

Wave Propagation and Scattering in Random Media and Rough Surfaces

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Invited Paper

Many natural and man-made media such as the atmosphere, oceans, geophysical media, biological media, and composite and disordered materials have random spatial inhomogeneities and vary randomly in time, and these media are called "random media." Microwaves, optical waves, and acoustic waves propagating in these media experience random fluctuations in space and time, and these fluctuations affect a broad range of practical problems such as communications, remote-sensing, imaging, and object identification. In addition, waves in random media present one of the most challenging problems to theoreticians. This paper reviews the historical development, basic theories, and some recent new developments and discoveries, including backscattering enhancement.

I. INTRODUCTION

Many natural and man-made media vary randomly in time and space, and therefore, electromagnetic, optical, or acoustic waves propagating in these media fluctuate also in time and space. Examples are microwave, optical and acoustic wave scattering by ocean waves, atmospheric turbulence, rain, fog, snow, terrain, vegetations, biological media, and composite materials. These fluctuations affect communications through these media, and the identification and remote-sensing of these media and of objects inside the random media.

In recent years, there has been a surging interest in wave propagation and scattering in random media. This is primarily due to the discovery of new multiple scattering phenomena and an increasing awareness that a common thread underlies the work of many researchers in such diverse fields as atmospheric optics, ocean acoustics, radio physics, astrophysics, condensed matter physics, plasma physics, geophysics, bioengineering, etc. In addition, waves

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in random media are one of the most challenging problems to theoreticians. Thus the field of wave propagation and scattering encompasses the most practical as well as the most theoretical questions. This paper reviews some of the historical development of the field, basic theories, and applications including the most recent developments and discoveries in this field. The strong interest in this subject is reflected in the launch of a new journal, *Waves in Random Media*, by the Institute of Physics, United Kingdom in 1991; international workshops on wave propagation in random media held in Tallin, USSR in 1988, on surface and volume scattering held in Madrid in 1988, and on modern analysis of scattering phenomena held in Aix en Provence, France in 1990; and an international meeting for wave propagation in random media to be held in Seattle, WA in 1992.

This paper is divided into four sections. General reviews and detailed expositions are given in several books [1]–[15]. Historically, early work on scattering by turbulence includes troposcatter and the Booker-Gordon formula in late 1940, ionospheric scintillations, optical fluctuation in the atmosphere, and thin screen theories and Dyson and Bethe-Salpeter equations for astrophysical applications. Early multiple scattering theories for randomly distributed particles were developed by Lax, Foldy, Twersky, and others. The relationships among multiple scattering theory, radiative transfer theory, and neutron transport theory have been investigated. The relationships with partially coherent waves and Wigner distributions are also noted. Rough surface theories date back to Rayleigh and Rice.

In Section II, we discuss wave propagation in turbulence and random continuum where the refractive index is a random function of space and time. Examples are optical propagation in the atmosphere, microwaves in the troposphere, ionosphere, planetary atmosphere, and solar wind, and acoustic scattering in the ocean turbulence.

Section III is devoted to the multiple scattering by random distributions of discrete scatterers. Examples are optical and microwave scattering by rain, fog, smog, snow,

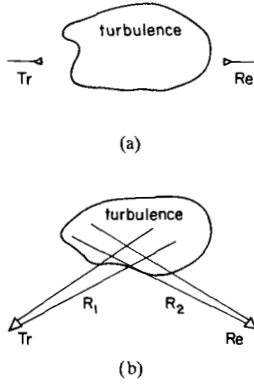


Fig. 1. Scattering by turbulence and random continuum. (a) Line-of-sight. (b) Scattering.

ice particles, and vegetation, optical and ultrasound scattering by tissues and blood, optical and acoustic scattering in the ocean, and scattering in composite materials.

In Section IV, we discuss scattering by rough surfaces and interfaces. Example are acoustic scattering by ocean surfaces, microwave and optical scattering by vegetation, terrain, and snow cover, and ultrasound scattering by rough interfaces in biological media.

In Section V, we discuss recent research on coherent backscattering enhancement phenomena.

II. WAVE PROPAGATION IN TURBULENCE AND RANDOM CONTINUUM

Consider an optical beam propagating through turbulent air. The refractive index $n(\bar{r}, t)$ of the air varies randomly in space and time, and therefore, the amplitude and the phase of the wave also vary randomly in space and time. Let us first consider the line-of-sight propagation (Fig. 1(a)). Suppose that a time-harmonic field with $\exp(-i\omega t)$ is incident on the medium. If we take a component E_x of the field vector \bar{E} , the scalar field $u(\bar{r}, t) = E_x$ is a random function of position \bar{r} and time t . We write u as follows:

$$u(\bar{r}, t) = \text{Re}[U(\bar{r}, t) \exp(-i\omega t)] \quad (1)$$

where $U(\bar{r}, t) = A(\bar{r}, t) \exp[i\phi(\bar{r}, t)]$ is called the complex envelope and A and ϕ are the random functions. Now we write U as the sum of the coherent field $\langle U \rangle$ and the fluctuating field U_f .

$$U(\bar{r}, t) = \langle U(\bar{r}, t) \rangle + U_f(\bar{r}, t) \quad (2)$$

where $\langle \rangle$ denotes the ensemble average. In theoretical work, we normally consider the ensemble average, but in practice, this can often be approximated by its spatial or time average.

The field U , neglecting the cross polarization, satisfies the scalar wave equation

$$[\Delta^2 + k^2(1 + \epsilon_1)]U = 0 \quad (3)$$

where the dielectric constant $\epsilon(\bar{r}, t) = n^2(\bar{r}, t)$ is a random function and

$$\epsilon = \langle \epsilon \rangle (1 + \epsilon_1)$$

$$k^2 = \frac{\omega^2}{c^2} \langle \epsilon \rangle. \quad (4)$$

ϵ_1 is the fluctuation and is assumed to be small.

Let us first consider the coherent field $\langle U \rangle$. From (3), we get

$$\nabla^2 \langle U \rangle + k^2 \langle U \rangle + k^2 \langle \epsilon_1 U \rangle = 0. \quad (5)$$

Note that ϵ_1 and U are correlated. The effective propagation constant K for the coherent field is defined by

$$(\nabla^2 + K^2) \langle U \rangle = 0. \quad (6)$$

An approximate expression for K is given by [16]

$$K^2 = k^2 \epsilon_e = k^2 + k^3 \int_0^\infty e^{ikr} \sin kr \langle \epsilon_1 \epsilon_2 \rangle dr \quad (7)$$

where $\langle \epsilon_1 \epsilon_2 \rangle = \langle \epsilon_1(\bar{r}_1) \epsilon_1(\bar{r}_2) \rangle$ is the correlation function of ϵ_1 and is a function of the separation distance $r = |\bar{r}_1 - \bar{r}_2|$. The imaginary part of K which represents the attenuation is given by

$$K_i = \frac{\pi^2 k^2}{2} \int_0^{2k} \Phi_\epsilon(k_s) k_s dk_s \quad (8)$$

where Φ_ϵ is the spectral density of the fluctuation ϵ_1 .

$$\Phi_\epsilon(k_s) = \frac{1}{(2\pi)^3} \int \langle \epsilon_1 \epsilon_2 \rangle e^{-j\bar{K}_s \cdot \bar{r}} dV. \quad (9)$$

In (8), $2K_i$ is equal to the scattering cross section per unit volume of the random medium, representing the power scattered by the randomness. Note that ϵ_e in (7) is the effective dielectric constant of the random medium and is, in general, complex even in a lossless medium. The imaginary part represents the attenuation due to scattering.

We next consider the propagation of the second moment $\Gamma(\bar{r}_1, \bar{r}_2)$, called the "mutual coherence function." Here since the wave scattering is confined within a narrow forward angular region, we make use of the parabolic equation and obtain the differential equation for Γ . In general, the higher order moment Γ_{nm} satisfies the following differential equation:

$$\left[2ik \frac{\partial}{\partial x} + (\Delta_1 + \dots + \Delta_n - \Delta'_1 - \dots - \Delta'_m) + \frac{ik^3}{4} F_{nm} \right] \Gamma_{nm} = 0 \quad (10)$$

where Δ_n and Δ'_m are the transverse Laplacian with respect to $\bar{\rho}_n$ and $\bar{\rho}'_m$, respectively and F_{nm} and Γ_{nm} are given by [2]

$$\begin{aligned} F_{nm} &= \sum_{i=1}^n \sum_{j=1}^n A(\bar{\rho}_i - \bar{\rho}_j) - 2 \sum_{i=1}^n \sum_{k=1}^m A(\bar{\rho}_i - \bar{\rho}'_k) \\ &\quad + \sum_{k=1}^m \sum_{l=1}^m A(\bar{\rho}'_k - \bar{\rho}'_l) \\ \Gamma_{nm} &= \langle U(x, \bar{\rho}_1) \dots U(x, \bar{\rho}_n) U^*(x, \bar{\rho}'_1) \dots U^*(x, \bar{\rho}'_m) \rangle \\ A(\bar{\rho}) &= 2\pi \int \Phi_\epsilon(\bar{K}) e^{i\bar{K} \cdot \bar{\rho}} d\bar{K} \end{aligned} \quad (11)$$

The moment (10) is the fundamental equation for all higher moments, but its exact analytical solution is available only for the second moment. The intensity fluctuation I_f is given by $I - \langle I \rangle$ where $I = UU^*$ and the normalized variance σ_I^2 of the intensity fluctuation is called the "scintillation index."

$$\sigma_I^2 = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}. \quad (12)$$

For a weak fluctuation region where $\sigma_I^2 < 0.4$, the scintillation index increases with frequency and distance. However, in the strong fluctuation region σ_I^2 saturates to approximately unity [2].

Formal perturbation methods based on Feynman diagrams have been applied to this problem and the fundamental equation for the coherent field $\langle G \rangle$ is called the "Dyson equation" [8].

$$\begin{aligned} \langle G(\bar{r}, \bar{r}') \rangle &= G_o(\bar{r}, \bar{r}') \\ &+ \iint G_o(\bar{r}, \bar{r}_1) M(\bar{r}_1, \bar{r}_2) \langle G(\bar{r}_2, \bar{r}') \rangle d\bar{r}_1 d\bar{r}_2 \end{aligned} \quad (13)$$

where G_o is the free space Green's function and M is the mass operator.

The mass operator can be expressed by the following "Bourret" approximation:

$$M = k^4 G_o(\bar{r}_1, \bar{r}_2) \langle \epsilon_1(\bar{r}_1) \epsilon_1(\bar{r}_2) \rangle. \quad (14)$$

The Dyson equation with (14) is called the first-order smoothing approximation. An improved equation can be obtained if the following "nonlinear" approximation is used:

$$M = k^4 \langle G(\bar{r}_1, \bar{r}_2) \rangle \langle \epsilon_1(\bar{r}_1) \epsilon_1(\bar{r}_2) \rangle. \quad (15)$$

Similarly, the second-order moment $\langle G(\bar{r}_1) G^*(\bar{r}_2) \rangle$ satisfies the "Bethe-Salpeter" equation with an intensity operator [8].

Solutions to the moment (10) have been obtained for the fourth moment [64] using functional integral [25] and path-integral techniques [5], [17], [35], and numerical simulation [18], [19]. The question of how the variation of the background profile affects the intensity fluctuation is also studied [20]. The correlation between the forward and backward waves contributes to the backscattering enhancement [21], [22], [23]. The scalar wave equation needs to be extended to include the cross polarization effects [3]. For weak fluctuation, the probability density function (PDF) is log normal. However, for strong fluctuation, the I-K distribution is shown to be applicable [26], [27].

Let us next consider scattering by turbulent air (Fig. 1(b)). We start with the radar equation for the case shown in Fig. 1(b). The received power P_r is given by

$$P_r = P_t \frac{\lambda^2}{(4\pi)^3} \int_{V_c} \frac{G_t G_r}{R_1^2 R_2^2} \sigma_{bi} e^{-\tau_1 - \tau_2} dV \quad (16)$$

where σ_{bi} is the bistatic scattering cross section per unit volume of the turbulence and is given by the spectral density Φ_ϵ of the turbulence, and τ is the optical distance:

$$\begin{aligned} \sigma_{bi} &= 2\pi^2 k^4 \Phi_\epsilon(k_s) \\ k_s &= 2k \sin\left(\frac{\theta}{2}\right) \\ \tau_1 &= \int_0^{R_1} 2K_i dR. \end{aligned}$$

K_i is given in (8). Note that the transmitted and the scattered waves attenuate through the turbulence with the attenuation constant of the coherent wave, and this process is called the first-order scattering or the distorted Born approximation. The cross section σ_{bi} is proportional to the spectral density at the turbulent eddy size l , $k_s = 2\pi/l = 2k \sin(\theta/2)$, representing Bragg scattering. This simple expression (16) must be extended to include multiple scattering to explain backscattering enhancement [24]. Even though most of the studies are directed to wave propagation in turbulence, similar techniques should be applicable to ultrasound scattering by tissues. As will be shown in the next section, there is a close similarity between scattering by continuum and scattering by discrete scatterers.

III. TRANSPORT THEORY AND SCATTERING BY DISCRETE SCATTERERS

Let us consider propagation and scattering of microwaves through rain, fog, snow, ice, or vegetation; optical and acoustic scattering and diffusion in tissues and whole blood; and optical scattering by vegetation.

In a random distribution of discrete scatterers, waves are scattered and absorbed due to the inhomogeneities and absorption characteristics of the medium. A mathematical description of the propagation and scattering characteristics of waves can be made in two different manners: Analytical theory and transport theory. In analytical theory [1], we start with Maxwell's equation, take into account the statistical nature of the medium, and consider the statistical moments of the wave. In principle, this is the most fundamental approach, including all diffraction effects, and many investigations have been made using this approach. However, its drawback is the mathematical complexities involved and its limited usefulness.

Transport theory [1], on the other hand, does not start with Maxwell's equations. It deals directly with the transport of power through turbid media. The development of the theory is heuristic and lacks the rigor of the analytical theory. Since both the analytical and transport theories deal with the same physical problem, there should be some relation between them. In fact, many attempts have been made to derive the transport theory from Maxwell's equations with varying degrees of success [6], [35]. In spite of its heuristic development, however, the transport theory has been used extensively, and experimental evidence shows that the transport theory is applicable to a large number of practical problems.

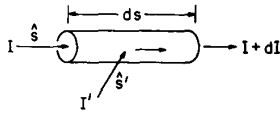


Fig. 2. Radiative transfer equation.

The fundamental quantity in transport theory is the radiance $\bar{I}(\bar{r}, \hat{s})$, which is also called the specific intensity in radiative transfer theory and the brightness in radiometry. Its unit is $\text{Watt m}^{-2} \text{sr}^{-1} \text{Hz}^{-1}$ and is the average power flux density in a given direction \hat{s} within a unit solid angle within a unit frequency band.

The fundamental differential equation for the radiance is the transport equation [1], [30].

$$\frac{d}{ds} I(\bar{r}, \hat{s}) = -\gamma_t I(\bar{r}, \hat{s}) + \frac{\gamma_t}{4\pi} \int p(\hat{s}, \hat{s}') I(\bar{r}, \hat{s}') d\omega' \quad (17)$$

where $\gamma_t = \gamma_s + \gamma_a$ is the extinction coefficient in m^{-1} , γ_s is the scattering coefficient, γ_a is the absorption coefficient. $p(\hat{s}, \hat{s}')$ is the phase function, and $d\omega'$ is the elementary solid angle about the direction \hat{s}' [1] (Fig. 2).

The quantity most useful is the energy density that is also called the "radiant energy fluence rate" and is denoted by $\psi(\bar{r})$. It is the sum of the radiance over all angles at a point \bar{r} and is measured in $\text{W m}^{-2} \text{Hz}^{-1}$.

$$\psi(\bar{r}) = \int_{4\pi} I(\bar{r}, \hat{s}) d\omega. \quad (18)$$

The flow of the power per unit area is represented by the flux $\bar{F}(\bar{r})$ defined by

$$\bar{F}(\bar{r}) = \int_{4\pi} I(\bar{r}, \hat{s}) \hat{s} d\omega \quad (19)$$

and is measured in $\text{W m}^{-2} \text{Hz}^{-1}$. Note that ψ and \bar{F} have the same dimension, but ψ is a scalar while \bar{F} is a vector.

From the transport equation (17), we can derive the following continuity equation

$$\nabla \cdot \bar{F} + \gamma_a \psi = 0 \quad (20)$$

where γ_a is the absorption coefficient (m^{-1}). Note that $\gamma_a \psi$ $\text{W m}^{-3} \text{Hz}^{-1}$ is the amount of power absorbed by a unit volume of the medium and (20) represents the total power flux $-\nabla \cdot \bar{F}$ entering a unit volume and is equal to the power absorbed within this volume.

Solutions of the radiative transfer equation have been obtained for a plane-parallel medium, first-order scattering, and the diffusion approximation [1], [28], [29]. The radiative transfer equation can be generalized to include all the polarization characteristics. This is done by using the Stokes parameters [I] [6].

$$[I] = \begin{bmatrix} \langle |E_x|^2 \rangle \\ \langle |E_y|^2 \rangle \\ 2 \text{Re} \langle E_x E_y^* \rangle \\ 2 \text{Im} \langle E_x E_y^* \rangle \end{bmatrix}. \quad (21)$$

We then obtain the vector radiative transfer equation with a 4×4 extinction matrix and a 4×4 Mueller matrix [31]. Note that, in general, the extinction coefficient is no longer scalar and must be replaced by the 4×4 extinction matrix. We can also obtain the vector radar equation including all the polarization. This approach is particularly useful for microwave remote sensing, polarimetric radars, and the polarization signature [6], [32], [34], [66].

The vector radiative transfer theory has been extended to the beam wave case [36] and to dense media where the correlation between particles needs to be included [37], [38]. Imaging through such random media is studied using the Modulation Transfer Function (MTF) [39]. Application of the radiative transfer theory to optical diffusion in tissues is discussed using the diffusion approximation [40].

When a wave enters a medium, the radiance can be expressed as the coherent intensity and the diffuse intensity. If the medium is mostly scattering, then the diffuse intensity tends to scatter almost isotropically and the diffuse radiance has a broad angular spread. Therefore, we can expand the diffuse intensity I_d in a series of spherical harmonics. The first two terms of the expansion constitutes the diffusion theory [1]:

$$I_d = \sum_{n=0}^{\infty} I_n = \frac{1}{4\pi} (\psi_d + 3\bar{F}_d \cdot \hat{s}) + \dots \quad (22)$$

The diffuse radiant energy fluence rate ψ_d satisfies the following diffusion equation:

$$(\nabla^2 - k^2) \psi_d = -Q. \quad (23)$$

$$Q = 3\gamma_s(\gamma_t + g\gamma_a) F_o e^{-r}$$

$$k^2 = 3\gamma_a \gamma_{tr}$$

$\gamma_{tr} = \gamma_s(1 - g) + \gamma_a$ where γ_{tr} is called the "transport coefficient." F_o is the incident flux density and g is the mean cosine of the scattering pattern [1, ch. 9]. Note that the transport coefficient is much smaller than the extinction coefficient, and therefore, the "transport mean free path" $l_{tr} = \gamma_{tr}^{-1}$ is much greater than the "mean free path" $l_t = \gamma_t^{-1}$.

Let us next consider the diffusion of a pulse in a turbid medium. First, the coherent intensity pulse $\psi_c(t)$ propagates through the medium with the velocity v of light in that medium ($v = c/n$, n is the refractive index). This coherent pulse is scattered by the medium and generates the diffuse pulse $\psi_d(t)$. The diffuse pulse $\psi_d(t)$ satisfies the following equation [41]:

$$\left[\nabla^2 - \frac{3}{v^2} \frac{\partial^2}{\partial t^2} - \frac{1}{D} \frac{\partial}{\partial t} - 3\gamma_a \gamma_{tr} \right] \psi_d(t) = 0 \quad (24)$$

where D is the diffusion coefficient given by

$$D = \frac{v}{3(\gamma_a + \gamma_{tr})}.$$

Equation (24) shows that the wave front of the diffuse pulse propagates with the velocity $v/\sqrt{3}$. It also shows that

the pulse front is followed by a long tail. At a given point in the medium, the coherent pulse ψ_c arrives with the velocity v , followed by the diffuse pulse ψ_d which is generated by ψ_c . Since ψ_d is generated at all points in the medium, the pulse has a broad peak in the neighborhood of $v/\sqrt{3}$, but it is not a sharp peak [40], [41].

Scattering by discrete scatterers and scattering by turbulence are similar in many respects. For example, we can calculate the optical depth τ using:

$$\tau = \int_0^{R_1} \rho \sigma_t dR \quad (25)$$

where ρ is the number density (number of particles per unit volume) and σ_t is the extinction cross section of a single particle. The bistatic cross section per unit volume can be replaced by

$$\sigma_{bi} = \rho \sigma_d \quad (26)$$

where σ_d is the differential cross section of a single particle. Then the radar equation equally applies to this problem. There are, however, differences. For example, if the fractional volume f , which is the percent of the volume occupied by the particles, is higher than about 10%, the correlation between the particles needs to be included in the formulations. This "dense" medium theory is still a challenging problem today [6], [37], [38].

IV. ROUGH SURFACE SCATTERING

Now let us consider the scattering due to a rough surface or a rough interface (Fig. 3). If the surface height is given by $z = f(x, y)$ and its variance is given by $\sigma^2 = \langle f^2 \rangle$, the surface is considered rough only if the standard deviation σ satisfies the following "Rayleigh criterion":

$$\sigma > \lambda / (8 \cos \theta_i) \quad (27)$$

where λ is the wavelength and θ_i is the incident angle. In general, the rough surface is characterized by the height variance σ^2 and the correlation distance l . If $\sigma \ll \lambda$ and $l \leq \lambda$, then the perturbation technique is applicable. If the radius of curvature of the surface is much greater than a wavelength, the Kirchhoff approximation is applicable (Fig. 4). In general, the specular reflection is reduced by the roughness while the diffuse scattering increases as the roughness increases. Detailed studies of the problem and several developments have been well documented [9], [10], [12], [13]. A definitive review of rough surface scattering was written by DeSanto and Brown [11] in which the spectral methodology and the connected diagram and smoothing expansions are discussed. Scattering by randomly perturbed quasi-periodic surfaces is addressed by Yueh, Shin, and Kong [42]. The regions of validity of the Kirchhoff and perturbation solutions are studied numerically by Chen and Fung [43]. Several new techniques have been introduced in recent years which are applicable to cases beyond the range of validity of the conventional perturbation and Kirchhoff approximations.

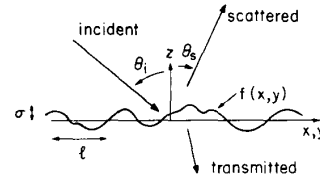


Fig. 3. Rough surface with rms height σ and correlation distance l .

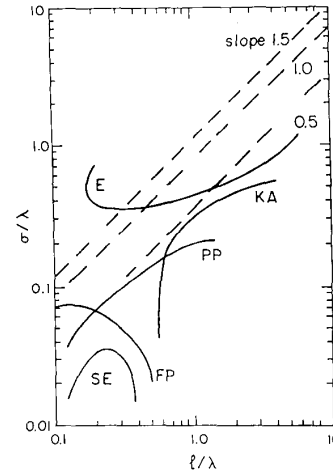


Fig. 4. Ranges of validity for Kirchhoff (KA), Phase Perturbation (PP), and Field Perturbation (FP) theories. Enhanced backscattering occurs in the range E. Also the enhancement due to surface wave modes occurs in SE. σ is the rms height and l is the correlation distance of the rough surface.

They include an equivalent surface impedance [44], the smoothing method [45], the phase perturbation technique [46], [47], [67], the full wave theory [48], and the integral equation method [68], [69].

V. BACKSCATTERING ENHANCEMENT

There has been considerable interest in the enhanced backscattering from randomly distributed scatterers and rough surfaces. This phenomenon is important in applications to lidars, surface optics, and electron localization in disordered media.

Backscattering enhancement phenomena have been observed for many years. It has been sometimes called the "retroreflectance" or the "opposition effect" [49]. An example is "glory" which appears around the shadow of an airplane cast on a cloud underneath the airplane. The moon is brighter at full moon than at other times. Many materials such as BaSO_4 or MgCO_3 are known to cause the enhancement of scattered light in the backward direction. Some soils and vegetation are also observed to cause backscattering enhancement. In spite of many observations, the several theories proposed to explain these phenomena are inadequate. For example, the shadowing theory is based on the fact that in the scattering direction other than in the

back direction, less light may be observed because of the shadowing. Also the scattering pattern of the surface or the scatterers such as Mie scattering may be often peaked in the back direction. The lens hypothesis is that the material may often act as corner reflectors or lenses resulting in peaked backscattering.

Recently, more quantitative experimental and theoretical studies of the enhancement have been reported. Watson [50] noted that the backscattered intensity is twice the multiple scattering and the first-order scattering. de Wolf [23] showed that the backscattered intensity from turbulence is proportional to the fourth-order moment and approximately twice the multiple scattered intensity [65]. An excellent review is given by Kravtsov and Saichev [51]. Kuga and Ishimaru reported on an experiment which showed that the scattering from latex microspheres is enhanced in the backward direction with a sharp angular width of a fraction of a degree [52]. It was explained theoretically by Tsang and Ishimaru that the enhanced peak is caused by the constructive interference of two waves traversing through the same particles in opposite directions [53]. Physicists have also recognized that the transport of electrons in a strongly disordered material is governed by multiple scattering and that multiple scattering leads to "weak Anderson localization" caused by "coherent backscattering" [54]–[58]. It is then shown that both electron localization in disordered material and photon localization in disordered dielectrics are governed by coherent backscattering which is caused by the constructive interference of two waves traversing in opposite directions. Our experimental work in 1984 was followed by several independent optical experiments showing that the backscattering enhancement is a weak localization phenomenon. The enhanced peak value is close to two and the angular width is governed by the diffusion length in the medium.

The experimental data on enhanced backscattering from rough surfaces was shown by O'Donnell and Mendez [59], and numerical studies were conducted by Nieto-Vesperinas *et al.* [60], [61]. The theoretical explanation of the enhancement was presented by Maradudin *et al.* [62] and Bahar [63]. The backscattering enhancement is important in several applications. For example, lidar target calibration requires consideration of the enhancement.

Let us consider a plane wave normally incident on a slab of randomly distributed particles. The first-order backscattering specific intensity I_1 is approximately given by:

$$I_1 = \frac{\gamma_b I_o}{4\pi} \int_0^d e^{-2\gamma_t z} dz$$

$$= \frac{\gamma_b I_o}{8\pi\gamma_t} (1 - e^{-2\gamma_t d})$$

where I_o is the incident intensity and γ_b and γ_t are the backscattering and extinction coefficients, respectively.

The multiple scattering I_m consists of two terms I_{ml} and I_{mc} . One I_{ml} corresponds to the wave multiply scattered through many particles, called the "ladder term."

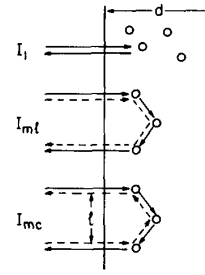


Fig. 5. The first-order scattering I_1 , the ladder I_{ml} , and the cyclical I_{mc} scattering. l is the mean free path for a thin slab and the transport mean free path for a thick slab.

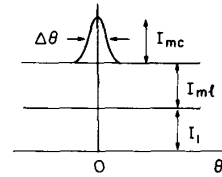


Fig. 6. Backscattering enhancement. (Back direction $\theta = 0$.)

The other I_{mc} corresponds to two waves traversing through the same particles in opposite directions. This is called the "cyclical" or the "maximally crossed" term and has the same magnitude as I_{ml} in the back direction, but diminishes away from the back direction (Fig. 5). The total intensity is therefore approximately shown in Fig. 6. The enhancement factor is given by:

$$\frac{I_1 + I_{ml} + I_{mc}}{I_1 + I_{ml}} = \frac{I_1 + 2I_{ml}}{I_1 + I_{ml}}$$

The enhancement factor is therefore between one and two.

The angular width $\Delta\theta$ is (a wavelength)/(mean free path) if the optical thickness of the slab is of the order of unity or less. If the optical thickness is much greater than unity and the particles are mostly scattering, then the wave is diffused and the angular width $\Delta\theta$ is (a wavelength)/(transport mean free path).

For rough surface scattering, there are two distinct enhancement phenomena shown by "E" and "SE" in Fig. 4. "E" is when the rms height is close to a wavelength and the slope is also close to unity. There the two waves scattered off the sloped surface interfere constructively in the back direction producing the enhanced peak. The angular width is broad and approximately proportional to the slope (Fig. 7).

The second enhancement, "SE", occurs when the rms height is much smaller than a wavelength, but the second medium supports a surface wave. This occurs when an optical beam is scattered from a slightly rough metallic surface. If the incident wave is p-polarized (parallel to the plane of incidence) and the dielectric constant of the second medium has a negative real part, then a surface wave is excited on the surface and two surface waves traversing on the surface in opposite directions interfere constructively

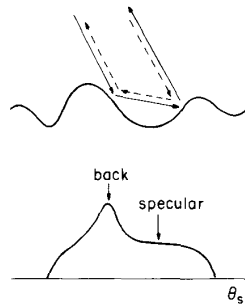


Fig. 7. Backscattering enhancement from rough surfaces.

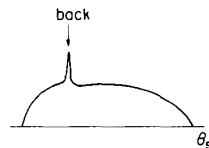


Fig. 8. Backscattering enhancement due to surface wave modes.

in the back direction, producing the enhancement (Fig. 8). The angular width is very small and is proportional to (a wavelength)/(decay distance of the surface wave).

VI. CONCLUSION

In this paper, we reviewed three areas of wave propagation and scattering in random media. They are random continuum (turbulence), randomly distributed particles, and rough surfaces. We included some recent studies of backscattering enhancement which have attracted considerable attention. The field of waves in random media encompasses broad areas of applications in the atmosphere, the ocean, the terrain, and biological media. It also poses one of the most challenging theoretical problems. In addition, interesting experimental observations were made on new multiple scattering phenomena such as coherent backscattering. For these problems of waves in random media, theoretical studies themselves are not sufficient, as they can often not predict new phenomena. Combined efforts of experimental, numerical, and theoretical studies are the key to the advancement of the field.

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