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MSDI Force Analysis

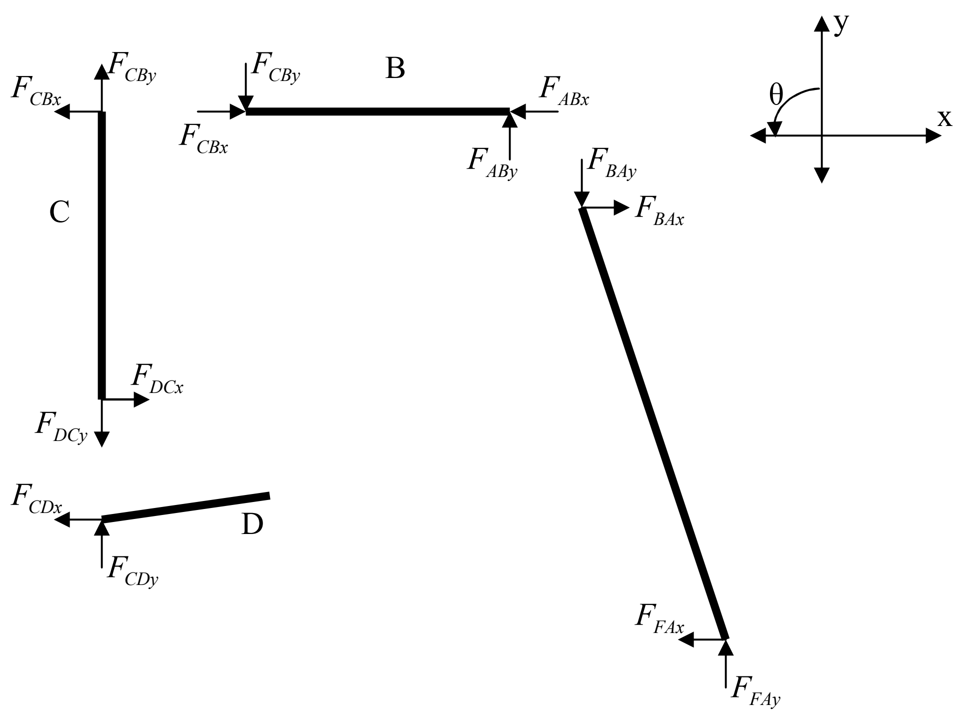
<u>Known:</u> Finger Geometry, Finger Mass/Weight

<u>Find:</u> Tension required in each cable to return fingers to straight position.

<u>Assumption:</u> Frictionless joints/ Cable, Light Cable

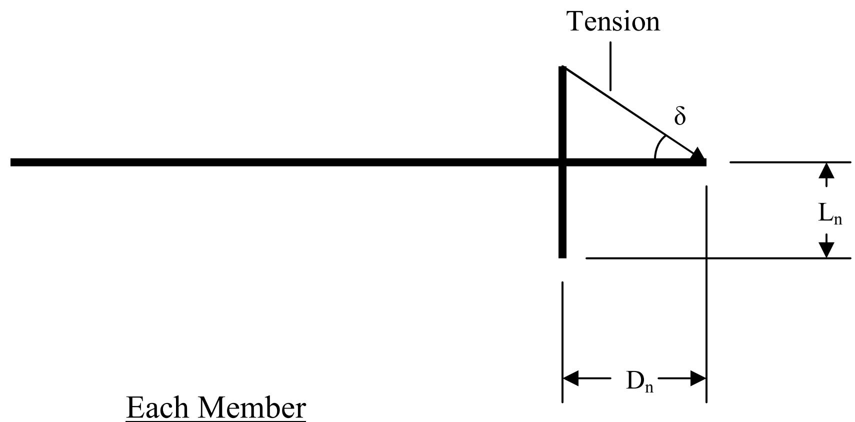
<u>Analysis:</u>

I. Define Forces/Geometry

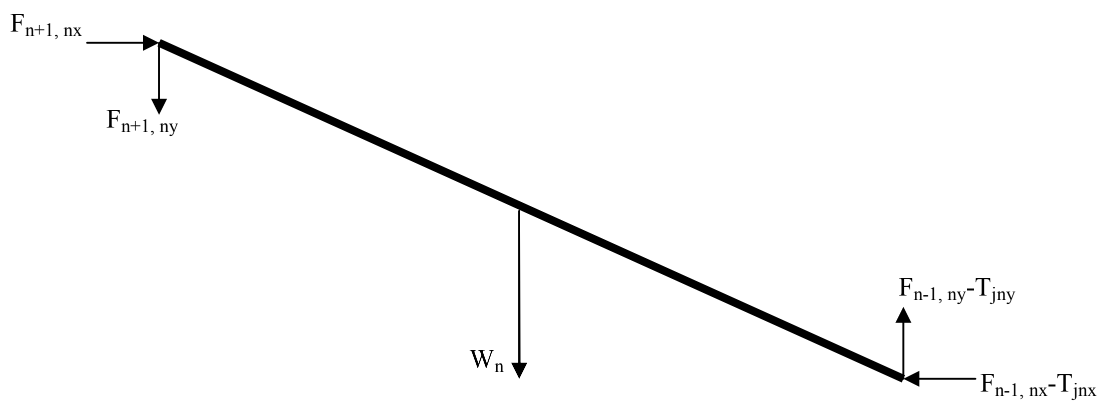


A Metacarpal
B Proximal Phalange
C Middle Phalange
D Distal Phalange

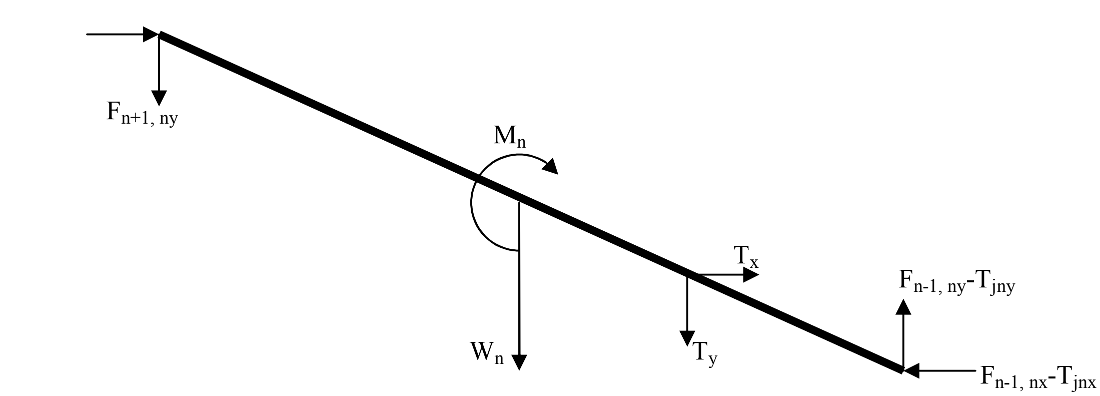
I. Define Forces Cont.



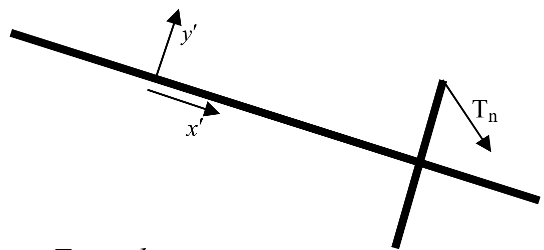
Each Member

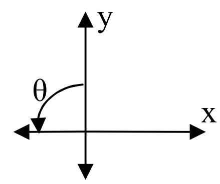


External Forces



Need to define M_n, T_{nx}, T_{ny}





$$T = unknown$$
$$T_{x'} = T\cos(\delta)$$
$$T_{y'} = T\sin(\delta)$$

Transpose x′, y′ to x, y

$$\boxed{\begin{aligned} T_x &= T_{x'}\cos(\theta_n) + T_{y'}\sin(\theta_n) \\ T_y &= T_{x'}\sin(\theta_n) + T_{y'}\cos(\theta_n) \end{aligned}}$$

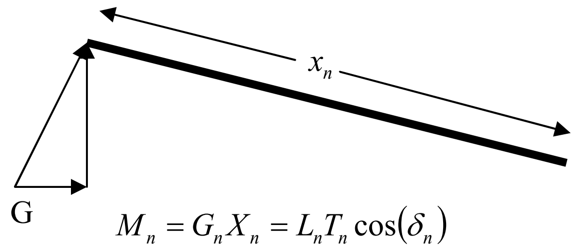
Global Components of Tension

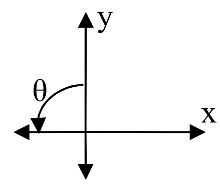
$$T_x = T_n\cos(\delta_n) + \sin(\theta_n) + T_n\sin(\delta_n)\cos(\theta_n)$$
$$T_y = T_n\cos(\delta_n)\cos(\theta_n) + T_n\sin(\delta_n)\sin(\theta_n)$$

Moment

$$M_n = L_n T_n \cos(\delta_n)$$

Change Moment to a Force





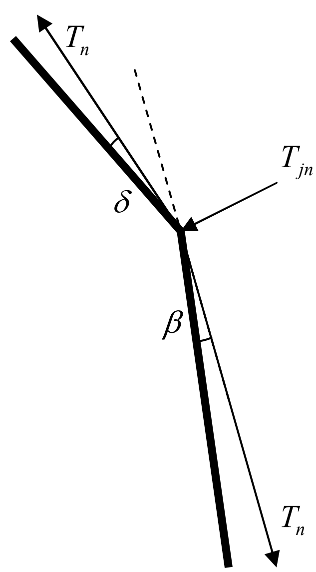
$$M_n = G_n X_n = L_n T_n \cos(\delta_n)$$

$$G_n = \frac{L_n T_n \cos(\delta_n)}{X_n}$$

Change To Global Coordinates

$$G_{ny} = G_n \sin(\theta_n)$$
$$G_{nx} = G_n \cos(\theta_n)$$

Tension Force on Joint



$$T_{jx'} = -T\cos(\delta_n)$$
$$T_{jy'} = -T\sin(\delta_n)$$
$$T_{jx'}\,|_{n-1} = -T\cos(\beta)$$
$$T_{jy'}\,|_{n-1} = -T\sin(\beta)$$
$$\beta = \tan^{-1}(L/x_{n-1})$$

Global Coordinates

$$T_{jx} = -T\cos(\delta_n)\sin(\theta_n) - T\sin(\delta)\cos(\theta_n) - T\cos(\beta)\sin(\theta_{n-1}) - T\sin(\beta)\cos(\theta_{n-1})$$
$$T_{jy} = -T\cos(\delta)\cos(\theta_n) - T\sin(\delta)\sin(\theta_n) - T\cos(\beta)\cos(\theta_{n-1}) - T\sin(\beta)\sin(\theta_{n-1})$$

II. Analyze Forces

See External Force Diagram

$$\sum F_y = 0$$

$$-F_{n+1,ny} + G_{ny} - W_n - T_{ny} + T_{njy} + F_{n-1,ny} = 0$$

$$\boxed{F_{n+1,ny} = G_{ny} - W_n - T_{ny} + T_{njy} + F_{n-1,ny}}$$

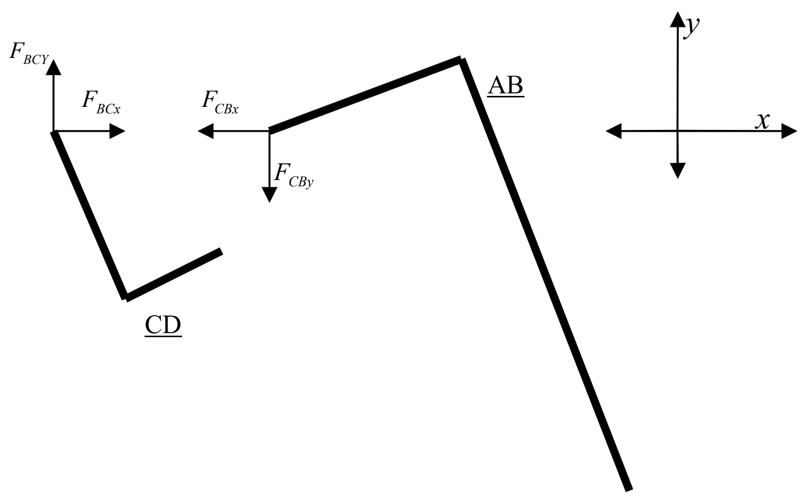
4 equations

4 unknowns

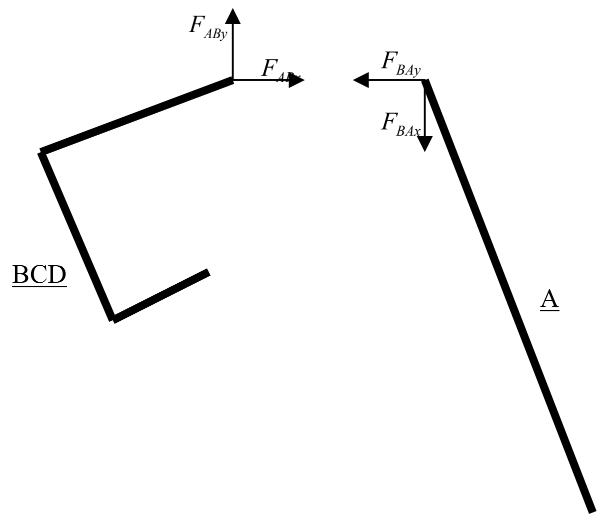
4 unknowns

4 Equations (1 for each member)
8 unknowns (4 Tensions, 4 Reaction Forces)

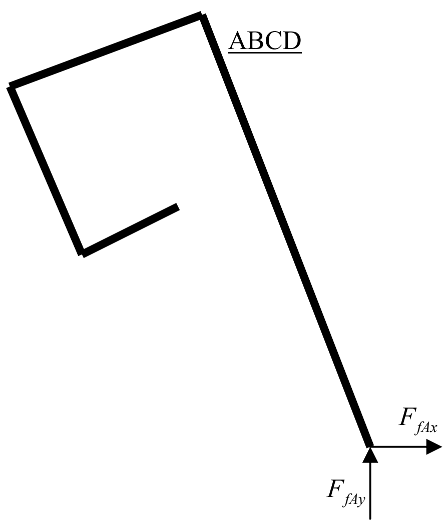
Split Up Finger Differently



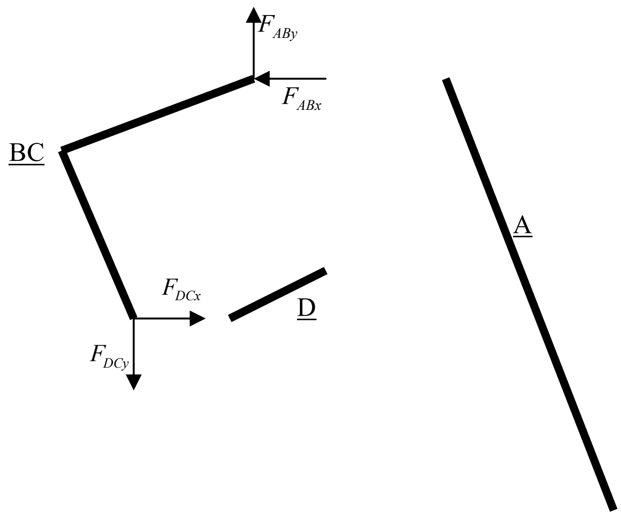
$$\boxed{F_{BCy} = \left(G_{Dy} - W_{Dy} - T_{Dy} - T_{jDy}\right) + \left(G_{Cy} - W_C - T_{yC} + T_{Cjy}\right)}$$



$$\boxed{F_{BA} = \left(G_{Dy} - W_d - T_{Dy} + T_{jDy}\right) + \left(G_{Cy} - W_C - T_{yC} + T_{jCy}\right) + \left(G_{By} - W_B - T_{yB} + T_{jB}\right)}$$



$$\boxed{-F_{fA} = \left(G_{Dy} - W_D - T_{yD} + T_{jD}\right) + \left(G_{Cy} - W_C - T_{yC} + T_{jC}\right) + \left(G_B - W_B - T_{yB} + T_{jB}\right) + \left(G_A - W_A - T_{yA} + T_{jA}\right)}$$



$$F_{DC} - F_{AB} = \left(G_{Cy} - W_C - T_{yC} + T_{jC}\right) + \left(G_{By} - W_{Cy} - T_{yC} + T_{jC}\right)$$

8 Equations
8 Unknowns

III. Solve Equations Simultaneously

$$\begin{Bmatrix} F_{Af} \\ F_{BA} \\ F_{CB} \\ F_{DC} \end{Bmatrix}^T = \left(\begin{Bmatrix} G_A \\ G_B \\ G_C \\ G_D \end{Bmatrix}^T - \begin{Bmatrix} W_A \\ W_B \\ W_C \\ W_D \end{Bmatrix}^T - \begin{Bmatrix} T_A \\ T_B \\ T_C \\ T_D \end{Bmatrix}^T + \begin{Bmatrix} T_{jA} \\ T_{jB} \\ T_{jC} \\ T_{jD} \end{Bmatrix}^T \right) \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ F_{AB} \end{Bmatrix}^T$$

Where

$$\begin{Bmatrix} G_A \\ G_B \\ G_C \\ G_D \end{Bmatrix}_y^T = \begin{Bmatrix} T_A \\ T_B \\ T_C \\ T_D \end{Bmatrix}^T \begin{bmatrix} C_A & 0 & 0 & 0 \\ 0 & C_B & 0 & 0 \\ 0 & 0 & C_C & 0 \\ 0 & 0 & 0 & C_D \end{bmatrix}$$

$$C_n = \frac{L_n \cos(\delta_n)\cos(\theta_n)}{X_n}$$

$$\begin{Bmatrix} T_A \\ T_B \\ T_C \\ T_D \end{Bmatrix}_y^T = \begin{Bmatrix} T_A \\ T_B \\ T_C \\ T_D \end{Bmatrix}^T \begin{bmatrix} H_A & 0 & 0 & 0 \\ 0 & H_B & 0 & 0 \\ 0 & 0 & H_C & 0 \\ 0 & 0 & 0 & H_D \end{bmatrix}$$

$$H_n = \cos(\delta_n)\cos(\theta_n) - \sin(\delta_n)\sin(\theta_n)$$

$$\begin{Bmatrix} T_{jA} \\ T_{jB} \\ T_{jC} \\ T_{jD} \end{Bmatrix}^T = \begin{Bmatrix} T_A \\ T_B \\ T_C \\ T_D \end{Bmatrix} \begin{bmatrix} J_A & 0 & 0 & 0 \\ 0 & J_B & 0 & 0 \\ 0 & 0 & J_C & 0 \\ 0 & 0 & 0 & J_D \end{bmatrix}$$

$$J_n = -\cos(\delta_n)\cos(\theta_n) - \sin(\delta_n)\sin(\theta_n) - \cos(\beta)\cos(\theta_{n-1}) - \sin(\beta)\sin(\theta_{n-1})$$

$$\left\lfloor \left(\underline{T}^T \underline{\underline{K}}\right)\underline{\underline{M}}_1 - \underline{W}^T \underline{\underline{M}}_1 \right\rfloor \underline{\underline{M}}_2^{-1} \underline{\underline{M}}_3 = \underline{T}^T \underline{\underline{K}} - \underline{W}^T$$

$$\underline{T}^T \underline{\underline{K}}\, \underline{\underline{M}}_1 \underline{\underline{M}}_2^{-1} \underline{\underline{M}}_3 - \underline{W}^T \underline{\underline{M}}_1 \underline{\underline{M}}_2 \underline{\underline{M}}_3 = \underline{T}^T \underline{\underline{K}} - \underline{W}^T$$

$$\underline{T}^T \left(\underline{\underline{K}}\, \underline{\underline{M}}_1 \underline{\underline{M}}_2^{-1} \underline{\underline{M}}_3 - \underline{\underline{K}}\right) = \underline{W}^T \left(\underline{\underline{M}}_1 \underline{\underline{M}}_2^{-1} \underline{\underline{M}}_3 - 1\right)$$

$$\boxed{T^T = \left(\underline{W}^T \left(\underline{\underline{M}}_1 \underline{\underline{M}}_2^{-1} \underline{\underline{M}}_3 - 1\right)\right)\left(\underline{\underline{K}}\, \underline{\underline{M}}_1 \underline{\underline{M}}_2^{-1} \underline{\underline{M}}_3 - \underline{\underline{K}}\right)}$$

T = *f* (known Values)

```
function forceanalysis()
%%%%Define Geometry and weight%%%%
%Length from cable guide to first joint
Xf=7;
%Radius of first cable guide measured using metacarpal as radius
Lf=1.5;
%L is the length of the moment arm
La=1;
Lb=1;
Lc=1;
Ld=1;
%D is the Distance from the joint to the disk
Da=1;
Db=1;
Dc=1;
Dd=1;
%X is the full length of the rod
Xa=5;
Xb=3;
Xc=2.5;
Xd=1;
%W is the weight of each member
Wa=9.8;
Wb=9.8;
Wc=9.8;
Wd=9.8;
%theta is the angle of the phalange with respect to the previous phalange
thetaa=0*pi/180;
thetab=thetaa+85*pi/180;
thetac=thetab+120*pi/180;
thetad=thetac+75*pi/180;
theta=[thetaa*180/pi thetab*180/pi thetac*180/pi thetad*180/pi]';
%%%%Define Geometry and weight%%%%

%%%%Define Constants%%%%
%gamma - extreme case
gammaa=atan(La/Da);
gammab=atan(Lb/Db);
gammac=atan(Lc/Dc);
gammad=atan(Ld/Dd);
%Beta
Betaf=atan(Lf/Xf);
Betaa=atan(La/Xa);
Betab=atan(Lb/Xb);
Betac=atan(Lc/Xc);
%Moment Constant
%Need to Come Back to This Section. Free Body Diagrams must be drawn for
%all components. This will change the sign on Many of these constants.
Ca=(La*cos(gammaa)*sin(thetaa))/(Xa);
Cb=(Lb*cos(gammab)*sin(thetab))/(Xb);
Cc=-(Lc*cos(gammac)*sin(thetac))/(Xc);
```

```
Cd=-(Ld*cos(gammad)*sin(thetad))/(Xd);
%Tension Component Constant
Ha=cos(gammaa)*cos(thetaa)+sin(gammaa)*sin(thetaa);
Hb=cos(gammab)*cos(thetab)+sin(gammab)*sin(thetab);
Hc=-cos(gammac)*cos(thetac)-sin(gammac)*sin(thetac);
Hd=-cos(gammad)*cos(thetad)-sin(gammad)*sin(thetad);
%Joint Tension Component Constant
Ja=-cos(gammaa)*cos(thetaa)-sin(gammaa)*sin(thetaa)-cos(Betaf)*cos(thetaa)-sin(Betaf) ↙
*sin(thetaa);
Jb=-cos(gammab)*cos(thetab)-sin(gammab)*sin(thetab)-cos(Betaa)*cos(thetab)-sin(Betaa) ↙
*sin(thetab);
Jc=(-cos(gammac)*cos(thetac)-sin(gammac)*sin(thetac)-cos(Betab)*cos(thetac)-sin(Betab) ↙
*sin(thetac));
Jd=(-cos(gammad)*cos(thetad)-sin(gammad)*sin(thetad)-cos(Betac)*cos(thetad)-sin(Betac) ↙
*sin(thetad));
%Now put them all together
Ka=Ca-Ha+Ja;
Kb=Cb-Hb+Jb;
Kc=Cc-Hc+Jc;
Kd=Cd-Hd+Jd;
%%%%Define Constants%%%%

%%%%Define Matrices and Weight Vector%%%%
K=[Ka 0 0 0; 0 Kb 0 0; 0 0 Kc 0; 0 0 0 Kd];
M1=[1 1 1 1; 0 1 1 1; 0 0 1 1 ; 0 0 0 1];
M2=[1 0 0 0; 0 1 0 0; 0 0 1 0; 0 1 0 1];
M3=[1 -1 0 0; 0 1 -1 0; 0 0 1 -1; 0 0 0 1];
W=[-Wa -Wb Wc Wd];
%%%%Define Matrices%%%%

%%%%Solve Matrix Equation%%%%
T=W*((M1*inv(M2)*M3-1))*inv(K*(M1*inv(M2)*M3-1));
Tensions=T';
%%%%Solve Matrix Equation%%%%

%%%%display the Answer%%%%
fprintf(1,'For Thetas with respect to the global y axis of\r')
disp(theta)
fprintf(1,'The Cable Tensions are\r');
disp(Tensions);
%%%%display the Answer%%%%
return
```