Fluid-Structure Interaction in Bluff-Body Aerodynamics and Long-Span Bridge Design: Phenomena and Methods

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by

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ABSTRACT

The interaction between a fluid flow and an embedded elastic bluff body is extremely complex. Different response modes and flow phenomena exist depending on the flow characteristics, the body geometry and the structural properties like stiffness and damping. This poses a particular challenge to the development of analytical and numerical models and renders experimental methods still the most reliable tool.

This report aims at introducing the various phenomena and at reviewing the most important analytical and numerical methods of analysis. It is largely based on a literature survey which has been carried out in preparation for a PhD degree on numerical methods for fluid-structure interaction analysis in long-span bridge design at the University of Cambridge.
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1 INTRODUCTION

An essential requirement in the design of modern suspension bridges is to assess the influence of the wind forces on the structural response. The need for such analyses arose from the evolution of designs to progressively longer and more slender structures. However, it was indeed not until the catastrophic failure of the first Tacoma Narrows Bridge (Figure 1) in 1940 that the susceptibility of flexible bridges to the dynamic effects of wind action was realised. Only then was revealed, that these violent aerodynamic oscillations could not have been predicted by pseudo-static analyses.

![Figure 1: Tacoma Narrows Bridge failure](image)

Today several different response phenomena in the field of fluid-structure interaction have been identified, largely grouped into response and stability problems. Design experience from recent long-span cable-supported bridges shows, that aerodynamic action can be the determining factor for stiffness requirements on the bridge deck. Even in regions prone to strong earthquakes has this been found to be the case.
This highlights the importance of understanding the phenomena, which determine the interaction between wind and structure and the need for reliable methods of analysis thereof. The mutual influence of structural dynamics and fluid flow in regions of moving boundaries makes this particularly challenging and the corresponding subject is termed aeroelasticity.

Serious studies on this were started in the early 1920's by aeronautical engineers. Although some part of the theory on the subject has gained classical stage, it is still a young science making rapid progress. Applications in structural engineering are numerous and mainly concentrate on tall buildings like towers, masts and skyscrapers and on bridges of flexible nature. The study of the collapse of the Tacoma Narrows Bridge, among them the works of von Kármán [2], [159] and Farquharson [34], who first applied the airfoil theory to bridge decks, introduced aeroelasticity as a new subject in structural engineering.

This report aims at giving an overview of the subject of aeroelasticity by introducing important phenomena as well as methods of analysis. A comprehensive review of recent publications on the topic covering both standard textbooks and journal papers is attempted.
2 HISTORY OF EARLY AERODYNAMICS

Already da Vinci (1452-1519) observed that air offered resistance to the movement of a solid object. He attributed this resistance to compressibility effects. He studied the flight of birds and through this his idea of the feasibility of a flying apparatus took shape and led to exhaustive research into the elements of air and water which also included observations on vorticity. Galilei (1564-1642) later undertook experiments and established the fact of air resistance. He arrived at the conclusion that the resistance was proportional to the velocity of the object passing through it. As a next step, in the late 17th century, Huygens and Newton determined that air resistance to the motion of a body was proportional to the square of the velocity. Newton considered the pressure acting on an inclined plate immersed in an airstream as arising from the impingement of particles on the side of the plate that faces the airstream. His formulation yielded the result that the pressure acting on the plate was proportional to the area of the plate, the product of the density of the air, the square of the velocity, and the square of the sine of the angle of inclination. Although this failed to account for the effects of the flow on the upper surface which causes a considerable lift, Newton’s work clearly marked the beginning of the classical theory of aerodynamics.

During the 18th and 19th century various discoveries led to a better understanding of the factors that have an influence on the movement of solid bodies through air. By the early 1800s the relationship between resistance and the viscous properties of a fluid had been discovered, but it was not until the experiments of Reynolds in the 1880s that the significance of viscous effects was fully appreciated. Parallel to this Strouhal [145] already investigated the vortex shedding process on a circular cylinder and formulated a dimensionless shedding frequency now widely known as Strouhal number, which he found to be constant over a certain range of Reynolds numbers.

The beginning of modern aerodynamics is considered to be at about the time that the Wright brothers made their first powered flight (1903). Only some years later Lanchester [73] and Rayleigh [114] proposed a circulation theory for the calculation of the lift of an airfoil of
infinite span and a vortex theory of the lift of a wing of finite span.

Prandtl, commonly regarded as the father of modern aerodynamics, arrived independently at the same hypotheses as Lanchester. Additionally, he developed the mathematical treatment and his work, refined and expanded by other investigators subsequently, formed the theoretical foundation of the field. He contributed significantly to the subject of airfoil theory which originated from Finsterwalder [37], emerging closely after the first successful flight attempts. Prandtl also advanced experimental methods and his book from 1934 [112] already includes a comprehensive collection of photos of flow phenomena.

In 1911 von Kármán, at this time working in Göttingen upon invitation of Prandtl, made an analysis of the alternating double row of vortices behind a bluff body fluid stream, now famous as von Kármán Vortex Street. He later applied this knowledge to explain the failure of the Tacoma Narrows suspension Bridge [159].

Airfoil theory, boosted by the demands of the first world war, fostered the developments in the field of aerodynamics and by the 1930's a comprehensive theory was established. Important theoretical contributions came from Kutta [71] and Joukowski [68], whose lift formula on the assumption of a lifting vortex is sometimes even regarded as the law of flight [48], and von Mises [160]. Extensive experimental work was carried out by Herrmann [58] and Everling [33].
3 TYPES OF AEROELASTIC PHENOMENA

3.1 Introduction

In this chapter the general principles of aeroelasticity shall be outlined with regard to applications in structural engineering. The main character of the structures is that they are usually not streamlined. The problems thus fall into the subject of bluff body aerodynamics.

The vibration phenomena found in bluff body aerodynamics are numerous and it is fruitful to group them by their origin and major characteristics. One such classification was proposed by Naudascher and Rockwell who distinguish 3 types of flow induced excitation as follows:

EIE: Extraneously induced excitation (e.g. periodic pulsation of oncoming flow);
IIE: Instability-induced excitation (flow instability inherent to the flow created by the structure under consideration), e.g. excitation induced by the von Kármán street;
MIE: Movement-induced excitation (fluid forces arising from the movement of the body or eventually of a fluid oscillator), e.g. galloping.

It should be noted that these can be acting simultaneously.

When a bluff body is embedded in a fluid flow it may cause a wake forming behind the body. In the wake the flow is turbulent but a distinguished pattern of vortices can usually be seen. These vortices are shed from the surfaces of the body and are then carried downstream. It is this shedding of vortices that induces an unsteady force on the body perpendicular to the undisturbed flow direction. The nature of the vortex shedding, particularly the frequency thereof, is determined largely by the geometry of the body, the speed of flow and the density and viscosity of the fluid.
If the body is not rigidly mounted but has a degree of freedom associated with a certain stiffness in the direction of the periodic force it will exercise an oscillation due to its inertia and the forcing action. If the natural frequency is close to the shedding frequency, resonance will occur. Examples of engineering significance are oil pipes, antennas, telephone wires and submarine periscopes which often encounter vibrational troubles of that origin. These vortex-induced vibrations can also be experienced by bridges of a flexible type as will be discussed later on. They may, generally, be overcome by either stiffening the structure to shift the natural frequency away or by increasing the damping.

Another aeroelastic phenomenon observed particularly with slender structures such as cables is called galloping. Under certain conditions particularly related to the cable cross section, these structures experience oscillations in the direction perpendicular to the flow with amplitudes much larger than the cross sectional dimensions. Early fundamental studies on galloping can be found in [44] whilst [77], [101] and [136] give a good overview of the state of the art on the topic. In bridge engineering galloping is an important issue in the design of stay cables. It has often been observed when ovalisation of the cross section due to ice occurred, thus requiring special precautions. Many investigations have been carried out to develop measures to prevent galloping of cables. Recent investigations on the galloping phenomenon in the context of long-span bridge design are described in [87] and [89]. Matsumoto et al. [89] clarified by wavelet analysis that cable-oscillations are not a stationary vibration with a fixed mode but a wave propagation mode. It was also found, that usually more than one mode contributes to the motion. Tests on various cable surface patterns being proposed as favourable, e.g. by Matsumoto [86], showed that, while the cables with dimples and similar patterns only had slightly better characteristics, cables with elliptical plates attached to them performed significantly more favourable. Still the most common method to prevent galloping of stay cables nowadays is to interconnect the cables by means of auxiliary ropes at non-equidistant points, thus efficiently suppressing higher mode oscillations. This was adopted for the Farø and the Normandie cable-stayed bridge as reported in [74] and [158] respectively. Also, various types of damping devices have been applied successfully. These include dashpot dampers between elements with relative movements, e.g. stay-cables and deck as applied to the Brotonne Bridge and the Sunshine Skyway Bridge, and tuned mass dampers fitted to the cables as was used for the long hangers of the Humber suspension
Bridge. Maeda et al. [87] concluded from their investigation on wake galloping response of multi-cable systems which have been used for cable-stayed bridges, that connecting two cables in a close and rigid arrangement can suppress the negative influence of the wake of the upstream cable. Although galloping is a design issue for the cables of long-span bridges, large-amplitude across-wind galloping of bridge decks themselves has not been reported to date [136]. Thus, the phenomenon will not be further discussed herein.

The two phenomena introduced so far involve a separation of the flow from the body and thus cause the periodic excitation. Separation is not necessary for the occurrence of flutter, which is also observed at airfoils which are streamlined such as to avoid flow separation. Investigations on flutter started early during World War II alongside the increase of speed of aircraft because the flow velocity is a determining factor in the occurrence of flutter. It is a self-excited oscillatory instability in which aerodynamic forces act to feed energy into the oscillating structure and progressively increase the amplitude of the motion. This can be thought of as negative damping and occurs at any flow velocity above the critical, which is also referred to as the flutter boundary. This clearly distinguishes it from resonance problems. Flutter is characterised by a harmonic and usually coupled motion in at least two degrees of freedom. While a body associated with only a transverse degree of freedom does not flutter, one degree of freedom torsional flutter is possible under certain angles of attack. Flutter can be suppressed (which means shifting the flutter boundary above the design wind speed) by increasing damping and stiffness where the interaction of the modal frequencies plays an important role.

It should be noted that the types of aeroelastic oscillations illustrated above can occur in a uniform flow without external disturbance. Therefore they are also termed self-excited. However, natural wind is not steady and if oscillations of a structure occur due to velocity fluctuations this is called buffeting. These fluctuations can also be due to the wake of another structure further upstream and this is then termed wake buffeting as investigated in [87].

It is important to appreciate that in stability problems the amplitude of the elastic deformations is indeterminate and only the type of response is of interest. Hence, it is possible to consider the deformations as infinitesimal around the equilibrium state. Therefore
geometrical linearisation is generally possible. In response problems, on the other hand, magnitudes of deformations and stresses are of interest. Since the fundamental equations of both fluid and solid mechanics are nonlinear, any linearisation is confined to certain conditions which needs to be kept in mind.

### 3.2 Vortex-induced vibrations

#### 3.2.1 The vortex shedding process

The process of vortex shedding can only be explained if the effect of viscosity is considered. Only a viscous fluid will satisfy a no-slip condition of its particles on the surface of a body immersed in the flow \[41\]. Even if the viscosity is very small this condition will hold but its influence on the flow regime will be confined to a small region: the boundary layer along the body. Within this boundary layer the velocity of the fluid changes from zero on the surface to the free-stream velocity of the flow as shown in Figure 2 for a flat plate.

![Boundary layer](Figure 2: Boundary layer)

Whilst the free stream is pulling the boundary layer forward the skin friction at the solid wall is retarding it. At surfaces with high curvature there can also be an adverse pressure gradient adding to the retarding action, which may cause the flow to be interrupted entirely and the boundary layer may detach from the wall. This is called separation. As concerns the adverse pressure gradient, streamlined bodies can still experience separation if the angle between free stream and surface is large enough.
It is clear from the physical understanding of the separation process, that viscosity and free stream velocity have an important influence and the can be described by the Reynolds number

\[
\text{Re} = \frac{Ul}{\nu}
\]  

(1)

wherein \( \nu = \mu/\rho \) is called the kinematic viscosity and \( l \) is the characteristic length. The Reynolds number thus expresses the ratio between the inertia force and the friction force acting on the fluid. If, for example, the flow past a circular cylinder is studied, a great variety of changes in the nature of the flow occur with an increasing Reynolds number. The dependence of drag coefficient and pressure distribution on it is shown in Figure 3 and Figure 4.

At a very low Reynolds number, say below 0.5, the inertia effects are negligible and the flow pattern is very similar to that for laminar flow, the pressure recovery being nearly complete. This means, that the pressure drag is also negligible and effective drag on the body is entirely due to skin friction.

At increased Re, approximately between 2 and 30, separation of the boundary layer occurs at two points at the back of the cylinder. There symmetrical eddies are formed which rotate in opposite direction. They remain fixed and the flow closes behind them.

Further increase of the Reynolds number elongates the fixed vortices, which then begin to oscillate until they break away at a Re of around 90. The breaking away occurs alternately from one and the other side and the eddies then travel downstream. This process is intensified with further increase of Re while the shedding of vortices from alternate sides of the cylinder is regular. This leads to the formation of the characteristic wake which is known as vortex street or von Kármán vortex street. The eddying motion is periodic both in space and time. The pressure drag at this stage is already larger than the profile drag. Having passed a transition range where the regularity of shedding decreases, above a Re of 300 vortex shedding is irregular. However, there still is a predominant frequency but the amplitude appears to be random.
At very high levels of Reynolds number from about $3 \cdot 10^5$ the separation point moves rearward on the cylinder. The drag coefficient decreases appreciably. The flow in the wake becomes so turbulent that the vortex street pattern is no longer recognisable.

![Figure 3: Reynolds number dependence of drag coefficient for circular cylinder [1]](image)

Generally, the process of vortex shedding and its dependence on the Reynolds number is highly complex which makes analytical as well as numerical treatment very challenging as will be shown in more detail later. A comprehensive overview of the vortex shedding phenomenon and its different modes has been presented by Zdravkovich [168].

Since the vortex shedding exerts a fluctuating force on the body, which is of particular interest
when the body can be excited to oscillations, Strouhal [145] defined a dimensionless shedding frequency, the Strouhal number, to characterise this process:

\[ \text{St} = \frac{f d}{U} \]  

(2)

in which \( f \) is the shedding frequency and \( d \) the across-flow dimension of the body. For circular cylinders the formula applies to \( 250 < \text{Re} < 2 \cdot 10^5 \).

After Strouhal’s observations subsequent investigations found the Strouhal number to be highly dependent on the cross-sectional geometry of the body and accordingly focussed on determining so called universal Strouhal numbers, which would be independent of the geometry. The most widely used is that proposed by Roshko [116], [117] which is based on his notched hodograph theory:

\[ S_k = \text{St}(D) \frac{1}{K} \left( \frac{D'}{D} \right) \]  

(3)

where \( K = \sqrt{1 - C_{pb}} \) is the velocity along the freestream line relative to that of the uniform oncoming flow and \( D' \) is the lateral distance between the two freestream lines as obtained from Roshko’s notched hodograph theory. It was successfully applied to a circular cylinder and a normal plate. Other universal Strouhal numbers were proposed by Goldburg et al. [45], Bearman [41], Gerrard [42] and Nakaguchi et al. [97] who all used certain geometrical characteristics of the wake and its formation as characteristic lengthscale.

As was stressed by many authors, e.g. Parkinson [106], the most important physical parameter of a two-dimensional body exhibiting vortex-induced oscillations is the size and shape of its afterbody which is the part of the cross-section downstream of the separation points. For vortex-induced or galloping type excitation the pressure loading occurs principally on the afterbody surface. Accordingly, a body with a very short afterbody, e.g. a semicircular cylinder with the flat face downstream, will only be weakly excited. On the contrary, the same cylinder mounted the other way round can experience considerable oscillations under the
same conditions [105]. Figure 5 shows a compilation by Deniz and Staubli [24], which compares results obtained in investigations on the effect of body geometry on the vortex shedding process. The sudden jumps of the Strouhal number occurring at elongation ratios of approximately $L/D=2–3$ and $L/D=4–7$ mark the limits of the three flow regimes as illustrated due to reattachment of the separated flow. A comprehensive set of data regarding the influence of the angle of attack on vortex formation can also be found in [88] and [25].

![Figure 5: Classes of vortex formation observed with increasing elongation of different prismatic bodies: Class I leading-edge vortex shedding; Class II impinging leading edge vortices; Class III trailing-edge vortex shedding [24]](image)

In a recent investigation of the vortex shedding process, Nakamura [98] carried out experiments to compare the different universal Strouhal numbers. He stressed the importance of an afterbody, the presence of which significantly alters the structure of the vortex formation region and seems to render Roshko's universal Strouhal number inapplicable.
3.2.2 Vortex-induced vibrations of structures, the lock-in phenomenon

The Strouhal number $St = f d / U$ describes the process of vortex shedding and depends on the body geometry and the Reynolds number. The frequency of the shedding $f$ is also that of the alternating forces acting transversely to the flow on the body whereas the forces in flow direction have a frequency of $2f$. It should, however, be noted, that this only describes the principal oscillating forces. The time-history actually applied on the body is much more complex with a rich frequency spectrum.

If the structure is elastically mounted the periodic force exerted by the process of vortex shedding gives rise to oscillations. These will also influence the flow pattern and a complex interaction takes place. If the structure is considerably deformable under the pressure forces it will not only act as a rigid body. Ovaling-oscillations have for example been observed at deformable steel shells [67]. Whilst in wind engineering of long-span bridges the oscillations in the cross-wind direction are of major interest, in marine applications along-flow vibrations have been found to be important [70], [166].

It is obvious that considerable excitation of the body only occurs at shedding frequencies close to the natural frequency of the body in across-flow direction. However, it is important to note, that even in the case of resonance the amplitude always remains limited, which was for example shown experimentally in studies of oscillating cylinders [35], [51], [50], [53]. Vortex-induced vibrations are thus a response problem as opposed to flutter being a stability problem. The aim is to predict the frequency of the aerodynamic forces and then to either design the structure for the thus caused oscillations or to make sure the characteristics of the structure are such that it will not be excited. In limit state terminology this type of oscillation can be considered as a serviceability problem because the levels of vibrations need to be limited to ensure comfort of the users and to avoid fatigue problems in the long term.

It had soon been realised by investigators, that the wake behind a bluff body is altered if the body exerts an oscillation. Early studies thus focussed on forced vibration tests to examine the shedding process under these circumstances [52], [55], [72]. The main finding was, that the oscillations alter the vortex pattern in that the spacing between vortices in the wake changes. This type of experiment, however, lacks in modelling the feedback of the flow on the
structure, which needs to be considered for an accurate modelling of the fully coupled flow-induced vibration problem. This was pointed out by Parkinson [106].

Subsequent investigations then studied elastically mounted bodies, mainly cylinders, e.g. [35], [51], [52]. An important phenomenon observed in those occurs at shedding frequencies close to resonance. Here the shedding process becomes controlled by the natural frequency of the structure even if variations in the flow velocity tend to shift it away. This is commonly referred to as lock-in and depicted in Figure 6. Numerous studies have been carried out investigating the vortex shedding and lock-in phenomenon to develop analytical methods of describing the problem. Simiu and Scanlan [136] provide a comprehensive compilation of references on this topic. Recent studies deal with the application of numerical methods (cf. section 5). An experimental response result of oscillations around the lock-in frequency used for calibration is shown in Figure 7.

![Figure 6: Lock-in phenomenon in vortex-induced oscillations [136]](image-url)
While the influence of the frequency of oscillation on the vortex shedding process is well investigated, the influence of the amplitude is less well known. This is surprising as the amplitude seems to have a significant influence on the nature of the shedding. Visualisation of the flow field around a transversally oscillating cylinder by Griffin and Ramberg [52] shows well-organised shedding for an amplitude of $0.5D$ and an oscillation frequency near the natural vortex-shedding frequency. An increase in amplitude to $1.0D$ at the same frequency of oscillation leads to a disorganisation of the wake. This can be thought of as a self-limitation of the vortex-induced excitation as described by Blevins [15] and Billah and Scanlan [13]. The underlying process of the nonlinear interactions which cause the modification of the flow field have been discussed thoroughly by Williamson and Roshko [165].
For further information and comprehensive reviews on vortex-induced vibrations the reader is referred to the publications by Sarpkaya [120], Bearman [9] and Parkinson [106].

3.2.3 Observed vortex-induced oscillations of bridges

Although vortex-induced oscillations should be a well-known phenomenon to bridge designers, cases have been reported where these occurred and retrofitting was needed to suppress them. It is interesting to note, that also the Tacoma Narrows Bridge suffered from these oscillations over several weeks before at heavy storms the response suddenly changed and it then collapsed due to flutter.

In [157] Vincent reports on torsional oscillations observed on the Golden Gate Bridge. These made it necessary to increase the torsional stiffness of the girder by adding lateral bracing between the lower chords of the stiffening truss. Wardlaw [162] reports on wind induced oscillations observed at the Thousand Islands, the Deer Isle, the Fykesund and the Bronx-Whitestone suspension bridges as well as at the Longs Creek and the Kessock cable-stayed bridges. Of all these only at the Longs Creek Bridge a retrofitting scheme of aerodynamic concept was applied [163]. Other measures taken were the application of tuned mass dampers and of various arrangements of additional stay cables.

Only recently vortex-induced vibrations have been observed on the Great Belt East suspension Bridge and reported in [38]. Guide vanes were mounted as a retrofitting scheme to suppress the motions.

3.3 Flutter

3.3.1 The nature of the flutter phenomenon

There are several types of flutter and only the classical flutter shall be treated herein as it is the most common type in bridge engineering. For coverage regarding further flutter classes the reader is referred to textbooks like [14] and [41].
Classical flutter is an aeroelastic phenomenon in which two degrees of freedom of a structure couple in a flow-driven, unstable oscillation. The motion is characterised by that the fluid regime feeds energy into the structure during each cycle thus counteracting the structural damping. If there is no flow in the wind tunnel any oscillation caused by a disturbance will decay due to the present damping. When the speed of flow is gradually increased, the rate of damping of the oscillation first increases. With further increase of flow speed, however, a point is arrived at, from where on the damping decreases again. The point where the effective damping equals zero is referred to as critical flutter condition. Here the oscillation just maintains its amplitude. Above the critical speed the flow causes any small disturbance of the section to grow and to initiate an oscillation of great amplitude.

Flutter analysis is commonly based on the assumption of linear elastic system behaviour [136]. This is justified because the oscillations of the structure are usually harmonic and because at the onset of flutter the amplitude is still limited.

Flutter instabilities are a major criterion in long-span bridge design. This subject has therefore been extensively covered in literature. General information on flutter with application to structural applications can be found in [41], [77] and [136] and. Reference to literature on methods of analysis will be made in subsequent sections of this report.

3.3.2 Observed flutter - the Tacoma Narrows Bridge failure

Certainly the best known case of bridge failure due to wind impact is the Tacoma Narrows suspension bridge, USA (Figure 1), which collapsed on November 7, 1940 only three months after its completion at a wind speed of 19m/s. This chapter will briefly outline the reasons for this disastrous failure which significantly contributed to learning in the field of aeroelasticity.

Some technical drawings of the bridge are shown in Figure 8. The fundamental weakness of the bridge was its extreme flexibility, both vertically and in torsion. The torsional stiffness of the Tacoma Narrows Bridge as compared to other bridges of that time is shown in Figure 9. The bridge’s narrowness, based on economic considerations and transportation studies, made the structure extremely sensitive to torsional motions created by aerodynamic forces. Since its
construction the bridge experienced considerable vertical oscillations and thus several methods were employed to reduce the motions of the bridge during its short life. The first solution involved the attachment of tie-down cables to the plate girders, which were anchoring to fifty-ton concrete blocks on the shore. This measure proved ineffective as the cables snapped shortly after installation. A second measure was to add a pair of inclined cable stays that connected the main cables to the bridge deck at mid-span. These remained in place until the collapse but were also ineffective at reducing the structural vibrations. Finally, the bridge was equipped with hydraulic buffers installed between the towers and the floor system of the deck to damp longitudinal motion of the main span. The effectiveness of the hydraulic dampers was, however, nullified because it was discovered that the seals of the unit were damaged when the bridge was sandblasted prior to being painted [128].

Figure 8: Technical specifications of the Tacoma Narrows Bridge and component drawings [2]
Initial suggestions as to the cause of the collapse came from the commission that was formed by the Federal Works Agency, the members of which included Ammann and von Kármán. Without drawing any definitive conclusions, the commission explored three possible sources of dynamic action:

- aerodynamic instability (negative damping) producing self-induced vibrations in the structure,
- eddy formations which might be periodic in nature and
- the random effects of turbulence, that is, the random fluctuations in velocity and direction of the wind.

Each source was considered separately in seeking the causes of the vertical and torsional oscillations. The commission appeared to have identified the leading possible contributors to the destructive oscillation, since all competing theories which followed to date fit into one of the above categories.

Von Kármán attributed the large amplitude oscillations to a resonance between the natural
frequency of the bridge with the vortex shedding frequency. Although scientifically sound, the lock-on theory proposed by von Kármán did not account for the fact that observations made show that the oscillation frequency of the torsional mode was only around 0.2 Hz, substantially different than the Strouhal frequency of 1 Hz. Thus, it does not seem likely that the power behind the destruction of the bridge can be wholly attributed to the natural vortex shedding of the structure. Even the Federal Works Administration report of the investigation concluded that "It is very improbable that resonance with alternating vortices plays an important role in the oscillations of suspension bridges" [118].

Scanlan's [13] explanation, which is today widely regarded as correct, attribute the behaviour of the bridge to the self-excited flutter phenomenon. The driving force for the oscillations is thus not purely a function of time, but is rather a function of bridge angle during torsional oscillation and the rate of change of that angle. For torsional motion, the behaviour is described mathematically by the relationship:

\[ I \left[ \ddot{\alpha} + 2\zeta \omega \dot{\alpha} + \omega^2 \alpha \right] = F(\alpha, \dot{\alpha}) \]  

(4)

where  

- \( I \)  inertia,
- \( \zeta \)  damping ratio,
- \( \omega \)  natural circular frequency of the system and
- \( \alpha \)  angle of torsional rotation.

The wind generated forces influence the overall damping of the structure, reversing the sign of the middle term in brackets in equation (4), thus producing a response whose solution increases without bound. For the case of the Tacoma Narrows Bridge the unstable torsional mode shown was pushed to destructive amplitude as a result of the interactive, self-excitation phenomenon.

A vast amount of literature is available about the Tacoma Narrows collapse and its explanations. The reader is referred to publications like [34], [46], [104], [109], [110], [118], [135], [142] and [159].
This chapter shall be concluded with a quote by Othmar Ammann, leading bridge designer and member of the Federal Works Agency Commission investigating the collapse of the Tacoma Narrows Bridge [2]:

".... the Tacoma Narrows bridge failure has given us invaluable information .... It has shown [that] every new structure which projects into new fields of magnitude involves new problems for the solution of which neither theory nor practical experience furnish an adequate guide. It is then that we must rely largely on judgment and if, as a result, errors or failures occur, we must accept them as a price for human progress."

4 ANALYTICAL APPROACHES

Analytical methods for describing the various phenomena in bluff-body aeroelasticity still are the most commonly used ones in practical design and they have also been widely adopted in the wind engineering of long-span bridges. As explained earlier, this report mainly focuses on the problems of vortex-induced oscillations and flutter and thus analytical approaches applicable to those will be concentrated on. Herein only well established standard procedures can and shall be described but for further information reference to more specialised literature is made also.

4.1 Vortex-induced vibrations

Attempts to establish analytical methods for determining the response of structures to vortex shedding have so far not been particularly successful. It has, on the other hand, found to be possible to set up empirical models, which can be fitted to reality by means of calibration in terms of a set of parameters.

If the Strouhal relationship (eqn. (2) in section 3.2.1) is considered and a constant amplitude sinusoidal forcing is assumed, an obvious approximation for the across-wind force acting on a bluff body per unit length is

\[ F = \frac{1}{2} \rho U^2 DC_{LS} \sin \omega t \]  

(5)

where \( \omega = 2\pi \text{St}U/D \) is the natural shedding frequency and \( C_{LS} \) is the lift coefficient. However, in real flow conditions it will be found, that the force increases with response of the structure and that a limiting amplitude exists. Considering only the transverse degree of freedom of the structure, its equation of motion can be written as
with \( f \) being the fluid-induced forcing function. Considerable effort has been made to find suitable forcing functions. These have led to an analogy of the wake consisting of alternately shed vortices with a separate oscillator coupled to the structure. These oscillator models emerged in the 1970s and numerous approaches have been published, e.g. [166], [70], [107], [56], [54], [137], [52], [60], [16] and [138]. They basically yield a new system composed of two coupled oscillators whose response can be calculated depending on the oscillator properties and the coupling. Perhaps the most widely accepted model is the one proposed by Van der Pol which is reviewed and compared with other models in [6].

A more simplistic but nonetheless useful forcing function for a single-degree of freedom system has been proposed by Simiu and Scanlan [136]:

\[
\frac{m}{2} \left[ \ddot{y} + 2\zeta \omega_0 \dot{y} + \omega_0^2 y \right] = \frac{1}{2} \rho U^2 D \left[ Y_1(K) \left( 1 - \varepsilon \frac{y^2}{D^2} \right) \frac{\dot{y}}{D} + Y_2(K) \frac{Y}{D} + C_L(K) \sin(\omega t + \phi) \right]
\]  

(7)

where \( D \) is the across-wind dimension of the body, \( K = D \omega_0 / U \) with \( \omega_0 \) as natural shedding frequency as above and \( Y_1, Y_2, \varepsilon \) and \( C_L \) are parameters to be determined from calibration tests. Although this model represents a lot of system characteristics, it is particularly interesting to apply it to the case of lock-in. Here, response is present with \( \omega \cong \omega_1 \) and \( Y_2 \cong 0 \), \( C_L \cong 0 \) because at lock-in the last two terms are found to be small as compared to the first one which represents the aerodynamic damping. If the amplitude of a corresponding steady state is sought, an expression for the energy, which needs to be conserved within the system during one cycle of length \( T \) in this case, can be used:

\[
\int_0^T \left[ 4m \ddot{y} \omega - \rho U Y \left( 1 - \varepsilon \frac{y^2}{D^2} \right) \right] \dot{y}^2 dt = 0.
\]  

(8)
If sinusoidal oscillation is assumed, the corresponding amplitude is found as

$$\frac{y_0}{D} = 2 \left[ \frac{Y_t - 8\pi S_c \text{St}}{\varepsilon Y_t} \right]^{1/2}$$  \hspace{1cm} (9)

where $S_c$ is the Scruton number defined as

$$S_c = \frac{\xi m}{\rho D^2}. \hspace{1cm} (10)$$

This can be used to predict prototype action from the response found at model tests which are used to determine the constants.

For structures with circular cross section, e.g. chimneys and towers, simple relationships resembling reality fairly well have been proposed by Vickery et al. [156] conditional upon two-dimensional flow-conditions. These are presented here to illustrate basic concepts of vortex-induced resonance.

If the forcing term from (7) is considered, it is firstly found, that the term $Y_t(K)y/D$ may in practice be ignored. The term

$$\frac{1}{2} \rho U^2 D Y_t(K) \left( 1 - \varepsilon \frac{y^2}{D^2} \right) \frac{\dot{y}}{D}$$  \hspace{1cm} (11)

is rewritten as

$$2\omega_p \rho D^2 K_{a0} \left( \frac{U}{U_{cr}} \right) \left( 1 - \frac{y^2}{(\lambda D)^2} \right) \dot{y}$$  \hspace{1cm} (12)

where $K_{a0}(U/U_{cr})$ is an aerodynamic coefficient and
\[ U_{cr} = \frac{\omega_l D}{2\pi St}. \] (13)

Since this term obviously has damping characteristics due to dependence of velocity, equating it with the structural damping term \(-2m\ddot{\xi}_s\omega_l\) yields the aerodynamic damping ratio

\[ \ddot{\xi}_a = -\rho D^2 \frac{K_{a0}}{m} \left( \frac{U}{U_{cr}} \right) \left( 1 - \frac{y^2}{(\lambda D)^2} \right). \] (14)

For \( \lambda = \sqrt{\frac{y^2}{D}} \) the damping term vanishes and thus the physical significance of \( \lambda \) is the ratio between the rms value of the aerodynamic response and \( D \). The damping term as shown above can be incorporated in the equation of motion by simply adding it to the structural damping \( \ddot{\xi} \).

Experimental results revealed the existence of flow regimes with distinctly different characteristics. Figure 10 shows the relationship between \( \lambda \) and a parameter \( K_s = m\ddot{\xi}/\rho D^2 \).

The regimes found are in order of decreasing \( K_s \): (1) vibrations forced mainly by the random nature of the vortex shedding process and the thus applied forces, (2) a transition zone with considerable increase of rms response and (3) self-induced vibrations (lock-in regime). Response patterns typical for these regimes are shown in Figure 11.
While the results discussed so far are obtained for smooth flow conditions, presence of turbulence has an influence on the interaction problem. Figure 12 shows the dependence of the aerodynamic coefficient $K_{a0}$ relative to its maximum value in smooth flow $K_{a0\text{max}}$ on $U/U_{c1}$ for different levels of turbulence. Apparently turbulence reduces the aerodynamic
coefficient in the region close to lock-in.

Figure 12: Influence of turbulence level on structural response (after [156])

Turbulence effects generally play an important role in analysis of aeroelastic phenomena although they are hard to incorporate into models. It has repeatedly been found that high levels of turbulence decreased the structural response [127], [122] in much the same way as a loss of span-wise coherence does. This lead to the conclusion, that both influences can be treated in a similar manner.

4.2 Flutter

The flutter phenomenon is the most-widely treated problem in aeroelasticity. This is perhaps due to its violent nature of instability, which makes predicting its occurrence a necessity in aircraft as well as in bridge design.

Analytical flutter analysis for design purposes today is still performed by means of relatively simple analytical methods the most important of which will be presented subsequently. However, as we shall see, some of these methods require information about the aerodynamic properties of the system to be analysed. These can only be obtained from experimental tests or numerical studies as they are dependent on the relatively complex geometry of the bluff body.

Figure 13 shows a cross section associated with two degrees of freedom.
The equations of motion for the translational and torsional degrees of freedom, respectively, can be expressed as follows:

\[ m\ddot{h} + 2m\zeta_\alpha \omega_\alpha \dot{h} + m\omega_\alpha^2 h = F_h(t) \]  
\[ I\dddot{\alpha} + 2I\zeta_\alpha \omega_\alpha \dot{\alpha} + I\omega_\alpha^2 \alpha = F_\alpha(t) \]

The driving forces \( F_h \) and \( F_\alpha \) for heave and pitch are governed by the aerodynamics of the body and thus also by the displacements of the section. Some popular solutions for this coupled system are presented in the next sections.

### 4.2.1 Theodorsen theory

A popular approach for flutter analysis is due to Theodorsen [152], [153], [154] who investigated flutter of aircraft wings. He developed the expressions for the aerodynamic forces on a flat plate. This has the advantage of being independent of the shape of the body, but naturally also neglects all effects that originate from the deviation from flat plate shape. The expressions are as follows:

\[ F_h(t) = -\rho b^2 U\pi \dot{\alpha} - \rho b^2 \pi \ddot{h} - 2\pi \rho C U h - 2\pi \rho C U b \frac{1}{2} \alpha \]  
\[ F_\alpha(t) = -\rho b^2 \frac{1}{2} U b \dot{\alpha} - \rho b^4 \pi \frac{1}{8} \ddot{\alpha} + 2\rho U b^2 \frac{1}{2} C U \alpha + 2\rho U b^3 \frac{1}{2} \pi C h^2 2 \rho \frac{1}{2} U b^3 \frac{1}{2} \pi C \alpha \]

\[ = \frac{1}{I} \left( \rho b^3 \pi \left( -\frac{U \dot{\alpha}}{2} - \frac{b}{8} \dot{\alpha} \right) + \rho U b^2 \pi C \left( \ddot{h} + U \alpha + \frac{b \dot{\alpha}}{2} \right) \right) \]
In the expression for the twisting force the two terms in the brackets correspond to the added mass and the aerodynamic force, respectively. The added mass term takes into account the extra volume of air around the section which participates in the motion and the aerodynamic force is derived as an approximation for small angle of attack.

Theodorsen’s circulation function $C$ is a function of the reduced frequency $k = \omega b / U$ as follows:

$$C(k) = F(k) + iG(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)} = \frac{K_1(ik)}{K_0(ik) + K_1(ik)}$$  \hspace{1cm} (19)$$

where $H$ and $K$ are the Hänkel and modified Bessel functions, respectively, and $b = B/2$. Therefore, the forcing functions generally are complex with, as explained in [31], the imaginary part catering for a lag between body motion and acting forces due to the motion.

For the quasi-static case the Theodorsen function is unity as the frequency is zero. This can be used for an approximate flutter analysis which still yields good results for low frequency motions.

4.2.2 Solution for Theodorsen theory

If we let

$$\dot{h} = v$$  \hspace{1cm} (20)$$

$$\dot{\alpha} = \Omega$$  \hspace{1cm} (21)$$

and introduce the following non-dimensional parameters:

$$h^* = \frac{h}{b}$$

$$\alpha$$
\[ a = \frac{\omega_b}{U} \]
\[ b = \frac{\rho h^2}{m} \]
\[ d = \frac{\omega_{\rho}}{\omega_p} \]
\[ e = \frac{b}{r_m}, \]

and furthermore use non-dimensional time as follows:

\[ t^* = \frac{U}{b} t \]

which yields

\[
\frac{d}{dt} = \frac{U}{b} \left( \frac{d}{dt^*} \right)
\]

and thus

\[ h = bh^* \]
\[ \dot{h} = Uh^* \]
\[ \ddot{h} = \frac{U^2}{b} h^*. \]

then a system of equations as follows is arrived at:

\[ v^* + 2\xi_h a v^* + a^2 h^* = -b\pi \Omega^* - b\pi v^* - 2b\pi C\alpha^* - 2\pi b C v^* - \pi b C \Omega^* \]

(22)

\[ \Omega^* + 2\xi_{\alpha} a \dot{\Omega}^* + a^2 \dot{\alpha}^* = -be^* \frac{\pi}{2} \Omega^* - be^* \frac{\pi}{8} \Omega^2 + be^* \pi C\alpha^* + be^* \pi C v^* + be^* \frac{\pi}{2} \Omega C \]

(23)
The system of equations then is

\[
\begin{pmatrix}
 h^+ \\
 v^+ \\
 \alpha^+ \\
 \Omega^+
\end{pmatrix} =
\begin{pmatrix}
 0 & 1 & 0 & 0 \\
 a_{21} & a_{22} & a_{23} & a_{24} \\
 0 & 0 & 0 & 1 \\
 a_{41} & a_{42} & a_{43} & a_{44}
\end{pmatrix}
\begin{pmatrix}
 h^+ \\
 v^+ \\
 \alpha^+ \\
 \Omega^+
\end{pmatrix}
\]

(24)

with

\[
\begin{align*}
 a_{21} &= -\frac{a^2}{1 + b\pi} \\
 a_{22} &= -2\frac{\xi_a a + b\pi C}{1 + b\pi} \\
 a_{23} &= -2b\pi C \\
 a_{24} &= -b\pi \frac{1 - C}{1 + b\pi} \\
 a_{41} &= 0 \\
 a_{42} &= -\frac{b e^2 \pi C}{1 + b e^2 \frac{\pi}{8}} \\
 a_{43} &= -\frac{a^2 d^2 + b e^2 \pi C}{1 + b e^2 \frac{\pi}{8}} \\
 a_{44} &= \frac{-2\xi_a ad + b e^2 \frac{\pi}{2} (C - 1)}{1 + b e^2 \frac{\pi}{8}}
\end{align*}
\]

Since the system apparently is of the form:

\[
\dot{X} = AX
\]

(25)

the solution for \(h(t)\) and \(\alpha(t)\) is of an exponential form. The Eigenvalues \(\lambda_i\) of the matrix A
characterise the response of the system as follows:

- positive real: increasing response
- negative real: decaying response
- imaginary part: oscillation

This means that as an Eigenvalue with a positive real part occurs, the system is unstable and prone to flutter.

As the imaginary part tends towards zero, the oscillatory part diminishes and the special phenomenon of the so called static divergence is observed. No oscillation occurs but the section experiences a pure heave or pitch motion which can be interpreted as a loss of vertical stiffness.

The flutter boundary is found by successively solving the system for an increasing wind speed \( U \) or a decreasing \( a \), respectively, until an Eigenvalue enters the positive real quadrant in the complex diagram as shown for in Figure 14. Here, flutter occurs before static divergence at a wind speed of 55 m/s.

![Figure 14: Theodorsen solution: paths of Eigenvalues for increasing \( U \)](image-url)
A program for solving the Theodorsen equations was written in Matlab and can be found in Appendix 1 to this report. Figure 15 shows the solutions for the flutter boundaries for various values of $\frac{\rho B^2}{m}$ and a number of levels of structural damping $\xi$.

If a quasi-static solution is sought the problem is straightforward since the Theodorsen function is unity and there is no coupling between the equations. However, for a general solution $C$ depends on the frequency of the deck motion and thus an iteration process is necessary to find the correct interaction solution. For any flutter boundary solution the corresponding frequency needs to be fed back into $C$ and the solution repeated until convergence is achieved between the frequency $\omega_h$ assumed for the motion and the frequency of the oscillation at the flutter boundary $\lambda$ (the Eigenvalue corresponding to Re=0).

![Figure 15: Theodorsen solution: flutter speed relations for Great Belt East Bridge properties (flat plate assumption)](image)

**4.2.3 Scanlan theory**

A popular set of expressions for the aerodynamic forces on a cross section in motion is the one proposed for bridge deck analysis by Scanlan [126], [121] and [136] in the 1970s, which is based on the assumption that the self-excited lift and moment on a bluff body may be
treated as linear in the structural displacement and rotation and their first derivatives as follows:

\[ F_h(t) = \frac{1}{2} \rho U^2 B \left[ KH_1(K) \frac{\dot{h}}{U} + KH_2(K) \frac{B\dot{\alpha}}{U} + K^2 H_3(K) \alpha + K^2 H_4(K) \frac{h}{B} \right] \]  
\[ F_\alpha(t) = \frac{1}{2} \rho U^2 B \left[ KA_1(K) \frac{\dot{h}}{U} + KA_2(K) \frac{B\dot{\alpha}}{U} + K^2 A_3(K) \alpha + K^2 A_4(K) \frac{h}{B} \right] \]

with

\[ h(t) \] vertical cross-wind motion,
\[ \alpha(t) \] section rotation,
\[ B \] chord length
\[ U \] wind velocity
\[ K = \frac{B\omega}{U} \] reduced frequency of motion of bridge

where the nondimensional coefficients \( H_i \) and \( A_i \) are referred to as aerodynamic or flutter derivatives. These are obtained by means of specially designed wind tunnel tests. The frequency of the bridge oscillation under aerodynamic forcing is also referred to as reduced frequency. As the derivatives are a function of this frequency they can only be measured if the bridge is in a sinusoidal oscillatory state.

If a harmonic motion of the bridge is assumed

\[ h = he^{i\omega t}, \]
\[ \alpha = \alpha e^{i\omega t} \]

and if the aerodynamic process is assumed to be linear, the motion induced forces can also be expected to be harmonic with identical frequency \( \omega \) but shifted relative to the motion by \( \varphi \).
Non-dimensionalising the forces and substituting the forcing functions and their derivatives into (26) and (27) yields

$$C_L e^{i(\omega t - \varphi)} = 2K^2 \left[ (iH_1^* + H_4^*) \frac{h}{B} + (iH_2^* + H_3^*) \alpha \right] e^{i\omega t}, \quad (29)$$

$$C_M e^{i(\omega t - \varphi)} = 2K^2 \left[ (iA_1^* + A_4^*) \frac{h}{B} + (iA_2^* + A_3^*) \alpha \right] e^{i\omega t}. \quad (30)$$

From these expressions the derivatives can be extracted,

$$H_1^* = -\left( \frac{U}{fB} \right)^2 \frac{C_L \sin \varphi}{2(2\pi)^3 (h/B)}. \quad (31)$$

$$H_2^* = -\left( \frac{U}{fB} \right)^2 \frac{C_L \sin \varphi}{2(2\pi)^3 \alpha}. \quad (32)$$

$$H_3^* = \left( \frac{U}{fB} \right)^2 \frac{C_L \cos \varphi}{2(2\pi)^3 \alpha}. \quad (33)$$

$$H_4^* = \left( \frac{U}{fB} \right)^2 \frac{C_L \cos \varphi}{2(2\pi)^3 (h/B)}. \quad (34)$$

$$A_1^* = -\left( \frac{U}{fB} \right)^2 \frac{C_M \sin \varphi}{2(2\pi)^3 (h/B)}. \quad (35)$$

$$A_2^* = -\left( \frac{U}{fB} \right)^2 \frac{C_M \sin \varphi}{2(2\pi)^3 \alpha}. \quad (36)$$

$$A_3^* = \left( \frac{U}{fB} \right)^2 \frac{C_M \cos \varphi}{2(2\pi)^3 \alpha}. \quad (37)$$
\[
A_i = \left(\frac{U}{fB}\right)^2 \frac{C_M \cos \varphi}{2(2\pi)^2 (h/B)^2}.
\] (38)

The procedure to calculate aerodynamic derivatives thus is:

- perform forced oscillation tests in either heave or pitch motion,
- calculate a best-fit harmonic of same forcing frequency to obtain lift coefficient and phase shift,
- calculate derivatives.

An application of this procedure can be found in [92], where aerodynamic derivatives were calculated for the Storebølt Bridge deck by means of forced vibration Discrete Vortex analyses.

The solution of the coupled system to evaluate the flutter boundary follows similar ideas to Theodorsen's solution, which was explained in section 4.2.2.

Due to the wide application of Scanlan's theory it has been covered extensively in literature, also in textbooks, e.g. [136] and [77]. Information on experimental studies to evaluate flutter derivatives can be found in [126], [125], [103], [62] (Golden Gate Bridge), [76] (Great Belt East Bridge), [148] (Höga Kusten Bridge), [69] (Akashi-Kaikyo Bridge), [21] (Messina Strait Bridge) and [111]. Aerodynamic derivatives for the Golden Gate Bridge, as obtained in [62] are shown in Figure 16.

Also, numerical analyses have been performed to evaluate aerodynamic derivatives, as reported in [79], [132], [76].
Several alternative formulations for aerodynamic derivatives have been proposed, e.g. in [84], [65] and [124]. In the latter Scanlan et al. also analysed the inter-relations among the flutter derivatives.

### 4.2.4 Selberg equation

The following equation, proposed by Selberg [130], is an empirical prediction of the flutter speed:

\[
\frac{U_{crit}}{f_\alpha B} = 4 \left[ 1 - \frac{f_h}{f_\alpha} \right] \left( \frac{mr_m}{\rho B^2} \right)^{1/2}.
\]  

(39)

It is that quoted in the 'Design Rules for Aerodynamic Effects on Bridges' by the Department of Transport [151].

### 4.3 Multi-modal structural response

So far analytical methods for flutter analysis of section models have been presented. These are essentially associated with 2 degrees of freedom and the corresponding stiffness is assumed
such as to resemble the stiffness of the structure. As the stiffness is associated with a certain
natural period of vibration, this means that the structural response thus predicted is confined
to 2 natural modes, i.e. normally the first vertical bending and torsional modes, respectively.
However, if the true structural response is regarded as superposition of the natural modes, i.e.
assuming linear behaviour, this can be used to predict the multi-modal response of the
structure to the aerodynamic loading.

After first having been proposed by Scanlan and Jones [123] this approach has been widely
adopted, e.g. in [12], [21], [30], [59], [69], [89], [155], [167] and, because often higher mode
contribution was found to be significant.

Let

\[ \zeta_i(t) \] the generalised coordinate,

\[ h_i(t), \alpha_i(t), p_i(t) \] the dimensionless representations of the mode shape,

\[ B \] section width.

Then the structural response is represented as

\[ h(x,t) = \sum_i h_i(x)B\zeta_i, \text{ (vertical)}, \]

\[ \alpha(x,t) = \sum_i \alpha_i(x)B\zeta_i, \text{ (twist)}, \quad (40) \]

\[ p(x,t) = \sum_i p_i(x)B\zeta_i, \text{ (sway)}. \]

The equation of motion of the system thus is

\[ I_i \left[ \dddot{\zeta_i} + 2\xi_i \omega_i \dot{\zeta_i} + \omega_i^2 \zeta_i \right] = Q_i(t) \quad (41) \]

where \( I_i \) is the generalised inertia of the \( i \)th mode, \( \omega_i \) the \( i \)th’ mode natural circular frequency,
\( \zeta_i \) the damping ratio and \( Q_i \) the generalised force.
If lift, drag and moment per unit span due to aeroelastic action are defined as \( L \), \( D \) and \( M \), respectively, the motion is assumed to be sinusoidal, and buffeting terms as used in [62] are neglected, the forcing can be expressed by means of aerodynamic derivatives as introduced in section 4.2.3:

\[
L = \frac{1}{2} \rho U^2 B \left[ KH_1 \frac{\dot{h}}{U} + KH_2 \frac{\dot{\alpha}}{U} + K^2 H_3 \alpha + K^2 H_4 \frac{h}{B} + KH_5 \frac{\ddot{p}}{U} + K^2 H_6 \frac{p}{B} \right],
\]

\[
D = \frac{1}{2} \rho U^2 B \left[ KP_1 \frac{\dot{p}}{U} + KP_2 \frac{\dot{\alpha}}{U} + K^2 P_3 \alpha + K^2 P_4 \frac{p}{B} + KP_5 \frac{\ddot{h}}{U} + K^2 P_6 \frac{h}{B} \right],
\]

\[
M = \frac{1}{2} \rho U^2 B \left[ KA_1 \frac{\dot{h}}{U} + KA_2 \frac{\dot{\alpha}}{U} + K^2 A_3 \alpha + K^2 A_4 \frac{h}{B} + KA_5 \frac{\ddot{p}}{U} + K^2 A_6 \frac{p}{B} \right],
\]

which yield the generalised force \( Q_i \) as

\[
Q_i(t) = \int (Lh_i B + Dp_i B + M \alpha_i) dx.
\]

If the aerodynamic derivatives are assumed constant and the modal integral is defined as

\[
\int q_i(x) r_j(x) \frac{dx}{I} = G_{q_i,r_j}
\]

with \( q_i \) and \( r_j \) being either \( h_i \), \( p_i \) or \( \alpha_i \) respectively, a new system of equations can be expressed in the Fourier-transform domain:

\[
E \zeta = 0
\]
where

\[
E_{ij} = -K^2 \delta_{ij} + iKA_{ij}(K) + B_{ij}(K),
\]
\[
A_{ij} = 2\xi K \delta_{ij} - \frac{\rho B^4 l K}{2I} \begin{bmatrix}
H_1^* G_{h,h} + H_2^* G_{h,\alpha} + H_5^* G_{h,p} + H_6^* G_{p,\alpha} + H_7^* G_{p,p} \\
H_2^* G_{p,h} + H_3^* G_{h,\alpha} + H_6^* G_{\alpha,h} + H_7^* G_{\alpha,p} + H_8^* G_{\alpha,\alpha}
\end{bmatrix},
\]
\[
B_{ij} = K^2 \delta_{ij} - \frac{\rho B^4 l K^2}{2I} \begin{bmatrix}
H_3^* G_{h,h} + H_4^* G_{h,\alpha} + H_5^* G_{h,p} + H_6^* G_{p,\alpha} + H_7^* G_{p,p} + H_8^* G_{p,p} \\
H_4^* G_{p,h} + H_5^* G_{h,\alpha} + H_6^* G_{\alpha,h} + H_7^* G_{\alpha,p} + H_8^* G_{\alpha,\alpha}
\end{bmatrix},
\]

and \( \delta_{ij} \) the Kronecker delta function.

The multi-mode flutter problem is solved by considering the homogeneous system (46) and determining the values of \( K \) and \( \omega \) for which both the real and imaginary parts of the determinant of matrix \( E \) become zero. The critical flutter speed is then obtained from using these values of \( K \) and \( \omega \). Explanations on how to calculate generalised displacements can be found in [62].
5 NUMERICAL APPROACHES

5.1 Introduction

Numerical methods for solving engineering problems have become increasingly popular over the past 10 years or so. This is due to various advantages that they have over the more established approaches like analytical and experimental methods. Only recently the term Computational Wind Engineering (CWE) was introduced. However, these methods are essentially derived from numerical methods commonplace in other fields of fluid mechanics. It should also be noted that fluid mechanics applications of numerical methods have always been among the most important ones contributing significantly to their evolution.

The advantages of numerical methods are particularly clear in Wind Engineering of structures where wind tunnel testing is prohibitively expensive and time-consuming which renders parametric studies almost impossible. It has also been mentioned earlier that scale effects are problematic. Analytical methods, on the other hand, usually fail to capture the full physical characteristics of complex fluid dynamics problems as present in bluff-body aeroelasticity.

The feasibility of computational methods is inevitably linked with the computing power available. Particularly in Computational Fluid Dynamics (CFD) the general solution algorithms need to be supplemented by additional algorithms that allow to account for small-scale processes that cannot be readily solved for because of computational limitations. In terms of application the art of CFD is, according to Denton [26], to construct an appropriate discrete representation of the continuum, whereby this is governed by

- physical behaviour of the continuum equations - conservation; the possible presence of characteristics; whether the problem is initial value or boundary value
- numerical behaviour of the discrete representation - accuracy; stability; false diffusion and convection
The most widely used methods of discretisation of the governing equations to date are:

- Finite Volume Methods,
- Finite Element Methods,
- Finite Difference Methods,
- Boundary Integral Methods.

Each of these methods is a topic of its own and a vast amount of literature is available on the subject of Computational Fluid Dynamics. It is clearly outside the scope of this work to give detailed descriptions of the general procedures. Subsequent sections will therefore only give a very brief introduction to the different approaches and then concentrate on recent applications in Wind Engineering to demonstrate the state of the art on this topic.

5.2 Discretisation Methods

5.2.1 Finite Volume Methods

The Finite Volume Method is based on the discretisation of the Navier-Stokes or Euler-equations in their conservation form. The domain is formulated using a number of finite control volumes. Every volume is contiguous with its neighbouring volumes so that all the flow which leaves one volume enters adjacent volumes. In this way, when a steady state is reached, no flow can be lost. If the flows entering and leaving every volume are not equal then the conditions inside the volume must be changing and the flow is not steady anymore. The same applies to the fluxes of momentum and energy. In a steady state solution the inlet and outlet mass flow rate will be obtained exactly equal and the change of momentum will equal the force exerted on solid boundaries. In this respect the Finite Volume method is superior to other methods with which exact conservation is difficult to achieve. Another advantage is the easy extendibility to three dimensions. Furthermore, unstructured mesh capabilities are usually available in FV codes rendering them highly applicable to problems with complex geometries.
It has been found practical to solve the unsteady equations even if the flow is steady. This originates from supersonic problems where the steady equations change from elliptic to hyperbolic nature whereas the unsteady equations are always hyperbolic. Solution of steady problems is performed by starting from an arbitrary initial guess of the flow field and marching the equations forward in time until the flow becomes steady.

5.2.2 Finite Element Methods

The Finite Element Method has its roots in structural analysis where it was developed from matrix solutions to stress and displacement calculations. The method is based on potential energy considerations of the system employing variational expressions. Few problems in fluid dynamics can be expressed in a variational form, but the Galerkin method is equivalent to the Ritz method for many situations and it is thus the most commonly used formulation in Finite Element methods in fluid mechanics [4].

The Galerkin method approximates the solution in terms of nodal unknowns from which the field is interpolated by means of shape functions. These functions which represent the spatial discretisation are usually chosen from low-order piecewise polynomials restricted to contiguous elements. For the solution the Galerkin method employs weighted residuals whereby their form is usually assumed akin to the shape functions.

5.2.3 Finite Difference Methods

This is the oldest method for numerical solution of partial differential equations, believed to have been introduced by Euler in the 18th century [36].

The conservation equations in differential form are approximated by replacing the partial derivatives by approximations in terms of the nodal values of the functions. This yields an algebraic equation for each grid node in which values of neighbouring nodes appear as unknowns. Taylor series expansions or polynomial fitting is usually used to obtain the derivatives of the functions with respect to the coordinates.
The advantage of this approach obviously is, that the error involved in the discretisation process is readily available. However, Finite Difference Methods have almost vanished completely which is mainly because conservation cannot be enforced and also due to stricter requirements on the grid. Although theoretically possible for unstructured grids, FD methods have only been applied to structured grids.

### 5.2.4 Boundary Element Methods

In common with the better-known Finite Element Method and Finite Difference Method, the boundary element method is essentially a method for solving partial differential equations. The boundary element method is derived through the discretisation of an integral equation that is mathematically equivalent to the original partial differential equation and that relates the boundary solution to the solution at points in the domain. The former is termed a boundary integral equation and in this formulation only the boundary of the domain of interest requires discretisation. Hence the computational advantages of the BEM over other methods can be considerable.

One example is the Discrete Vortex Method, which is derived from the knowledge that in a high-Reynolds number flow there are three distinct regions: the viscous, rotational boundary layer, the wake and an inviscid outer region which is usually irrotational. The idea is to introduce the vorticity at a certain region or point and then to trace it through the flow by using the vorticity equation derived from the Navier-Stokes equation. Good introductions to this method covering the most important aspects are the reports by Lewis [83], Spalart [141], Leonard [82] and Sarpkaya [119].

The main difference to the other methods is that the Discrete Vortex Method is grid-free and thus data input is much facilitated. The computational effort can, however, be immense since all mutual vortex interactions have to be considered at each time step thus making the cost proportional to the square of the number of vortices in the domain.
5.3 Vortex shedding simulations

5.3.1 Introduction

An integral part of calculations in bluff-body aerodynamics is the modelling of the vortex shedding process whose physics have been described in section 3.2.1. Whilst for analytical approaches this is included by means of some empirical model, the aim in computational aerodynamics is to model the fluid flow such that by the global formulation vortex shedding is readily catered for.

Since vortex shedding is such a complex process many studies in the past have concentrated on the shedding from a circular cylinder for which the solution is well established and thus calibration can be done. Some of these studies shall be presented in the next section before moving on to applications with different geometries and in bridge aerodynamics.

5.3.2 Shedding from a circular cylinder

The first numerical studies of the flow past a cylinder were performed by Son and Hanratty [140] as early as 1969. They published solutions for the unsteady two-dimensional Navier-Stokes equations for flow conditions of Re<500. Although these results were very promising and pioneering they suffered from limited computer resources available and comprised very coarse meshes.

Serious improvement could be achieved in the 1980s and a very comprehensive report was then published by Braza et al. [17]. They used the Finite Volume Method to study the vortex shedding from circular cylinders for Re<1000. It is interesting to note that they had to introduce perturbations to trigger the vortex shedding. Lecointe and Piquet [81] applied the Finite Difference scheme to steady and unsteady incoming flow. Both contributions give detailed evaluation of the results and comparison with experiments.

Tamura [146] carried out extensive numerical investigations on the vortex shedding from bluff bodies of various shapes. Also forced and self-induced oscillations have been simulated.
Different flow regimes in terms of Reynolds number have been considered and a comprehensive set of results is presented including vorticity distributions and time-histories of lift and drag. However, only few information are given as to what technique was used for the computations. No comments are made regarding modelling aspects.

Frandsen [38] performed calibration studies for the Spectrum FE code on the vortex shedding from a circular cylinder using a mesh provided by Oksta et al. [102]. Different Reynolds number flows were considered. Two-dimensional calculations were performed using elements of hexahedral shape and the boundary layer was modelled by a layer 5 cells thick. In this, the thickness of the boundary layer was approximated using the formula by Zdravkovich [169]

$$\delta = \frac{D_{\text{circ}}}{\sqrt{Re}}$$

although the mesh was kept unaltered for changed Reynolds numbers. No turbulence model was used. A stable flow regime could be established after a time corresponding to the flow crossing the domain twice. Good results were achieved, which reproduced the main physical processes described in section 3.2.1. Results for Reynolds numbers of 200 and 1000 are compared with the results from other recent studies in Table 1. Whilst Mendes and Bronco used a Finite Element formulation with no turbulence model applied, the results by Walther were obtained with the Discrete Vortex Method and Franke et al. applied the Finite Volume Method without turbulence model.

Figure 17: Computational mesh for vortex shedding analysis, Frandsen [38]
Figure 18: Computational mesh for vortex shedding analysis, Franke et al. [38]

| Source                  | $\Delta t$ | $c_D$ | $|c_L|$ | St |
|-------------------------|------------|-------|--------|----|
| Frandsen [38]           | 0.05       | 1.433 | 0.620  | 0.195 |
| Mendes and Bronco [90]  | -          | 1.399 | 0.726  | 0.202 |
| Walther [161]           | -          | 1.319 | 0.7     | 0.19 |
| Franke et al. [40]      | 0.05       | 1.31  | 0.650  | 0.194 |

| Source                  | $\Delta t$ | $c_D$ | $|c_L|$ | St |
|-------------------------|------------|-------|--------|----|
| Frandsen [38]           | 0.1        | 1.52  | 1.320  | 0.221 |
| Behr et al. [11]        | 0.05       | 1.52  | 1.458  | 0.241 |
| Shakib [134]            | 0.1        | -     | -      | 0.217 |
| Franke et al. [40]      | 0.012      | 1.47  | 1.36   | 0.236 |

Table 1: Results for circular cylinders at Re of 200 and 1000 (after [38])

Frandsen also performed sample calculations with Re=$10^6$ and the same mesh. The results were found to be unsatisfactory because the mesh was too coarse for such high Reynolds number. This stresses the relationship between suitable mesh density, time step and Reynolds
Incompressible Finite Volume calculations applying a pressure correction method extended to hybrid grids have been performed by Schulz and Kallinderis [129]. The pressure correction formulation stores primitive variables in a non-staggered fashion and artificial dissipation was introduced into the momentum equations to suppress oscillatory solutions. They successfully simulated vortex shedding from a single and double circular cylinder arrangement and comparison with experimental data is given.

Anagnostopoulos [3] simulated laminar vortex shedding behind a circular cylinder at a Reynolds number of 106 using the Finite Element technique. The report deals extensively with flow visualisation techniques and superposition of different visualisation patterns. These could be applied to vortex strength calculations confirming that the highest strength occurs at the end of the formation region, a fact which has recently been show experimentally by Green and Gerrard [49]. A vorticity balance technique was used to compare the circulation influx into the wake with the vortex strengths. Furthermore, pressure distributions and vortex velocities in the wake were investigated.

Vortex shedding from a circular cylinder was also used by Dawes [22] as a test case for his Finite Volume code NEWT. This program has been successfully applied to various problems in turbomachinery, e.g. [23]. The solver is based on an adaptive method incorporating capabilities of solution-dependent refinement and derefinement of the unstructured mesh. This enables the solution to be adapted to the local time scales of the unsteady flow thus capturing its significant processes. Figure 19 shows a normal and a solution-adapted mesh for a vortex shedding simulation. It is obvious that the adaptive meshing enables more economical solutions by using a suitable mesh size at any point in the domain.
Vortex shedding from a rotationally oscillating circular cylinder in a uniform flow has been studied by Lu and Sato [85]. A range of Reynolds number flow regimes has been considered and the rotation amplitude $\alpha = \omega R/U$ and frequency $f_e/f_o$ with $f_o$ being the vortex shedding frequency have been varied. Basic modes of vortex shedding could be identified which are in good agreement with available experimental data. One such mode is shown in Figure 20. It is interesting to note that the vortex shedding process can be either governed by the incoming flow or the rotation of the cylinder. If lock-in to the rotational oscillation occurs, this leads to a distinctly different wake pattern. A diagram of these modes is shown in Figure 21. In addition, the influence of the oscillation frequency and amplitude on the forces acting on the cylinder has been investigated. An incompressible Finite Element formulation was used for this study.
The Discrete Vortex Method has also been applied to the vortex shedding from circular cylinders. The fundamental work by Lewis [83] studies both applications of vortex cloud modelling and methods with fixed separation points. Also methods of catering for viscous effects are discussed. He and Su [57] presented results for the circular cylinder using an approach similar to Lewis’. They mainly describe the effects of different modelling techniques on the pressure distribution on the bluff body. The vortex-in-cell method was used by Zhou et al. [171] for modelling of shedding from a cylinder in forced oscillatory motion. This method, first proposed by Christiansen [20], projects the vorticity from the vortices onto a fixed grid.
Numerical approaches

on which vorticity is thus discretised. Then the velocity field can be calculated through the use of the Poisson equation for the stream function. Thus the computationally expensive Biot-Savart law for the mutual vortex interactions is bypassed. This method has been very successful in predicting the force, the vortex pattern and the vortex-shedding frequency for the two-dimensional viscous flow around a rigid structure, e.g. [29], [91], [139], [143] and [170]. Zhou et al. [171] investigated pressure fluctuations for different ratios of forced oscillatory frequency and shedding frequency $f_n/f_*^*$. Figure 22 shows various wake patterns obtained.

![Wake patterns for various $f_n/f_*^*$ ratios](image)

**Figure 22:** Wake patterns for various $f_n/f_*^*$ ratios [171]

Based on the Finite Element Method a program for arbitrarily moving boundaries was developed by the group of Tezduyar and presented in [149]. Its main applications are unsteady flows with complex interfaces, fluid-structure interaction, airdrop systems and contaminant dispersion. At each time step the locations of the boundaries and interfaces are determined as part of the overall solution where an interface-tracking method is employed.
Various examples including free-surface flow past a circular cylinder are presented in [149].

### 5.3.3 Shedding from sharp edged bodies, applications in bridge aerodynamics

Even though the circular cylinder case is of interest for calibration and test purposes particularly because of the high Reynolds number dependence of the solution, it is directly applicable only to few civil engineering structures like towers and chimneys. Furthermore, the flow around bluff bodies with sharp edges shows much higher pressure gradients and thus poses a different challenge on modelling techniques.

With the exception of the Messina Strait Crossing for which a deck of elliptical shape is envisaged, the box girders of modern long-span bridges are usually of hexahedral shape. This makes investigations on rectangular cylinders more relevant and some of these shall be presented subsequently.

A Finite Element code for two-dimensional incompressible viscous flow based on an adaptive h-version mesh refinement-recovery method was developed by Choi and Yu [19]. In this, elements with a variable number of midside nodes are used to form the transition zone between the refined and unrefined mesh. The method is described in [19] and a vortex shedding example for a rectangular body is given. Interestingly, it was found to be necessary to initiate the vortex shedding by some disturbances. Good results could, however, be obtained by the method.

In his paper [146] mentioned before, Tamura also presented results for rectangular cylinders and bluff plates. For the bluff plate two distinct modes of vortex formation have been identified as shown in Figure 23 by the vorticity contours and in Figure 24 by time histories for lift and drag involving both modes. In the case of a weak roll-up of vortices from the body both lift and drag are significantly lower than for a strong roll-up.
Large Eddy Simulations were carried out by Rodi [115]. He compares LES and Reynolds-averaged Navier-Stokes (RANS) calculations of vortex shedding flow past a square cylinder at Re=22000 to study their feasibility and computational cost. It is stressed that the complex flow phenomena involved in bluff body aerodynamics like separation and reattachment, unsteady vortex shedding, high turbulence, large-scale turbulent structures and curved shear layers require the adequate modelling of turbulence effects in numerical simulations. Statistical models have difficulties especially when large-scale eddy structures dominate the turbulent transport, when unsteady processes like vortex shedding and bistable behaviour prevail or when dynamic loading or spatial influences like curvature is of importance. Here,
the LES approach is more suitable as it captures the large-scale unsteady motions and requires modelling only of the small-scale unresolvable turbulent motion. In his study Rodi compares results from a variety of LES and RANS methods for the square cylinder case with experimental results. As for the RANS models the $k-\varepsilon$ model yielded poor results suffering from an overproduction of turbulence in the stagnation region, a problem which can be resolved by the Kato-Launder modification. Further improvement could be obtained when the model was combined with a two-layer model resolving the near-wall region. Also, the Reynolds-stress model predicted reasonable results. The LES models applied generally lead to better results, particularly in capturing the turbulence fluctuations. However, the results are not uniformly good and the computational cost was approximately 900% higher than for the best RANS method.

The applicability of Large Eddy Simulation on the vortex shedding from square cylinders has also been investigated by Murakami et al. first for 2-dimensional [93] and later for 3-dimensional [95] simulations of fixed bodies. Papers [96], [94] report on 3-dimensional modelling of oscillating bodies. Reynolds numbers for the simulations were in the order of 20000. Generally it was found [95] that 3-dimensional analysis shows the best agreement with experimental results by Bearman et al. [10] and Nakamura et al. [99]. For forced as well as for free oscillation analyses the lock-in phenomenon could be simulated. In [94] different kinds of LES models are presented and their advantages are discussed with respect to the computational effort involved.

LES calculations have also been performed by Selvam et al. [132], [131], [133] and Tamura et al. [147] using FE and FD approaches, respectively. In both cases 2D simulations delivered poor results as opposed to good results using 3D formulations. It is interesting to note that Selvam’s model needed only 2% of the nodes of Tamura’s model, which is explained by a more accurate modelling of the convection terms by the FE method.

Comprehensive studies on the application of Finite Element modelling to long-span bridge aerodynamics have been carried out by Frandsen [38], [39]. The commercial code Spectrum by Ansys Inc. [5] was used employing an integrated formulation capable of solving the fluid and structural response simultaneously. Initial investigations on the vortex shedding process
Numerical approaches were performed on a stationary 2-dimensional model of the Great Belt East suspension bridge. Laminar flow was assumed throughout due to computational limitations. Figure 25 shows the process of extracting aerodynamic properties from the calculations and obtained results as compared with results from other investigations are shown in Table 2.

![Figure 25: Vortex shedding analysis for Great Belt East Bridge section [38]](image)

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta t$</th>
<th>$c_D$</th>
<th>$c_L$</th>
<th>St</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frandsen [38] (FE)</td>
<td>0.01</td>
<td>0.42</td>
<td>-0.21</td>
<td>0.25</td>
</tr>
<tr>
<td>Jenssen and Kvelmsdal [66] (FV)</td>
<td>0.025</td>
<td>0.45</td>
<td>0.04</td>
<td>0.16</td>
</tr>
<tr>
<td>Enevoldsen et al. [32] (FV)</td>
<td>0.51</td>
<td>0.08</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>Frandsen [38] (DVM)</td>
<td>0.097</td>
<td>0.57</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>DMI and SINTEF [27] (section model)</td>
<td>0.54</td>
<td>0.01</td>
<td>0.11-0.15</td>
<td></td>
</tr>
<tr>
<td>Larose [75] (Taut strip model 1:300)</td>
<td>0.72</td>
<td>-0.08</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Frandsen [38] (full-scale)</td>
<td></td>
<td></td>
<td></td>
<td>0.08-0.11</td>
</tr>
<tr>
<td>Frandsen [38] (section model)</td>
<td></td>
<td></td>
<td></td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 2: Results for Great Belt East Bridge aerodynamic simulations (after [38])

Vortex induced oscillations could not be modelled successfully although different mesh
densities were examined. Nevertheless, unsteady aeroelastic simulations were performed which are discussed in section 5.4 of this report.

The Discrete Vortex Method has also been applied to vortex shedding from sharp edged bodies. Steggel and Rockliff [144] investigated rectangular cylinders of different aspect ratios and at different Reynolds numbers using a vortex-in-cell (VIC) method as proposed by Graham [47]. The Strouhal number dependence on the aspect ratio already mentioned in section 3.2.1 could be modelled and Reynolds number dependence of the processes is discussed as well.

Larsen and Walther [79], [76] reported on flow simulations of bridge cross sections using the DVM code DVMFLOW by Walther [161]. Various cross sectional shapes were considered and aerodynamic properties like lift and drag coefficients and Strouhal number calculated (cf. Figure 26). By applying forced oscillations, aerodynamic derivatives were determined and compared with experimental results by Scanlan and Tomko [126]. Good agreement was achieved but since only few information regarding the method are given, the results are difficult to judge.
5.4 Aeroelastic simulations

In their study [129] mentioned earlier, Schulz and Kallinderis used an incompressible FV program for deformable hybrid grids to study the fluid-structure interaction for circular cylinders. A loosely coupled interaction procedure was used and vortex-induced oscillations studied. The lock-in phenomenon could be modelled and the limited amplitude characteristics predicted. Also, two elastically mounted tandem cylinders were considered and their interaction simulated. The computational model has been validated by comparisons with experimental data, but the paper supplies few results.
Murakami et al. [96] also applied their code featuring an LES model to free oscillations of square cylinders. Lock-in was simulated, but at least for the range of results presented, no limit in amplitude is observed. No comments have been made as to whether the 3-D LES model worked favourably as compared to a 2-D formulation.

In her studies using the FE code Spectrum, Frandsen [38] used the fully coupled fluid-structure formulation to investigate both vortex-induced vibrations and flutter for the Great Belt East Bridge deck. Difficulties arose in modelling the former, which was deemed to be due to lack in mesh density. However, lock-in could be modelled with a surprisingly coarse mesh, when the free stream velocity corresponded to the resonance case as predicted by the Strouhal number of the fluid-only model. The influence of guide-vanes as applied to the real bridge as a retrofit measure to suppress vortex induced vibrations was simulated. The results are somewhat ambiguous but the shedding pattern agreed with smoke visualisation tests. Furthermore, flutter analyses were performed. These were successful in that they predicted a flutter boundary close to results from other investigators. Also the shape of the flutter mode was similar to that predicted by the flat-plate Theodorsen solution. The simulations were performed by starting off with very high structural damping values in order for the solution to settle in. The damping was then decreased to the correct value and observation as to whether the response grows or decays indicates whether flutter occurs. Interestingly, these calculations were done using a mesh which had not even developed a wake for vortex shedding analyses at lower flow velocities. This seems to show, that numerical locking of the flow at the trailing edge can be overcome by displacements of the bluff body which then act as perturbation.

The DVMFLOW code by Larsen and Walther [79], [76] has been applied by Frandsen [38] to compare with the FE results obtained for the Great Belt Bridge. Vortex induced oscillations at a flow velocity of 8m/s are predicted by the DVM with considerably smaller amplitudes which are, however, closer to full-scale measurements. Also the Strouhal number of 0.09 is much smaller than other numerical predictions (cf. Table 2) but compares well with full-scale measurements. Flutter simulations could also be performed successfully predicting a flutter speed of 65m/s. However, this is different from the result obtained by Larsen and Walther [78] with the same program.
Larsen and Walther [76], [80] also applied their code DVMFLOW to simulate fluid-structure interaction problems. Mainly predictions of flutter were undertaken on various bridge deck shapes like Tacoma Narrows and a proposed section for the Gibraltar Straits crossing. Reasonable results could be reported. The discrete vortex code by Taylor and Vezza [150] was applied to study the flutter behaviour of Storebølt Bridge. Good agreement with wind tunnel tests was established and also the influence of an active control mechanism was simulated successfully.

In [171] Zhou et al. applied the DVM in its hybrid Vortex-In-Cell formulation to the fluid-structure interaction of a circular cylinder. The cylinder was assumed mounted as a spring-damper-mass system associated with 2 perpendicular translational degrees of freedom. The interaction process is studied in great detail and vortex patterns before and at lock-in are discussed. Transversal vibrational amplitudes up to 0.57D are found and when the amplitude of the cylinder reaches its maximum value, the wake undergoes significant changes in the vortex spacing. Furthermore it is shown, that the vibration level not only depends strongly on the reduced damping, but also on the mass ratio $M^*$. 

For their studies on oscillations of deep water riser pipes, Graham and his co-workers [43], [108], [164] developed a novel technique for quasi three-dimensional modelling based on the DVM approach. The method, termed Vortex Lattice Method, introduces a mesh for the coupling of 2-D vortex simulations in the region of the wake. This makes use of a Finite Element solution for the unstructured grid, which combines the streamfunction-vorticity and the velocity-vorticity methods. This quasi-3D approach is considerably faster than applying all 3-dimensional vortex-vortex interactions and it is also argued, that the grid based solution if of higher order and thus more accurate. Multi-mode vortex-induced vibrations could be successfully simulated.

Preidikmann and Mook [113] used the DVM to design a passive damping system for long-span bridges and to proof its efficiency. The system uses light-weight airfoils mounted under the bridge and connects them to each other as well as to the bridge deck through a series of springs and dampers. The flow past the deck and the wings was modelled using predefined fixed separation points where the vortices are shed. Validation of the model was done by
comparison with results for flat plates by Fung [41] and prediction of the damping characteristics of the wings was successful. However, it seems as if applications to genuine bridge decks would require a more accurate modelling of the vortex shedding from the deck.

A DVM model using fixed separation points has also been applied by Jadic et al. [61] who studied flow-induced vibrations of airfoils. Flutter was predicted putting particular emphasis on the nonlinear structural response when limit cycles occur. Also, changes in the wake pattern were revealed at the flutter speed where a definite curvature could be found.
6 SUMMARY

This report gave an overview of the phenomena involved in bluff body aerodynamics and fluid-structure interaction. Furthermore an account of established methods and recent work on aerodynamic and aeroelastic analysis was given to outline the state of the art on the subject.

A vast number of publications are available on the subject, which highlights the importance that aeroelastic analysis has today. Reference was made to specialised literature, both textbooks and journal papers.
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