CALCULATIONS AND ANALYSIS

See Stress Calculation Spreadsheet for sources of equations, sources of constants and material properties, and additional calculations.

Impact Analysis

Direct wheel impact at max speed

- By using the deflection equation, \( s = \frac{FL^3}{192EI} \) (based upon two fully constrained rod ends), solving for F, and using a basic kinematic equation \( v_f^2 = v_o^2 = 2a(\Delta y) \) to solve for s in terms of F, the force of impact can be determined (227505 N)
- Utilizing shaft stress equations shown below the stress can be determined (400 MPa)
- When comparing this to the shaft’s yield strength, a factor of safety of 1.33 is calculated
Direct pulley impact at max speed

- Utilizing this same force and finding the stress on the shaft due to bending,
  \[ \sigma = \frac{Mc}{I} = 8510 \text{ MPa} \]
- This means the shaft will permanently bend due to the moment applied on it.
- The way to avoid this catastrophic failure is to ensure the chassis protects these open gears by extending past its edges or enclosing it completely. While this may not completely ensure the module’s safety, it will fix nearly every probable scenario.

Shaft Stress Calculations

Shaft 1 (Diameter=3/8")
- Material: 1045 Steel, Yield Strength (S_y) = 530 MPa, Ultimate Strength= 625MPa
- Max Stress
  - The shaft is keyed for a 3/32” key, thus a close approximation for the actual yield strength is \( \frac{3}{4} \) the materials yield strength (Keyed Yield Strength=398 MPa)
  - Loading is comprised of three components
    - Moment-Based on cantilevered distance from bearing and radial load exerted on shaft from the miter gear (2.1 N-m)
    - Force- Based on axial load exerted on shaft from miter gear (156.12 N)
    - Torque- Exerted by the stall torque of the motor, through a gear ratio of 2:1 (9.64 N-m)
  - Stress Calculation
    - \( \sigma_{\text{max}} = \frac{4}{\pi d^2} [(8M + Fd)^2 + 48T^2]^{1/2} = 102 \text{ MPa} \)
    - \( \tau_{\text{max}} = \frac{2}{\pi d^3} [(8M + Fd)^2 + 64T^2]^{1/2} = 58.4 \text{ MPa} \)
  - Factors of Safety
    - \[ n = \frac{S_y}{\sigma_{\text{max}}} = 3.9 \]
    - \[ n = \frac{S_y}{2\tau_{\text{max}}} = 3.4 \]
- Fatigue Life
  - Infinite Life- 2000RPM (Average operating speed)=33.3 cycles/second
    - 5 year life @ 1 hour operating time (2 hr per week)-approximately 1,908,000 seconds of use
    - \( 33.3 \times 1,908,000 = 6.4E7 \) cycles to failure for infinite life
  - The endurance strength can be calculated using the stress concentration factors from the keyway (197 MPa)
    - \( \sigma'^{\text{I}} = S_{ut} + 345 \text{ MPa} = 970 \text{ MPa} \)
\[ b = -\frac{\log(\sigma' / S_e)}{\log(2N_c)} = -0.109915548 \]
\[ f = \frac{\sigma'_F}{S_{ut}} (2 \cdot 10^3)^b = 0.673 \]
\[ a = \left( \frac{f \cdot S_{ut}}{S_e} \right)^2 = 900 \text{ MPa} \]

- Loads are based on typical operating conditions, not max conditions
  - Moment-Based on cantilevered distance from bearing and radial load exerted on shaft from the miter gear (2.1 N-m)
  - Force- Based on axial load exerted on shaft from miter gear (156.12 N)
  - Torque- Exerted by the operating torque of the motor, through a gear ratio of 2:1 (2.82 N-m)

\[ \sigma_a = \frac{4}{\Gamma d^3} [(8M + Fd)^2 + 48T^2]^{1/2} = 39.4 \text{ MPa} \]

\[ N = \left( \frac{\sigma_a}{a} \right)^{1/2} = 2.25 \times 10^{12} \text{ cycles to failure} \]

**Shaft 2 (Diameter=1/2")**
- Material: 1045 Steel, Yield Strength= 530 MPa, Ultimate Strength= 625MPa
- Max Stress
  - The shaft is keyed for a 1/8” key, thus the actual yield strength can be equated to 3/4 the materials yield strength (Keyed Yield Strength=398 MPa)
  - Loading is comprised of three components
    - Moment-Based on the axle length between bearings and radial load exerted on shaft from the miter gear (4.28 N-m)
    - Force- Based on axial load exerted on shaft from miter gear (156.12 N)
    - Torque- Exerted by the stall torque of the motor, through a gear ratio of 2:1 (9.64 N-m)
- Stress Calculation-
  - \[ \sigma_{max} = \frac{4}{\Gamma d^3} [(8M + Fd)^2 + 48T^2]^{1/2} = 47.2 \text{ MPa} \]
  - \[ \tau_{max} = \frac{2}{\Gamma d^3} [(8M + Fd)^2 + 64T^2]^{1/2} = 26.5 \text{ MPa} \]
- Factors of Safety-
  - \[ n = \frac{S_y}{\sigma_{max}} = 8.4 \]
  - \[ n = \frac{S_y}{2\tau_{max}} = 7.5 \]
- Fatigue Life
  - Infinite Life- 1000RPM=16.67 cycles/second
- 5 year life @ 1 hour operating time (2 hr per week)-approximately 1,908,000 seconds of use
  - 16.67*1,908,000=3.2E7 cycles to failure for infinite life
  - The endurance strength can be calculated using the stress concentration factors from the keyway (197 MPa)
  - \( \sigma' = S_{ut} + 345 \) MPa = 970 MPa
  - \( b = -\frac{\log(\sigma' / S_c)}{\log(2N_c)} = -0.109915548 \)
  - \( f = \frac{\sigma'_F}{S_{ut}} (2.10^3)^b = 0.673 \)
  - \( a = \left( \frac{f \cdot S_{ut}}{S_c} \right)^2 = 900 \) MPa

  - Loads are based on typical operating conditions, not max conditions
    - Moment-Based on the axle length between bearings and radial load exerted on shaft from the miter gear (4.28 N-m)
    - Force- Based on axial load exerted on shaft from miter gear (156.12 N)
    - Torque- Exerted by the operating torque of the motor, through a gear ratio of 2:1 (2.82 N-m)
  - \( \sigma_a = \frac{4}{\Pi d^3} \left[ (8M + Fd)^2 + 48T^2 \right]^{1/2} = 25.6 \) MPa
  - \( N = \left( \frac{\sigma_a}{a} \right)^{\frac{1}{p}} = 1.15 \times 10^{14} \) cycles to failure

**Shaft 3 (Diameter=3/4")**
- Material: 1045 Steel, Yield Strength= 530 MPa, Ultimate Strength= 625 MPa
- Max Stress
  - The shaft is keyed for a 3/16” key, thus the actual yield strength can be equated to ¾ the materials yield strength (Keyed Yield Strength=398 MPa)
  - Loading is comprised of three components
    - Moment-Based on the axle length between bearings and the force exerted by the weight of the system (21.53 N-m)
    - Force- Based on axial load exerted on the shaft from turning forces (235.44 N)
    - Torque- Exerted by the stall torque of the motor, through a gear ratio of 8:1 (38.56 N-m)
  - Stress Calculation-
    - \( \sigma_{max} = \frac{4}{\Pi d} \left[ (8M + Fd)^2 + 48T^2 \right]^{1/2} = 59.0 \) MPa
    - \( \tau_{max} = \frac{2}{\Pi d^3} \left[ (8M + Fd)^2 + 64T^2 \right]^{1/2} = 32.7 \) MPa
  - Factors of Safety-
\[ n = \frac{S_y}{\sigma_{\text{max}}} = 6.7 \]

\[ n = \frac{S_y}{2\tau_{\text{max}}} = 6.1 \]

- **Fatigue Life**
  - Infinite Life- 500RPM=8.34 cycles/second
    - 5 year life @ 1 hour operating time (2 hr per week)-approx 1,908,000 seconds of use
    - 8.34*1,908,000=1.6E7 cycles to failure for infinite life
  - The endurance strength can be calculated using the stress concentration factors from the keyway (197 MPa)
  - \( \sigma'_{\text{f}}=S_{\text{ut}}+345\text{MPa}= 970 \text{ MPa} \)
  - \( b = -\frac{\log(\sigma'_{\text{f}}/S_e)}{\log(2N_e)} =-0.109915548 \)
  - \( f = \frac{\sigma'_{\text{f}}}{S_{\text{ut}}} (2 \cdot 10^5)^b =.673 \)
  - \( a = \left(\frac{f \cdot S_{\text{ut}}}{S_e}\right)^2 =900 \text{ MPa} \)
  - Loads are based on typical operating conditions, not max conditions
    - Moment-Based on the axle length between bearings and the force exerted by the weight of the system (21.53 N-m)
    - Force- Based on axial load exerted on the shaft from turning forces (235.4 N)
    - Torque- Exerted by the operating torque of the motor, through a gear ratio of 8:1 (11.28 N-m)
  - \( \sigma_a = \frac{4}{\Pi d^3} [(8M + Fd)^2 + 48T^2]^{1/2} = 35.6 \text{ MPa} \)
  - \( N = \left( \frac{\sigma_a}{a} \right)^{\frac{1}{2}} = 5.7E12 \text{ cycles to failure} \)

**Steering Shaft (Diameter=1/4")**
- Material: 303 Stainless Steel, Yield Strength= 240 MPa, Ultimate Strength= 620 MPa
- **Max Stress**
  - Loading is based on torque alone (0.745 N-m)
  - Stress Calculation-
    - \( \sigma_{\text{max}} = \frac{4}{\Pi d} [(8M + Fd)^2 + 48T^2]^{1/2} = 25.7 \text{ MPa} \)
    - \( \tau_{\text{max}} = \frac{2}{\Pi d^3} [(8M + Fd)^2 + 64T^2]^{1/2} = 14.8 \text{ MPa} \)
  - Factors of Safety-
    - \( n = \frac{S_y}{\sigma_{\text{max}}} = 9.4 \)
\[ n = \frac{S_y}{2\tau_{\text{max}}} = 8.1 \]

- Fatigue Life
  - \( \sigma' = S_{ut} + 345 \text{MPa} = 965 \text{E6 MPa} \)
  - \( b = -\frac{\log(\sigma' / S_e)}{\log(2N_e)} = -0.07772 \)
  - \( f = \frac{\sigma'}{S_{ut}} (2 \cdot 10^3)^b = 0.862 \)
  - \( a = \frac{(f \cdot S_{ut})}{S_e} = 914 \text{MPa} \)
  - Load is comprised of torque alone (.745 N-m)
  - \( \sigma_a = \frac{4}{\pi d^3} [(8M + Fd)^2 + 48T^2]^{1/2} = 25.7 \text{ MPa} \)
  - \( N = \left( \frac{\sigma_a}{a} \right)^{1/5} = 9.3 \text{E19 cycles to failure} \)

Spur Gears (Calculated using ANSI standards)

**Driving Spur**

- Material - Carbon Steel, Yield Strength=76900 psi, Modulus of Elasticity=30E6 psi, Poisson’s Ratio=.29, Brunell Hardness 179
- Max Bending Stress
  - \( W' = \frac{33000 \cdot H}{V} = 306.8 \text{ lbf} \)
  - \( K_o = 1.25 \) - Overload Factor, based on light shocks encountered
  - \( K_v = 1.15 \) - Dynamic Factor, based on quality and velocity of gears
  - \( K_s = 1 \) - Size Factor
  - \( P_d = .833'' \) – Pitch diameter
  - \( F = .25'' \) – face width
  - \( K_m = 1.20 \) – Load-Distribution factor, based on geometry
  - \( K_R = 1 \) – Rim Thickness factor, based on geometry
  - \( J = .325 \) - Geometry factor, based on number of teeth of gears
  - \( \sigma = W' K_o K_v K_s \frac{P_d}{F} K_m K_R = 5357.1 \text{ psi} \)
  - \( n = \frac{S_y}{\sigma_{\text{max}}} = 9.0 \)

- Endurance Stress
  - \[ C_p = \left[ \frac{1}{\pi \left( \frac{1 - v_p^2}{E_p} + \frac{1 - v_G^2}{E_G} \right)} \right]^{1/2} = 2284.7 \text{ lbf/in}^2 \]
o $C_f=1$

o $I=0.08$ - Geometry Factor

$\sigma = C_p (W'K_o K_f K_m C_f)^{1/2} = 56972.4$ psi

o $Sc= 180000$ psi - Repeatedly applied contact strength @ $10^7$ cycles, material property

o $Z_n=.59$ - Stress cycle life factor, based on hardness and number of cycles

o $C_H=1$ - Hardness ratio factor

o $K_T=1$ - Temperature factor

o $K_R=1$ – Reliability factor

o $S_H = \frac{S_Z Z_K C_{H}}{I} = 1.9$

o Comparable factor of safety $S_H^2 = 3.5$

**Driven Spur**

- Material- Carbon Steel, Yield Strength=76900 psi, Modulus of Elasticity=30E6 psi, Poisson’s Ratio=.29, Brunell Hardness 179

- Max Bending Stress

  o $W' = \frac{33000 \cdot H}{V} = 306.7$ lbf

  o $K_o= 1.25$ - Overload Factor, based on light shocks encountered

  o $K_v= 1.15$ - Dynamic Factor, based on quality and velocity of gears

  o $K_s= 1$ - Size Factor

  o $P_d= 1.667''$ – Pitch diameter

  o $F= .25''$ – face width

  o $K_m= 1.19$ – Load-Distribution factor, based on geometry

  o $K_R=1$ – Rim Thickness factor, based on geometry

  o $J= .389$ - Geometric factor, based on number of teeth of gears

  o $\sigma = W'K_o K_v K_s \frac{P_d K_m K_R}{FJ} = 9011.0$ psi

  o $n = \frac{S_n}{\sigma_{max}} = 5.4$

- Endurance Stress

  o $C_p = \left[ \frac{1}{\pi \left( \frac{1-v_p^2}{E_p} + \frac{1-v_G^2}{E_G} \right) } \right]^{1/2} = 2284.7$ lbf/in$^2$

  o $C_f=1$

  o $I=0.08$ - Geometry Factor

  o $\sigma = C_p (W'K_o K_f K_m C_f)^{1/2} = 40010.7$ psi
Sc= 180000 psi- Repeatedly applied contact strength @ 10⁷ cycles, material property

Zₙ=.60 - Stress cycle life factor, based on hardness and number of cycles

Cₜ=1 - Hardness ratio factor

Kₜ= 1 - Temperature factor

Kₚ= 1 – Reliability factor

\[ S_H = \frac{S_N C_H}{(K_T K_R)} = 2.70 \]

Comparable factor of safety= Sₚ=7.2

Ring/Pinion Gears (Calculated using ANSI standards)

**Steering Spur**

- Material- 2024-T4 Aluminum, Yield Strength=47000 psi, Modulus of Elasticity=10.4E6 psi, Poisson’s Ratio=.333
- Max Bending Stress
  - \( W' = \frac{33000. H}{V} = 39.3 \text{ lbf} \)
  - Kₒ= 1.25 - Overload Factor, based on light shocks encountered
  - Kᵥ= 1.10 - Dynamic Factor, based on quality and velocity of gears
  - Kₜ= 1 - Size Factor
  - Pₜ= .4375” – Pitch diameter
  - F= .125” – face width
  - Kₚ= 1.20 – Load-Distribution factor, based on geometry
  - Kₚ= 1 – Rim Thickness factor, based on geometry
  - J=.24- Geometry factor, based on number of teeth of gears
  - \( \sigma = W' K_o K_v P_d K_p = 951.8 \text{ psi} \)
  - \( n = \frac{S_H}{\sigma_{max}} = 44.1 \)

**Steering Ring**

- Material- 2024-T4 Aluminum, Yield Strength=47000 psi, Modulus of Elasticity=10.4E6 psi, Poisson’s Ratio=.333
- Max Bending Stress
  - \( W' = \frac{33000. H}{V} = 39.3 \text{ lbf} \)
  - Kₒ= 1.25 - Overload Factor, based on light shocks encountered
  - Kᵥ= 1.10 - Dynamic Factor, based on quality and velocity of gears
  - Kₜ= 1 - Size Factor
  - Pₜ= 3.125” – Pitch diameter
  - F= .125” – face width
  - Kₚ= 1.8 – Load-Distribution factor, based on geometry
  - Kₚ= 1 – Rim Thickness factor, based on geometry
  - J=.4- Geometry factor, based on number of teeth of gears
\[
\sigma = W'K_oK_vK_xP_dK_mK_B F = 3996.0 \text{ psi}
\]
\[
n = \frac{S_v}{\sigma_{max}} = 10.5
\]

Miter Gears (Calculated using ANSI standards)

**Both Miters (At max torque)**

- Material - Medium Carbon Steel, Yield Strength=76900 psi
- Max Bending Stress
  - \(P_d = 1.25''\) – Pitch diameter
  - \(W' = \frac{2T}{P_d} = 84.7 \text{ lbf}\)
  - \(K_o= 1.25\) - Overload Factor, based on light shocks encountered
  - \(K_v= 1\) - Dynamic Factor, based on quality and velocity of gears
  - \(K_x= .5\) - Size Factor
  - \(F= .27''\) – face width
  - \(K_m= 1.10\) – Load-Distribution factor, based on geometry
  - \(J= 0.175\) - Geometry factor, based on number of teeth of gears
  - \(K_x= 1\), Lengthwise curvature factor
  - \(\sigma = \frac{W'}{F}P_dK_oK_vK_xK_mK_B = 14922.2 \text{ psi}\)
  - \(n = \frac{S_v}{\sigma_{max}} = 5.15\)

**Both Miters (At max speed)**

- Material - Medium Carbon Steel, Yield Strength=76900 psi
- Max Bending Stress
  - \(P_d = 1.25''\) – Pitch diameter
  - \(W' = \frac{2T}{P_d} = 4 \text{ lbf}\)
  - \(K_o= 1.25\) - Overload Factor, based on light shocks encountered
  - \(K_v= 1.28\) - Dynamic Factor, based on quality and velocity of gears
  - \(K_x= .5\) - Size Factor
  - \(F= .27''\) – face width
  - \(K_m= 1.10\) – Load-Distribution factor, based on geometry
  - \(J= 0.175\) - Geometry factor, based on number of teeth of gears
  - \(K_x= 1\), Lengthwise curvature factor
  - \(\sigma = \frac{W'}{F}P_dK_oK_vK_xK_mK_B = 901.6 \text{ psi}\)
  - \(n = \frac{S_v}{\sigma_{max}} = 85.3\)
Forces
- Knowing max torque on miter (9.63 N-m), we can find the max tangential force by dividing by half the pitch diameter: \( F_{\tan} = 606.6 \) N
- \( \alpha = 20 \) degrees - pressure angle
- \( d = 45 \) degrees
- \( F_n = \frac{F_{\tan}}{\cos \alpha} = 645.5 \) N
- \( F_1 = \frac{F_n}{\sin \alpha} = 220.8 \) N
- \( F_{axial} = F_{radial} = F_1 \sin d = 156.1 \) N

Retaining Rings
- On 3/8” shaft
  - Ring can withstand 542.7 N of axial force
  - Miter gear provides axial load = 156.1 N
  - Factor of safety = 3.48
- On 1/2” shaft
  - Ring can withstand 542.7 N of axial force
  - Miter gear provides axial load = 156.1 N
  - Factor of safety = 3.48
- On 3/4” shaft
  - Ring can withstand 631.6 N of axial force
  - Axial load is from turning
    - Assume wheel instantaneously turns 90 degrees, the max force that can be applied axially would be equivalent to the frictional force
    - \( F_{axial} = F_{friction} = W \cdot \mu = 235.4 \) N (assuming \( \mu = 0.6 \))
  - Factor of safety = 2.68

Mechanical Brake

Max Temperature
- Assuming all kinetic energy is converted directly into heat energy,
  \[ \frac{1}{2} m_{veh} v_{veh}^2 = m_{plate} C_v \Delta T \]
- Assume emergency brake will not be used continuously, but rather for one cycle during the emergency
- Assume initial temperature of 23 ° Celcius
- Solving the above equation for \( T_{final} \) we find it to be 38.9 °Celcius

Heat Dissipation
- Assuming Free Convection, the time required for heat dissipation can be calculated
- Utilizing the properties of air at room temperature, the Rayleigh number, Nusselt number, and convection heat transfer coefficient can be calculated
Using this information the heat transfer rate is determined

\[ q = \frac{\Delta E}{\Delta t} \]

gives the time to dissipate the heat (4.8 minutes)

This resultant was later verified by the manufacturer of the brake

**Keys**

**On 3/8” Shaft**
- Key is High carbon steel, Yield Strength 427 MPa, 3/32” square
- Knowing the diameter of and torque on the shaft, the shear force on the key can be calculated (2024.1 N)
- Assuming a factor of safety of 4, the required length of the key is calculated (.63”)

**On 1/2” Shaft**
- Key is High carbon steel, Yield Strength 427 MPa, 1/8” square
- Knowing the diameter of and torque on the shaft, the shear force on the key can be calculated (1518.1 N)
- Assuming a factor of safety of 4, the required length of the key is calculated (.35”)

**On 3/4’’ Shaft**
- Key is High carbon steel, Yield Strength 427 MPa, 3/16” square
- Knowing the diameter of and torque on the shaft, the shear force on the key can be calculated (4048.3 N)
- Assuming a factor of safety of 4, the required length of the key is calculated (.63”)

**Keyways**

- Keyway analysis was done using Cosmos FEA software
- By utilizing shaft diameters and torques, forces on keyway surfaces were calculated and input into the program
- Factor of Safety
  - Driving Miter=16
  - Driven Miter=15
  - Driving Pulley=3.4
  - Driven Pulley=8.4
  - Driven Spur=8.1
  - Wheel=1.8, but in reality, failure would result in the slip of a pressed insert, rather than physical failure of the wheel

**Set Screws**

**To connect spur gear to 5/16” drive motor shaft**
- By choosing a screw size and quantity (2- #8’s), the maximum force at the shaft surface can be calculated (3425.1 N)
- The torque and diameter of the shaft is used to determine the actual force seen at this shaft surface (1214.5 N)
- By comparing these two values the factor of safety is determined (2.82)

**To connect spur gear to 8mm steering motor shaft**
- By choosing a screw size and quantity (2-#6’s), the maximum force at the shaft surface can be calculated (2224.1 N)
- The torque and diameter of the shaft is used to determine the actual force seen at this shaft surface (234.6 N)
- By comparing these two values the factor of safety is determined (9.5)

**Timing Belt and Pulleys**
- Utilizing MITCalc simulation software and inputting various parameters including distance between centers, power applied to belt, operating speeds, and other operating conditions a belt type and specific model was selected
- From this the 5M Powergrip GT2 belt was chosen and matched with pulleys of 18 and 72 teeth
- The selection of these parts was also verified with an engineer at the supplier sdp-si.com

**Bearings**

\[
\begin{align*}
C_{10} &= \text{Catalog Load Rating (lb$_f$)} \\
L_R &= \text{Rating Life (hrs)} \\
n_R &= \text{Rating Speed (RPM)} \\
F_D &= \text{Desired Radial Load (lb$_f$)} \\
L_D &= \text{Desired Life (hrs)} \\
n_D &= \text{Desired Speed (RPM)} \\
F_R &= \text{Radial Force (lb$_f$)} \\
F_A &= \text{Axial Force (lb$_f$)} \\
F_e &= \text{Equivalent Radial Load (lb$_f$)} \\
C_0 &= \text{Static Load Rating (lb$_f$)} \\
X_2 &= \text{Factor dependent on bearing geometry} \\
Y_2 &= \text{Factor dependent on bearing geometry} \\
V &= \text{Rotation Factor} \\
a &= 3 \text{ (ball bearing)} \\
e &= \text{abscissa}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Lower Drive Bearing</th>
<th>Upper Drive Bearing</th>
<th>Center Bearing</th>
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</thead>
<tbody>
<tr>
<td>$C_{10}$ (lbs)</td>
<td>1171</td>
<td>1187</td>
</tr>
<tr>
<td>$L_R \times n_R$</td>
<td>1.0E+06</td>
<td>1.0E+06</td>
</tr>
<tr>
<td>$F_D$ (lbs)</td>
<td>44.125</td>
<td>84.6</td>
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<tr>
<td>$n_D$ (RPM)</td>
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<td>2000</td>
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<tr>
<td>$a$</td>
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<td>4000</td>
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<tr>
<td>$n_D$ (RPM)</td>
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</table>
\[
L_D = \left( \frac{C_{10}}{F_D} \right)^{\frac{a}{n_D}} \left( \frac{L_R \cdot n_R}{n_D} \right)
\]

\[
F_c = X_2 V F_R + Y_2 F_A
\]

**Screws**

**Bolts connecting Yoke to Turntable**

Bolt type: 4 * 10-32 (SAE)

Torque applied to the turntable

\( T = 2.38 \text{ N-m} \)

Converted ASTM

\( T = 21 \text{ lb-in} \)

Resultant load on each bolt

\[
V = \frac{T}{r} = 21 \text{ lb-in} \cdot \frac{1}{2.59 \text{ in}} = 8.108 \text{ lb}
\]

\( M = T = 21 \text{ lb-in} \)

Primary Shear Load per Bolt is

\[
F' = \frac{V}{n} = \frac{8.108}{4} = 2.027 \text{ lb}
\]

Since the secondary shear Forces are equal we have

\[
F'' = \frac{Mr}{4r^2} = \frac{M}{4r} = \frac{21}{4 \cdot 2.59} = 2.027 \text{ lb}
\]
The resultant force is

\[ F_r = 2.867\, lb \]

\[ F_t = F_a = F_b = F_c = F_d = 2.867\, lb \]

Maximum Shear Stress

\[ A_s = 0.155 \]

\[ \tau = \frac{F_r}{A_s} = \frac{2.867}{0.155} = 18.497 \, psi \]

Bolts connecting Brake to Brake plate

Bolt type: 4 * 8-32 (SAE)

Torque applied to the turntable

\[ T = 15 \, lb\cdot in \]

Resultant load on each bolt

\[ V = \frac{T}{r} = \frac{15}{1.125} = 13.33\, lb \]

\[ M = T = 15 \, lb\cdot in \]

Primary Shear Load per Bolt is

\[ F_r = \frac{V}{n} = \frac{13.33}{4} = 3.33\, lb \]

Since the secondary shear Forces are equal we have

\[ F'' = \frac{Mr}{4r^2} = \frac{M}{4r} = \frac{15}{4*1.125} = 3.33\, lb \]

The resultant force is

\[ F_r = 4.714\, lb \]
\[
F_t = F_a = F_b = F_c = F_d = 4.714 \text{ lb}
\]

**Maximum Shear Stress**

\[
A_s = \frac{F_t}{A_s} = \frac{2.867}{.0992} = 28.9 \text{ psi}
\]

**Yoke**

- Yoke stress analysis was done using Cosmos FEA software
- \textbf{Loading}
  - Weight vertically loads lower bearing holes (196.2 N each)
  - Turning force loads inside wall (158.3 N)
  - Driven Miter axial force loads inside wall (156 N)
  - Driven Miter radial force loads upper bearing holes
Driving Miter axial force loads upward on top plate

- Minimum factor of safety = 20

**Brake Plate**

- Brake plate stress analysis was done using Cosmos FEA software
- Loading
  - Outside edge was fixed, as it is welded to the motor mount assembly
  - Each brake mounting hole was loaded with a force corresponding to the brake’s torque output and the holes distance from center
- Minimum factor of safety = 200

**Turntable**

- Capable of withstanding 750 lbs, or 340 kg
- Actual weight is about 40 kg per module
- Factor of Safety = 8.5
### Figure X: Shaft Stress and Fatigue Strength Calculations

<table>
<thead>
<tr>
<th>Shaft 1</th>
<th>Shaft 2</th>
<th>Shaft 3</th>
<th>Shaft 4</th>
<th>Shaft 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Material</td>
<td>Material</td>
<td>Material</td>
<td>Material</td>
</tr>
<tr>
<td>Yield Strength</td>
<td>Yield Strength</td>
<td>Yield Strength</td>
<td>Yield Strength</td>
<td>Yield Strength</td>
</tr>
<tr>
<td>355 MPa</td>
<td>360 MPa</td>
<td>365 MPa</td>
<td>380 MPa</td>
<td>390 MPa</td>
</tr>
<tr>
<td>Yield Point</td>
<td>Yield Point</td>
<td>Yield Point</td>
<td>Yield Point</td>
<td>Yield Point</td>
</tr>
<tr>
<td>270 MPa</td>
<td>280 MPa</td>
<td>290 MPa</td>
<td>300 MPa</td>
<td>310 MPa</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>Modulus of Elasticity</td>
<td>Modulus of Elasticity</td>
<td>Modulus of Elasticity</td>
<td>Modulus of Elasticity</td>
</tr>
<tr>
<td>207 GPa</td>
<td>209 GPa</td>
<td>211 GPa</td>
<td>213 GPa</td>
<td>215 Gpa</td>
</tr>
<tr>
<td>Hardness</td>
<td>Hardness</td>
<td>Hardness</td>
<td>Hardness</td>
<td>Hardness</td>
</tr>
<tr>
<td>179</td>
<td>181</td>
<td>183</td>
<td>185</td>
<td>187</td>
</tr>
</tbody>
</table>

**Driving Spur 1: Carbon Steel**
- **Yield Strength**: 700 MPa
- **Modulus of Elasticity**: 207 GPa
- **Hardness**: 179

**Driving Spur 2: Carbon Shaft**
- **Yield Strength**: 700 MPa
- **Modulus of Elasticity**: 207 GPa
- **Hardness**: 179

### Driving Spur 1

<table>
<thead>
<tr>
<th>Metric</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Diameter</td>
<td>mm</td>
<td>0.55</td>
</tr>
<tr>
<td>Operating Pitch Diameter</td>
<td>mm</td>
<td>1.66</td>
</tr>
<tr>
<td>Rotational Speed</td>
<td>rpm</td>
<td>2521.54</td>
</tr>
<tr>
<td>Velocity</td>
<td>m/min</td>
<td>57.11</td>
</tr>
<tr>
<td>Power</td>
<td>kW</td>
<td>0.21</td>
</tr>
<tr>
<td>Transmitted Load</td>
<td>Nm</td>
<td>350.54</td>
</tr>
<tr>
<td>Overload Factor</td>
<td></td>
<td>1.25</td>
</tr>
<tr>
<td>Quality Class</td>
<td></td>
<td>Commercial</td>
</tr>
</tbody>
</table>

### Driving Spur 2

<table>
<thead>
<tr>
<th>Metric</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Diameter</td>
<td>mm</td>
<td>0.55</td>
</tr>
<tr>
<td>Operating Pitch Diameter</td>
<td>mm</td>
<td>1.66</td>
</tr>
<tr>
<td>Rotational Speed</td>
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<tr>
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<tr>
<td>Transmitted Load</td>
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<td>350.54</td>
</tr>
<tr>
<td>Overload Factor</td>
<td></td>
<td>1.25</td>
</tr>
<tr>
<td>Quality Class</td>
<td></td>
<td>Commercial</td>
</tr>
</tbody>
</table>

### Figure X: Drive Motor Spur Gear Stress Calculation

- **Load/Distribution Factor**: 1.25
- **Yield-Thickness Factor**: 1
- **Geometry Factor**: 0.8
- **Bending Stress**: 126500
- **Bending Factor of Safety**: 52.42
- **Endurance**: 1
- **Transverse Pressure Angle**: 14.6°
- **Operating Pitch Diameter**: 0.848 in
- **Rotational Speed**: 2913.741 rpm
- **Velocity**: 872.352 ft/min
- **Transmitted Load**: 6.026086 lbf
- **Bending Stress**: 126500
- **Bending Factor of Safety**: 52.42

---

**Figure X: Shaft Stress and Fatigue Strength Calculations**

**Figure X: Drive Motor Spur Gear Stress Calculation**
## Miter Gears

<table>
<thead>
<tr>
<th>Torque</th>
<th>95 in-lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Diameter</td>
<td>1.25 in</td>
</tr>
<tr>
<td>Transmitted Load</td>
<td>136 lb</td>
</tr>
<tr>
<td>Net Face Width</td>
<td>0.22 in</td>
</tr>
<tr>
<td>Overload Factor</td>
<td>1.35</td>
</tr>
<tr>
<td>Quality Class</td>
<td>5</td>
</tr>
</tbody>
</table>

## Forces

| Pressure angle | 20 deg |
| Pinion Pitch Diameter | 0.03175 m |
| Pinion Torque | 3.63 N-m |
| Tangential Force | 608.65142 N |
| Frictional Force | 168.1218 N |
| Frictional Force | 168.1218 N |

## Figure X: Miter Gear Stress and Force Calculation

### Capacity

<table>
<thead>
<tr>
<th>Diameter Shaft</th>
<th>Thrust Load (lbs)</th>
<th>Thrust Load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/8&quot;</td>
<td>122</td>
<td>542 883.84</td>
</tr>
<tr>
<td>1/2&quot;</td>
<td>122</td>
<td>542 883.84</td>
</tr>
<tr>
<td>3/4&quot;</td>
<td>142</td>
<td>631 467.52</td>
</tr>
</tbody>
</table>

### Shaft 1

| Axial Load (N) | 156.12 Due to Bevel |
| Factor of Safety | 3.48 |

### Shaft 2

| Axial Load (N) | 156.12 Due to Bevel |
| Factor of Safety | 3.48 |

### Shaft 3

| Axial Load (N) | 313.68 Due to Wheel |
| Factor of Safety | 2.01 |

## Figure X: Retaining Ring Calculations

### Impact on Wheel

| Force | 113752.5802 N |
| Time of collision | 0.004219485 s |
| Yield Strength | 5.30E+08 |
| Shear Stress | 2.00E+08 |
| Factor of Safety | 2.66 |

### Impact on Pulley (Bending on shaft)

| Bending length | 1 m |
| Moment | 2093.31567 N-m |
| Distance from neutral axis | 0.0096241 m |
| Stress | 4.20E+09 Pa |
| Factor of Safety | 0.12 |

## Figure X: Impact Calculations

### Brake Temperature and Heat Dissipation Calculations

| Max Veh. Speed | 3 m/s |
| Veh. Mass | 180 kg |
| Max Energy | 720 |

### Brake

| Mass of friction plates | 0.05 kg |
| Specific Heat of Plates | 450 J/kg K |
| # Breaking wheels | 2 |
| Initial Temp | 25 Celsius |
| Max Temp | 300.93 Celsius |
| AIR Temp | 400 K |
| k | 0.333 W/m K |
| y | 0.0000324 m²/s |
| alpha | 0.0000385 m²/s |
| Pr | 0.68 |
| Beta | 0.05 |
| g | 9.81 m/s² |

| h bar | 22.2365417 W/m² K |
| q | 2.51986428 W |
| Time to cool | 285.2747437 s |
| 4.77 minutes |
### Key Strength

<table>
<thead>
<tr>
<th>Key Width</th>
<th>Shaft Dia (in)</th>
<th>Shaft Dia (mm)</th>
<th>Shaft Dia (in)</th>
<th>Shaft Dia (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/32 in</td>
<td>0.09525 m</td>
<td>0.09525 m</td>
<td>0.09525 m</td>
<td>0.09525 m</td>
</tr>
<tr>
<td>1/16 in</td>
<td>0.093175 m</td>
<td>0.093175 m</td>
<td>0.093175 m</td>
<td>0.093175 m</td>
</tr>
<tr>
<td>3/32 in</td>
<td>0.093175 m</td>
<td>0.093175 m</td>
<td>0.093175 m</td>
<td>0.093175 m</td>
</tr>
</tbody>
</table>

### Material Properties

<table>
<thead>
<tr>
<th>Approx. Yield Str.</th>
<th>High Carbon Plain Steel</th>
<th>High Carbon Plain Steel</th>
<th>High Carbon Plain Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (N)</td>
<td>4370-60 Pa</td>
<td>Approx. Yield Str.</td>
<td>4370-60 Pa</td>
</tr>
<tr>
<td>F (N)</td>
<td>2024.14682</td>
<td>1518.11</td>
<td>4540.294</td>
</tr>
</tbody>
</table>

### Factor of Safety

| Factor of Safety | 4 | 4 | 4 |
| Required Length | 0.035969 m | 0.035969 m | 0.035969 m |
| Required Length | 0.63 in | 0.63 in | 0.63 in |

### Set Screw Holding Power

<table>
<thead>
<tr>
<th>Set Screw</th>
<th>Holding Power (bf)</th>
<th>Holding Power (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M6</td>
<td>65</td>
<td>2024.14682</td>
</tr>
<tr>
<td>M8</td>
<td>85</td>
<td>26313443</td>
</tr>
<tr>
<td>M10</td>
<td>120</td>
<td>3347178</td>
</tr>
<tr>
<td>M12</td>
<td>160</td>
<td>4151718</td>
</tr>
<tr>
<td>M14</td>
<td>200</td>
<td>500</td>
</tr>
<tr>
<td>M16</td>
<td>250</td>
<td>652.233</td>
</tr>
<tr>
<td>M18</td>
<td>200</td>
<td>808544</td>
</tr>
<tr>
<td>M20</td>
<td>290</td>
<td>1113555</td>
</tr>
<tr>
<td>M24</td>
<td>250</td>
<td>1568777</td>
</tr>
</tbody>
</table>

**Figure X: Key and Set Screw Analysis**
Figure X: Timing Belt and Pulley Analysis
Figure X: Driven Miter Gear Keyway Analysis

Figure X: Driving Miter Gear Keyway Analysis
Figure X: Driven Pulley Keyway Analysis

Figure X: Driving Pulley Keyway Analysis (Representation)
Figure X: Driven Spur Gear Keyway Analysis

Note: Actual wheel utilizes press fit keyway insert, thus failure during stall will result in slip of this insert, rather than the physical failure of a mechanical part.

Figure X: Wheel Keyway Analysis (Representation)

Note: Actual wheel utilizes press fit keyway insert, thus failure during stall will result in slip of this insert, rather than the physical failure of a mechanical part.
Figure X: Yoke Displacement