**Fundamental Equations of Dynamics**

**KINEMATICS**

**Particle Rectilinear Motion**

<table>
<thead>
<tr>
<th>Variable $a$</th>
<th>Constant $a = a_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = \frac{dv}{dt}$</td>
<td>$v = v_0 + a_d t$</td>
</tr>
<tr>
<td>$v = \frac{ds}{dt}$</td>
<td>$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$</td>
</tr>
<tr>
<td>$a ds = v dv$</td>
<td>$v^2 = v_0^2 + 2a_c (s - s_0)$</td>
</tr>
</tbody>
</table>

**Particle Curvilinear Motion**

<table>
<thead>
<tr>
<th>$x, y, z$ Coordinates</th>
<th>$r, \theta, z$ Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_x = \dot{x}$</td>
<td>$a_x = \ddot{x}$</td>
</tr>
<tr>
<td>$a_x = \ddot{x}$</td>
<td>$a_r = \ddot{r} - r \dot{\theta}^2$</td>
</tr>
<tr>
<td>$v_y = \dot{y}$</td>
<td>$a_y = \ddot{y}$</td>
</tr>
<tr>
<td>$v_\theta = \dot{\theta}$</td>
<td>$a_\theta = \ddot{\theta} + 2\dot{r} \dot{\theta}$</td>
</tr>
<tr>
<td>$v_z = \dot{z}$</td>
<td>$a_z = \ddot{z}$</td>
</tr>
</tbody>
</table>

**n, t, b Coordinates**

$v = \dot{s}$

$\dot{a}_t = \ddot{v} = r \frac{dv}{ds}$

$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{[d^2y/dx^2]}$

**Relative Motion**

$v_B = v_A + v_{B/A}$

$a_B = a_A + a_{B/A}$

**Rigid Body Motion About a Fixed Axis**

<table>
<thead>
<tr>
<th>Variable $\alpha$</th>
<th>Constant $\alpha = \alpha_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = \frac{d\omega}{dt}$</td>
<td>$\omega = \omega_0 + \alpha_c t$</td>
</tr>
<tr>
<td>$\omega = \frac{d\theta}{dt}$</td>
<td>$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c \theta_0^2$</td>
</tr>
<tr>
<td>$\omega d\alpha = \alpha d\theta$</td>
<td>$\omega = \omega_0^2 + 2 \alpha_c (\theta - \theta_0)$</td>
</tr>
</tbody>
</table>

**For Point P**

$s = \delta r$

$v = \omega r$

$\dot{a}_r = \ddot{r}$

$\dot{a}_w = \alpha \ddot{r}$

<table>
<thead>
<tr>
<th>Relative General Plane Motion—Translating Axes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_B = v_A + v_{B/A}(pie)$</td>
</tr>
</tbody>
</table>

**Relative General Plane Motion—Trans. and Rot. Axis**

$v_B = v_A + \Omega \times r_{B/A} + (v_{B/A})_{xyz}$

$a_B = a_A + \Omega \times r_{B/A} + \Omega \times (\Omega \times r_{B/A}) + 2\Omega \times (v_{B/A})_{xyz} + (a_{B/A})_{xyz}$

**KINETICS**

<table>
<thead>
<tr>
<th>Mass Moment of Inertia $I = \int r^2 dm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel-Axis Theorem $I = I_G + md^2$</td>
</tr>
<tr>
<td>Radius of Gyration $k = \frac{I}{\sqrt{m}}$</td>
</tr>
</tbody>
</table>

**Equations of Motion**

<table>
<thead>
<tr>
<th>Particle</th>
<th>$\Sigma F = ma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid Body</td>
<td>$\Sigma F_x = m(a_x)_x$</td>
</tr>
<tr>
<td>(Plane Motion)</td>
<td>$\Sigma F_y = m(a_y)_y$</td>
</tr>
<tr>
<td></td>
<td>$\Sigma M_G = I_G \omega$ or $\Sigma M_P = \Sigma (M_k)<em>{P</em>{ij}}$</td>
</tr>
</tbody>
</table>

**Principle of Work and Energy**

$T_1 + U_{1-2} = T_2$

| Kinetic Energy | $T = \frac{1}{2} mv^2$ |
| Rigid Body (Plane Motion) | $T = \frac{1}{2} mv_0^2 + \frac{1}{2} I_G \omega^2$ |
| Work | $U_F = \int F \cos \theta \ ds$ |
| Variable force | $U_F = (F_s \cos \theta) \Delta s$ |
| Constant force | $U_F = W \Delta y$ |
| Weight | $U_e = -\left(\frac{1}{2} k_s x^2 - \frac{1}{2} k_s x^2 \right)$ |
| Spring | $U_e = M \Delta \theta$ |

**Power and Efficiency**

$P = \frac{dU}{dt} = F \cdot \nu$

$\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$

**Conservation of Energy Theorem**

$T_1 + V_1 = T_2 + V_2$

**Potential Energy**

$V = V_s + V_e$, where $V_s = \pm Wy$, $V_e = \mp \frac{1}{2} k_s x^2$

**Principle of Linear Impulse and Momentum**

<table>
<thead>
<tr>
<th>Particle</th>
<th>$mv_1 + \Sigma \int F \ dt = mv_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid Body</td>
<td>$m(v_G)_1 + \Sigma \int F \ dt = m(v_G)_2$</td>
</tr>
</tbody>
</table>

**Conservation of Linear Momentum**

$\Sigma (syst. \ mv)_1 = \Sigma (syst. \ mv)_2$

**Coefficient of Restitution**

$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

**Principle of Angular Impulse and Momentum**

<table>
<thead>
<tr>
<th>Particle</th>
<th>$(\mathbf{H}_0)_1 + \Sigma \int \mathbf{M}_d \ dt = (\mathbf{H}_0)_2$</th>
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<tbody>
<tr>
<td>$H_0 = (d)(mv)$</td>
<td>$H_0 = I_G \omega$</td>
</tr>
<tr>
<td>Rigid Body (Plane motion)</td>
<td>$(\mathbf{H}_0)_1 + \Sigma \int \mathbf{M}_d \ dt = (\mathbf{H}_0)_2$</td>
</tr>
<tr>
<td>$H_0 = I_G \omega$</td>
<td>$H_0 = I_G \omega$</td>
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**Conservation of Angular Momentum**

$\Sigma (syst. \mathbf{H})_1 = \Sigma (syst. \mathbf{H})_2$