived and compared with experimental data. By separating coherent components of scattered fluxes, the transport equation can be represented in terms of each order scattering flux, and the equations for each order scattering flux have a simplified integration term of scattering contribution. Also, actual phase function of scatterers can be used to describe the flux distribution in a multiple scattering medium.

It is found that the successive order scattering transport approximation can be calculated easily and can produce an angular dependent flux in a specific direction in any scattering geometries. The method is very effective for highly anisotropic scatterers, because only lower-order scatterings need to be considered. For whole blood scattering media, the first-order scattering approximation alone can produce a good agreement with experiment.

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