P11412 UV Water Treatment Team

Flywheel Design Analysis

Comparison of Design Concepts

Current System Preliminary Analysis

Proposed System Preliminary Analysis

Initial 2nd Order System Model of Mechanical System

Gearing Analysis

Detailed 1st Order System Model of Mechanical System

Detailed 1st Order System Model of Mechanical System

General 1st Order System Model

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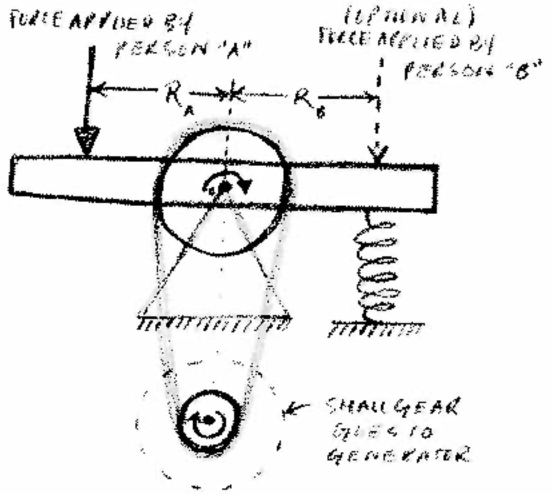
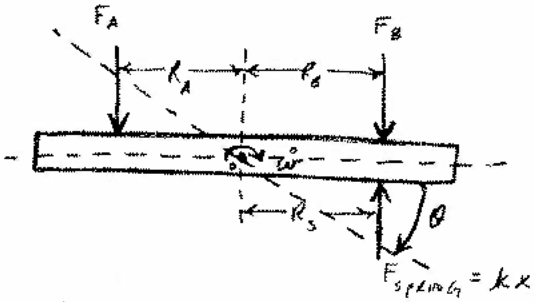
PROPOSED DESIGNS

A, "SEE - SAW" IDEA





$\Sigma M_{o} = F_A R_A - F_B R_B + F_S R_S = -I\alpha$

ASSUME $F_A = m_A g$

$F_B = m_B g$

$\alpha \neq 0$

$F_S = kx$, x = spring compression

RECTANGULAR CROSS SECTION

$\Sigma M_{o} = F_A R_A - F_B R_B + F_S R_S = -I\alpha$

$m_A g R_A - m_B g R_B + kx R_S = -\frac{1}{12} b h^2 \frac{d}{dt}(\omega)$

x = SPRING COMPRESSION

$\tan(\theta) = \frac{x}{R_S} \rightarrow x = R_S \tan(\theta)$

$m_B g R_B - m_A g R_A - k R_S \tan(\theta) R_S = \frac{1}{12} b h^3 \frac{d}{dt}(\omega)$

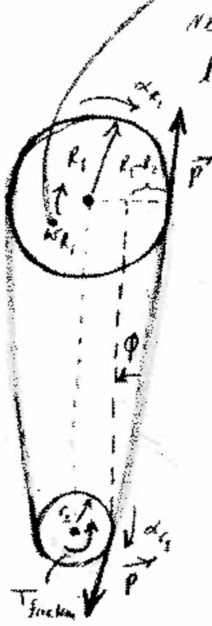
$\Rightarrow \frac{d}{dt}(\omega) = 12 \left\{ \frac{m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta)}{b h^3} \right\}$ } ASSUME F_A, F_B, F_S are all constant / θ = constant

↳ INTEGRATE WITH RESPECT TO "t"

$$\boxed{\omega = \frac{12}{bh^3} \left\{ m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta) \right\} \cdot t}$$

angular velocity caused purely by $F_A, F_B, \& F_S$

NEED TO CONSIDER FRICTION INDUCED BY GENERATOR



look at small gear, r_2

$\Sigma M_{o_{r_2}} = -P r_2 + \underbrace{T_{fric}}_{\text{PRODUCED BY GENERATOR}} = -I_{r_2} \alpha_{r_2}$

$-P r_2 + \frac{P_{gen}}{2\pi f_{gen}} = -\frac{1}{2} m_{r_2} r_2^2 \alpha_{r_2} \Rightarrow P = \frac{P_{gen}}{2\pi r_2 f_{gen}} + \frac{1}{2} m_{r_2} r_2 \alpha_{r_2}$

$\Sigma M_{o_{R_1}} = \underset{\uparrow \text{ SUBSTITUTE FOR } P}{P} R_1 - M_{\theta_1} = -I_{R_1} \alpha_{R_1}$

$\left\{ \frac{P_{gen}}{2\pi r_2 f_{gen}} + \frac{1}{2} m_{r_2} r_2 \alpha_{r_2} \right\} R_1 - \left\{ m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta) \right\} = -\frac{1}{2} m_{R_1} R_1^2 \alpha_{R_1}$

$\frac{P_{gen}}{2\pi f_{gen}} \left(\frac{R_1}{r_2} \right) + \frac{1}{2} m_{r_2} R_1 r_2 \alpha_{r_2} - m_B g R_B + m_A g R_A + k R_S^2 \tan(\theta) = -\frac{1}{2} m_{R_1} R_1^2 \alpha_{R_1}$

$\left. \begin{array}{l} \frac{d}{dt}(\theta_1 R_1 = \theta_2 r_2) \\ \omega_1 R_1 = \omega_2 r_2 \\ \alpha_1 R_1 = \alpha_2 r_2 \end{array} \right\} \therefore \alpha_2 = \left(\frac{R_1}{r_2} \right) \alpha_1 = (GR)\alpha_1, \; GR = \frac{R_1}{r_2}$

A_1 "SEE-SAW" IDEA

$$\frac{P_{gen}}{2\pi f_{gen}}\left(\frac{R_1}{r_2}\right) + \frac{1}{2} m_{r_2} R_1 r_2 \alpha_{r_2} - m_B g R_B + m_A g R_A + k R_S^2 \tan(\theta) = -\frac{1}{2} m_{r_1} R_1^2 \alpha_{r_1}$$

SUBSTITUTE $\alpha_2 = \left(\frac{R_1}{r_2}\right)\alpha_1$

$$-\frac{P_{gen}}{2\pi f_{gen}}\left(\frac{R_1}{r_2}\right) - \frac{1}{2} m_{r_2} R_1 r_2 \left(\frac{R_1}{\cancel{r_2}}\right)\alpha_{r_1} + m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta) = \frac{1}{2} m_{r_1} R_1^2 \alpha_{r_1}$$

$$\frac{-P_{gen}}{2\pi f_{gen}}\left(\frac{R_1}{r_2}\right) - \frac{1}{2} m_{r_2} R_1^2 \alpha_{r_1} + m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta) = \frac{1}{2} m_{r_1} R_1^2 \alpha_{r_1}$$

$$\frac{-P_{gen}}{\cancel{2}\pi f_{gen}} \cdot \frac{\cancel{R_1}}{r_2} \cdot \frac{\cancel{2}}{m_{r_1} R_1^{\cancel{2}}} - \cancel{\tfrac{1}{2}} m_{r_2} \cancel{R_1^2} \alpha_{r_1} \cdot \frac{\cancel{2}}{m_{r_1} \cancel{R_1^2}} + m_B g R_B \cdot \frac{2}{m_{r_1} R_1^2} - m_A g R_A \cdot \frac{2}{m_{r_1} R_1^2} - k R_S^2 \tan(\theta) \cdot \frac{2}{m_{r_1} R_1^2} = \alpha_{r_1}$$

$$\frac{-P_{gen}}{R_1 \pi m_{r_1} r_2 f_{gen}} - \frac{m_{r_2}\alpha_{r_1}}{m_{r_1}} + \frac{2}{m_{r_1} R_1^2}\left\{m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta)\right\} = \alpha_{r_1}$$

$$\frac{-P_{gen}}{R_1 \pi m_{r_1} r_2 f_{gen}} + \frac{2}{m_{r_1} R_1^2}\left\{m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta)\right\} = \underbrace{\left\{1 + \frac{m_{r_2}}{m_{r_1}}\right\}}_{\frac{m_{r_1}+m_{r_2}}{m_{r_1}}} \alpha_1$$

$$\frac{-\cancel{m_{r_1}}}{m_{r_1}+m_{r_2}} \cdot \frac{P_{gen}}{R_1 \pi \cancel{m_{r_1}} r_2 f_{gen}} + \frac{\cancel{m_{r_1}}}{m_{r_1}+m_{r_2}} \cdot \frac{2}{\cancel{m_{r_1}} R_1^2}\left\{m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta)\right\} = \alpha_1$$

$$\boxed{-\frac{P_{gen}}{R_1 \pi r_2 f_{gen}(m_{r_1}+m_{r_2})} + \frac{2}{R_1^2(m_{r_1}+m_{r_2})}\left\{m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta)\right\} = \alpha_1}$$

ANGULAR ACCELERATION OF INPUT SHAFT / GEAR 1

$\frac{d\omega}{dt} = \alpha \rightarrow$ ASSUME θ = constant

$$\boxed{\omega_1 = \left\{\frac{2}{R_1^2(m_{r_1}+m_{r_2})}\left\{m_B g R_B - m_A g R_A - k R_S^2 \tan(\theta)\right\} - \frac{P_{gen}}{\pi R_1 r_2 f_{gen}(m_{r_1}+m_{r_2})}\right\} t}$$

ANGULAR VELOCITY OF INPUT SHAFT / GEAR 1

ASSUME $R_A = R_B = R \rightarrow$ SOLVE FOR R

$$\omega_1 = \frac{2}{R_1^2(m_{r_1}+m_{r_2})} R (m_B g - m_A g) t - \frac{2}{R_1^2(m_{r_1}+m_{r_2})} k R_S^2 \tan(\theta) t - \frac{P_{gen}}{R_1 \pi r_2 f_{gen}(m_{r_1}+m_{r_2})} t$$

$$\frac{2}{R_1^2(m_{r_1}+m_{r_2})} (m_B g - m_A g) t \cdot R = \omega_1 + \frac{2}{R_1^2(m_{r_1}+m_{r_2})} k R_S^2 \tan(\theta) t + \frac{P_{gen}}{R_1 \pi r_2 f_{gen}(m_{r_1}+m_{r_2})} t$$

$$R = \frac{R_1^2(m_{r_1}+m_{r_2})}{2(m_B g - m_A g) t} \cdot \omega_1 + \frac{\cancel{R_1^2(m_{r_1}+m_{r_2})}}{\cancel{2}(m_B g - m_A g)\cancel{t}} \cdot \frac{\cancel{2}}{\cancel{R_1^2(m_{r_1}+m_{r_2})}} k R_S^2 \underbrace{\tan(\theta)}_{\frac{x}{R_S}} \cancel{t} + \frac{R_1^{\cancel{2}}\cancel{(m_{r_1}+m_{r_2})}}{2(m_B g - m_A g)\cancel{t}} \cdot \frac{P_{gen}}{\cancel{R_1}\pi r_2 f_{gen}\cancel{(m_{r_1}+m_{r_2})}} \cancel{t}$$

$$\boxed{R = \frac{R_1^2(m_{r_1}+m_{r_2})}{2(m_B g - m_A g)} \cdot \frac{\omega_1}{t} + \frac{k R_S x}{(m_B g - m_A g)} + \frac{R_1 P_{gen}}{2(m_B g - m_A g)\pi r_2 f_{gen}}}$$

CHECK UNITS: $\frac{m^2 \cdot kg}{N} \cdot \frac{rad/s}{s} = \frac{m^2 \cdot kg}{kg \cdot m/s^2} \cdot \frac{1}{s^2} = m^2 \cdot \frac{s^2}{m} \cdot \frac{1}{s^2} = m$ ✓ $\quad \frac{\frac{N}{m} \cdot m^2}{N} = m$ ✓ $\quad \frac{m \cdot \frac{Nm}{s}}{N \cdot \frac{1}{s}} = \frac{Nm^2}{s} \cdot \frac{s}{N} = m$ ✓

A, "SEE-SAW" IDEA

USE AN ITERATIVE METHOD TO DETERMINE RADIUS, R
ASSUME THE FOLLOWING VALUES

$m_{g_1} \equiv$ mass of large gear $= 10\,lb / 32.2\,ft/s^2 = 0.31055\,slug$
$m_{g_2} \equiv$ mass of little gear $= 1\,lb / 32.2\,ft/s^2 = .0310559\,slug$
$R_1 \equiv$ radius of large gear $= 1\,ft$
$r_2 \equiv$ radius of little gear $= 0.1\,ft$
} 1:10 Gear Ratio

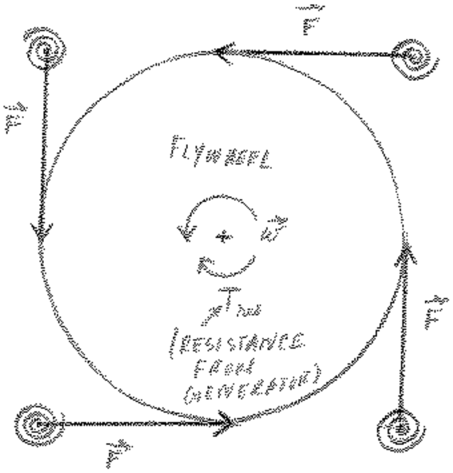
$k \equiv$ spring constant $= 240\,lb/ft$
$R_s \equiv$ Radius to spring $= 1\,ft$
$m_A g \equiv$ Weight of person "A" $= 50\,lb$
$m_B g \equiv$ Weight of person "B" $= 90\,lb$
} assume a 40 lb force differential applied by users

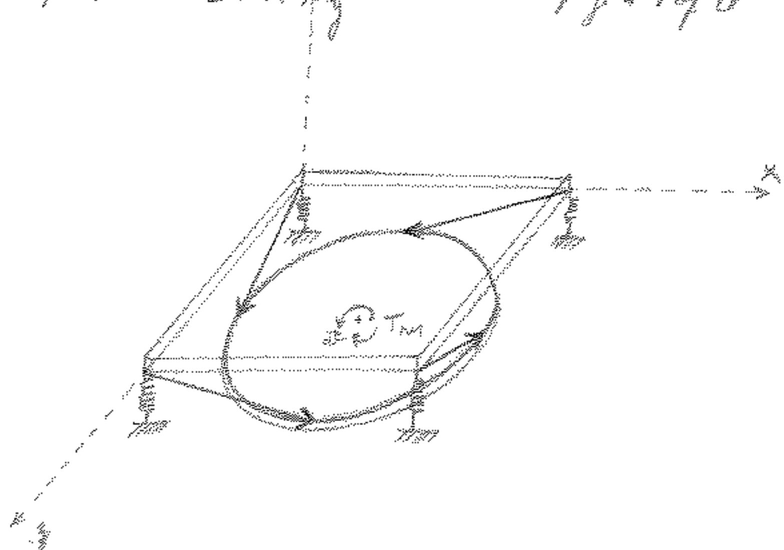
$\omega_1 \equiv$ angular velocity of large gear $= 2\pi(5\,Hz) = 31.4159\,\frac{1}{s}$
$x \equiv$ spring compression length $= 4\,in = 0.333\,ft$
$f_{gen} \equiv$ frequency of the generator $= 50\,Hz$
$P_{gen} \equiv$ power output of the generator $= 1\,kW = 12.5886\,\frac{lb \cdot ft}{s}$
$t \equiv$ time to reach $\omega_1 = 10\,s$

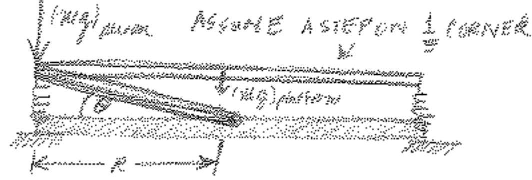
$$R = \underbrace{\frac{(1ft)^2(0.31055\,slug + .031055\,slug)(31.4159\,\tfrac{1}{s})}{2(90\,lb - 50\,lb)(10\,s)}}_{.013419\,ft} + \overbrace{\frac{(240\,lb/ft)(1ft)(0.333\,ft)}{(90\,lb - 50\,lb)}}^{1.998\,ft} + \underbrace{\frac{(1ft)(12.5886\,lb \cdot ft/s)}{2(90\,lb - 50\,lb)\pi(0.1ft)(50\,\tfrac{1}{s})}}_{.009978\,ft} = 2.02319\,ft$$

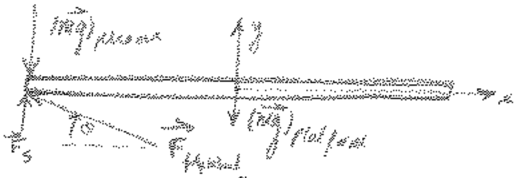
$\boxed{R = 2.0\,ft}$ $\therefore$ PLATFORM LENGTH $= \underline{4\,ft}$

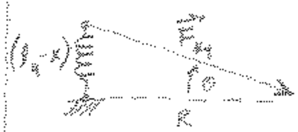
B, "SPRING-FLOOR" IDEA



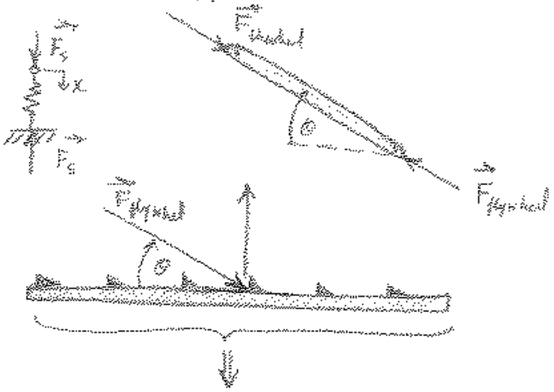








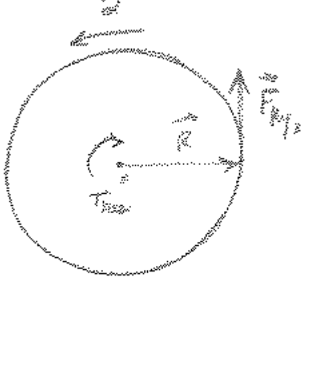
$$\left\{ \begin{array}{l} F_{fly_y} = -F_{fly}\sin(\theta) \\ F_{fly_x} = -F_{fly}\cos(\theta) \end{array} \right.$$



$$\Sigma F_y = -m_{hum}g - m_{plat}g + F_s + F_{fly}\sin(\theta) = 0$$

$$F_{fly} = \frac{m_{hum}g + m_{plat}g - kx}{\sin(\theta)} = \frac{m_{tot}g - kx}{\sin(\theta)}$$

$$\therefore F_{fly_x} = -\frac{m_{tot}g - kx}{\tan(\theta)}$$ DRIVING FORCE ON FLYWHEEL $$\Longrightarrow F_{fly_x} = \frac{-m_{tot}g + kx}{\frac{(l_u - x)}{R}} = \frac{(-m_{tot}g + kx)R}{(l_u - x)}$$



$$\Sigma M_o = RF_{fly_x} - T_{res} = I\alpha$$

$$R \cdot \frac{(-m_{tot}g + kx)R}{(l_u - x)} - \frac{P_{gen}}{2\pi f_{gen}} = \frac{1}{2} m_{fly} R^2 \alpha$$

$$\frac{(-m_{tot}g + kx)}{(l_u - x)}R^2 - \frac{P_{gen}}{2\pi f_{gen}} = \frac{1}{2} m_{fly} R^2 \alpha$$

$$\boxed{\frac{2(-m_{tot}g + kx)}{(l_u - x)m_{fly}} - \frac{P_{gen}}{\pi m_{fly} R^2 f_{gen}} = \alpha}$$ ANGULAR ACCELERATION OF FLYWHEEL

$$d\omega = \left(\frac{2(-m_{tot}g + kx)}{(l_u - x)m_{fly}} - \frac{P_{gen}}{\pi m_{fly} R^2 f_{gen}} \right) dt$$ } assume P_{gen}, f_{gen}, m_{tot}, m_{fly}, R, k, x are all constants

$$\boxed{\omega = \left(\frac{2(-m_{tot}g + kx)}{(l_u - x)m_{fly}} - \frac{P_{gen}}{\pi m_{fly} R^2 f_{gen}} \right) t}$$ ANGULAR VELOCITY OF FLYWHEEL

B, "SPRING - FLOOR" IDEA

$$\omega_{fly} = \frac{2(-m_{tot}g + kx)}{m_{fly}(l_n - x)}t - \frac{P_{gen}}{\pi m_{fly} R^2 f_{gen}}t$$

SOLVE FOR $\underline{\underline{k}}$

$$\omega_{fly} = -\frac{2 m_{tot} g}{m_{fly}(l_n - x)}t + \frac{2kx}{m_{fly}(l_n - x)}t - \frac{P_{gen}}{\pi m_{fly} R^2 f_{gen}}t$$

$$\frac{2kx}{m_{fly}(l_n - x)}t = \omega_{fly} + \frac{2 m_{tot} g}{m_{fly}(l_n - x)}t + \frac{P_{gen}}{\pi m_{fly} R^2 f_{gen}}t$$

$$k = \frac{m_{fly}(l_n - x)}{2xt}\omega_{fly} + \frac{\cancel{m_{fly}(l_n - x)}}{\cancel{2}x\cancel{t}}\cdot\frac{\cancel{2}m_{tot}g\,\cancel{t}}{\cancel{m_{fly}(l_n - x)}} + \frac{\cancel{m_{fly}}(l_n - x)}{2x\cancel{t}}\cdot\frac{P_{gen}\,\cancel{t}}{\pi \cancel{m_{fly}} R^2 f_{gen}}$$

$$\boxed{k = \left(\frac{l_n - x}{x}\right)\frac{m_{fly}\,\omega_{fly}}{2t} + \frac{m_{tot}g}{x} + \left(\frac{l_n - x}{x}\right)\frac{P_{gen}}{2\pi R^2 f_{gen}}}$$

SPRING CONSTANT NECESSARY FOR $\alpha \neq 0$ and GEARING $\omega_{fly} \neq \omega_{gen}$

Check units:

$$\left[\frac{N}{m}\right] = \left[\frac{m}{m}\cdot\frac{kg\cdot rad/s}{s}\right] + \left[\frac{N}{m}\right] + \left[\frac{m}{m}\cdot\frac{W}{m^2\cdot 1/s}\right]$$

$$= \left[\frac{kg\cdot rad}{s^2}\right] + \left[\frac{N}{m}\right] + \left[\frac{N\cdot m}{s}\cdot\frac{s}{m^2}\right]$$

$$= \left[\frac{N\cdot s^2\,rad}{m}\cdot\frac{1}{s^2}\right] + \left[\frac{N}{m}\right] + \left[\frac{N}{m}\right]$$

$$= \left[\frac{N}{m}\right] \checkmark$$

USE AN ITERATIVE METHOD TO DETERMINE REALISTIC K-VALUES
ASSUME THE FOLLOWING VALUES:

$l_n \equiv$ natural length $= 4$ in $= 0.3333$ ft
$x \equiv$ compression length $= 1$ in $= .08333$ ft
$R \equiv$ Radius of Flywheel $= 1.5$ ft
$m_{fly} \equiv$ mass of flywheel $= 20$ lb$/32.2$ ft/s² $= 0.62112$ slug
$m_{tot}g \equiv$ mass of person + platform $= 150$ lb
$P_{gen} \equiv$ Power of Generator $= 17$ W $= 12.5386$ $\frac{lb\cdot ft}{s}$
$f_{gen} \equiv$ Frequency of the Generator $= 50$ Hz
$\omega_{fly} \equiv$ angular velocity of the flywheel $= 2\pi(5$ Hz$) = 31.41592$ $\frac{rad}{s}$ } 1:10 GEARING
$t \equiv$ time to reach $\omega_{fly} = 5$ sec

$$k = \underbrace{\left(\frac{.3333\,ft}{.08333\,ft} - 1\right)\frac{(0.6211\,slug)(31.41592\,\frac{rad}{s})}{2(5\,s)}}_{5.73522\ \frac{lb}{ft}} + \underbrace{\frac{150\,lb}{.08333\,ft}}_{1800\ \frac{lb}{ft}} + \underbrace{\left(\frac{.3333\,ft}{.08333\,ft} - 1\right)\frac{12.5386\,\frac{lb\cdot ft}{s}}{2\pi(1.5\,ft)^2(50\,\frac{1}{s})}}_{.053216\ \frac{lb}{ft}} \cong 1805\ \frac{lb}{ft}$$

$\underline{\underline{k = 150\ \frac{lb}{in}}}$ TRUE SINCE INPUT $m_{tot}g = 150$ lb, $x = 1$ in ✓

$\therefore$ $\underline{\underline{\text{DESIGN IS POSSIBLE}}}$

COST ANALYSIS (PRELIMINARY)

IDEA A "SEE-SAW" IDEA pages 1-3

IDEA B "SPRING-FLOOR" IDEA pages 4-5

IDEA C HAND CRANK + FLYWHEEL w/ COLOR LEDs
↳ this is the most affordable idea, which is just an enhancement to the current design to reduce physical activity.
NOTE: ADD COLOR LED's to FLYWHEEL, SO THAT COLORS ARE LIT WHILE USING THE HAND CRANK — COLORS WILL "SPIN"

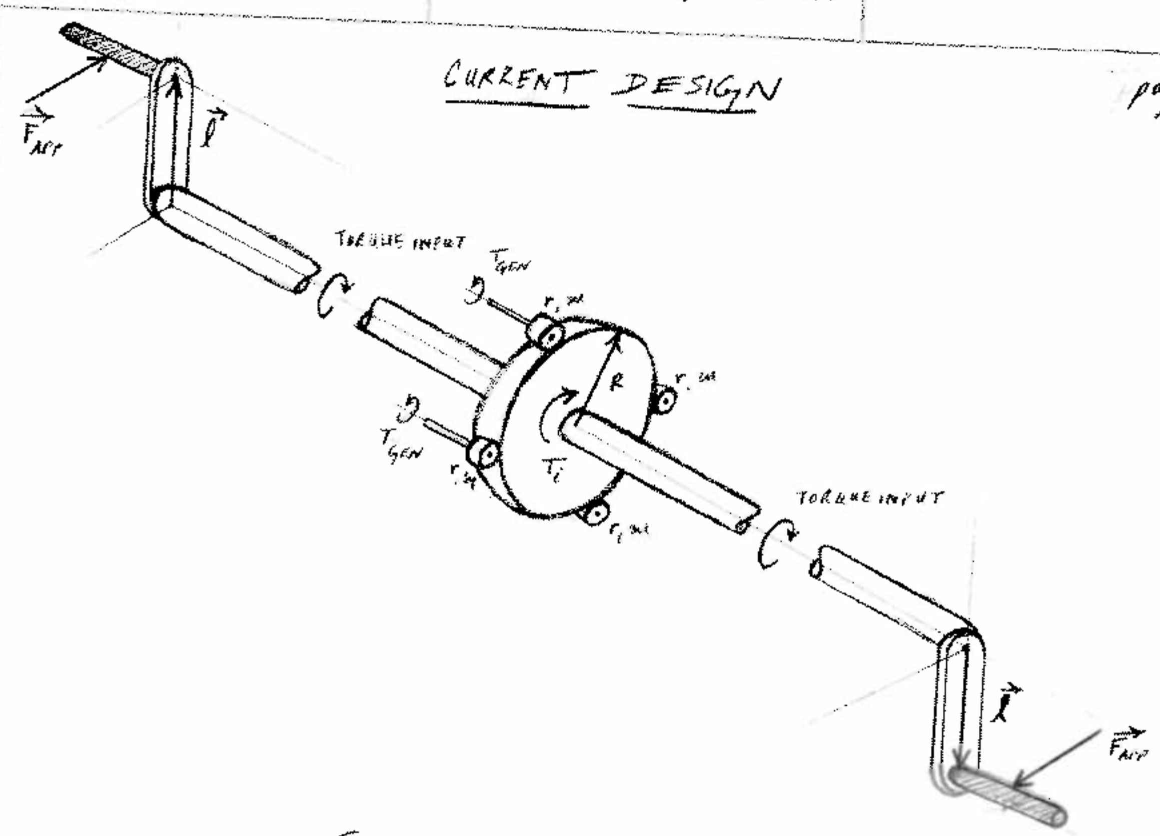
IDEA "A" SEE-SAW IDEA		IDEA "B" SPRING FLOOR IDEA		IDEA "C" ENHANCED HANDCRANK	
SPRING	$50	SPRINGS	$30/6	FLYWHEEL	NEED EST
SPROCKET + CHAIN	$20	GEARS/SPROCKET & CHAIN	$20	LEDs (4)	$1.60/400
2x4 TREATED	$10	PLYWOOD	$10		
BALLBEARING	$5	2x4 TREATED	$10		
		MOTION ARMS	$12		
		BALL BEARING	$5		
		FLYWHEEL w/ GROOVE	NEED EST.		
TOTAL	$81	TOTAL	$50.25 + EST	TOTAL	$1.60 + EST

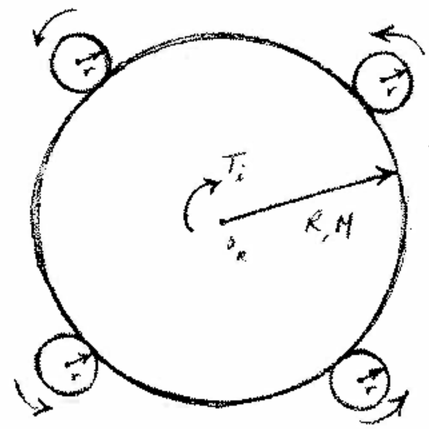
WE NEED AN ESTIMATE FOR A FLYWHEEL w/ GROOVE PATTERN "BEVELED GEAR" CONCEPT ON TOP SURFACE.

NEED AN ESTIMATE ON HOLLOW PLASTIC FLYWHEEL (fill w/ sand & dirt upon installation)

D = 1.5 ft

CURRENT DESIGN





$\sum M_{o_R} = T_i - 4\underbrace{F_R R}_{\text{Resistive Torque due to generator internal resistance}} = I_{ROT}\,\alpha_R \, , \; I_{ROT} = \sum I_{SHAFT}$

$$\underline{\underline{2F_{APP}\,l - 4T_{RES}}} = (I_{SHAFT} + I_R)\,\alpha_R$$

UNDER NORMAL STEADY STATE OPERATING CONDITIONS : $\alpha_R \approx 0$ (constant angular velocity to power the pump on the R-9 Water treatment system)

CURRENT DESIGN: WHAT HAPPENS WHEN YOU LET GO OF THE HANDLES? When T_i goes to zero, the angular velocity quickly goes to zero $\Longrightarrow$ implies very high angular acceleration in the <u>negative</u> direction

$$\cancel{2F_{AP}\,l}^{\,0} - 4T_{RES} = \left(I_{SHAFT} + \frac{MR^2}{2}\right)\alpha_R$$

$$\therefore \; \alpha_R = \underline{\underline{\frac{-4T_{RES}}{\left(I_{SHAFT} + \frac{MR^2}{2}\right)}}}$$

$T_{RES} \equiv$ CONSTANT (WORST CASE)
$I_{SHAFT} \equiv$ MOMENT OF INERTIA OF ROTATING SHAFT
$\frac{MR^2}{2} \equiv$ MOMENT OF INERTIA OF LARGE GEAR

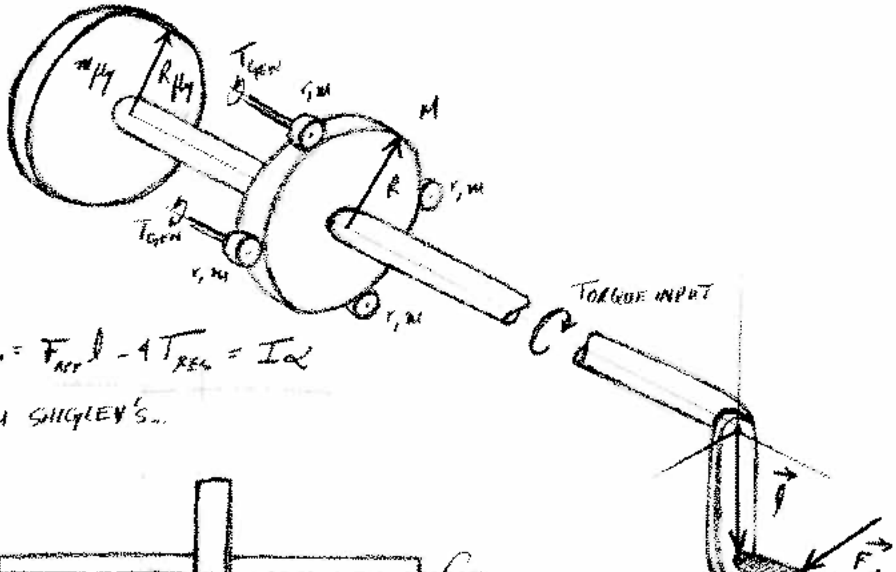
PROPOSED DESIGN: ADD A FLYWHEEL TO THE SHAFT TO INCREASE <u>MOMENTUM</u> OF THE SYSTEM $\longrightarrow$ inertial energy storage device

$$\|\vec{\alpha}_{CURRENT}\| \gg \|\vec{\alpha}_{PROPOSED}\|$$

$$\alpha_R(I_{TOTAL}) = \frac{-4T_{RES}}{I_{TOT}}$$

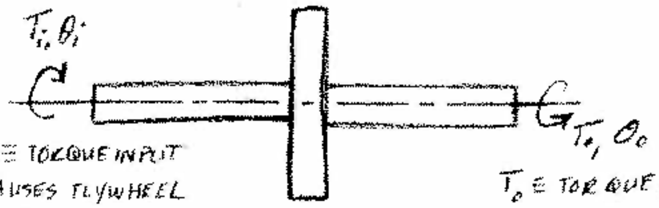
INCREASE I_{TOT} to decrease $\|\vec{\alpha}_R\|$ by adding a flywheel to the shaft (rotating member)

PROPOSED DESIGN



$$\Sigma M_o = F_{APP}\, l - 4T_{REG} = I\alpha$$

FROM SHIGLEY'S...



$T_i \equiv$ TORQUE INPUT CAUSES FLYWHEEL SPEED TO INCREASE

I_{fly}, θ

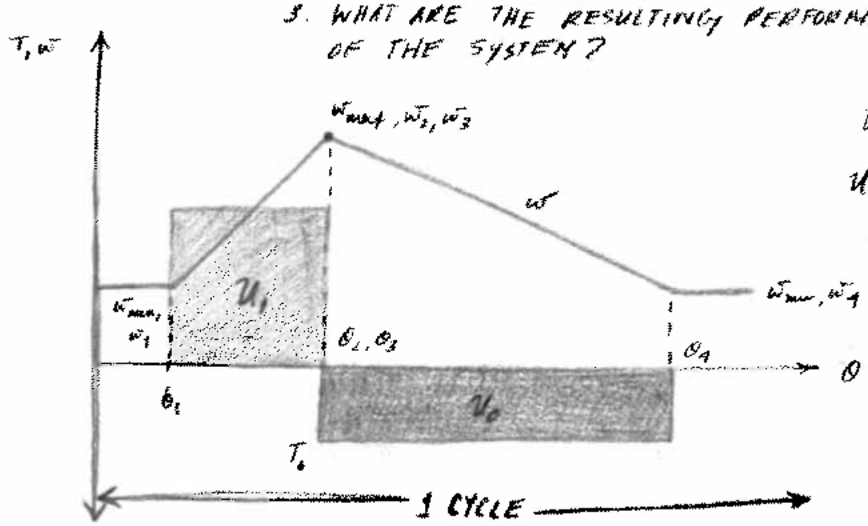
$T_o \equiv$ TORQUE OUTPUT OR LOAD ABSORBS ENERGY FROM THE FLYWHEEL AND CAUSES IT TO SLOW DOWN

⟹ DESIGN A FLYWHEEL SO AS TO OBTAIN A SPECIFIED AMOUNT OF SPEED REGULATION. IN OUR CASE, MAINTAIN ANGULAR VELOCITY THAT POWERS THE B-9 BETTER WATER MAKER.

$$\therefore \Sigma M_{o_{fly}} = T_i(\theta_i, \dot{\theta}_i) - T_o(\theta_o, \dot{\theta}_o) = I_{fly}\ddot{\theta}$$

if T_i & T_o are known, we can solve for θ, ω, and α as functions of time, but we are not really interested in the instantaneous values. Need to answer...

1. WHAT SHOULD I_{fly} be?
2. HOW DO WE MATCH THE POWER SOURCE TO IT'S LOAD?
3. WHAT ARE THE RESULTING PERFORMANCE CHARACTERISTICS OF THE SYSTEM?



$U_i = T_i(\theta_2 - \theta_1) \equiv$ WORK INPUT

$U_o = T_o(\theta_4 - \theta_3) \equiv$ WORK OUTPUT

BASED ON EXPERIMENTAL DATA...

$\omega = 2\,\frac{rev}{s}$ to power the pump (Green light comes on)

$\omega = 2\,\frac{rev}{s} \Longrightarrow P = 17\,W$

$\omega_R = 2\,\frac{rev}{s} \cdot \frac{2\pi\,rad}{1\,rev} = 12.5664\,\frac{rad}{s} \cdot \frac{180°}{\pi\,rad} = 720°/s$

$R = 3''$, $r = 0.125''$ $\Big|$ $\frac{R}{r} = \frac{3''}{.125''} = 24 \equiv$ GEAR RATIO

$\theta_R \cdot R = \theta_r\, r \Longrightarrow \theta_r = \left(\frac{R}{r}\right)\theta_R$

$\therefore\ \omega_R \cdot R = \omega_r\, r \Longrightarrow \omega_r = \left(\frac{R}{r}\right)\omega_R$

$\therefore\ \alpha_R\, R = \alpha_r\, r \Longrightarrow \alpha_r = \left(\frac{R}{r}\right)\alpha_R$

TO POWER THE UNIT THE GENERATORS ARE SPINNING AT WHAT VELOCITY?

$\omega_r = \left(\frac{R}{r}\right)\omega_R = 24 \cdot 2\,\frac{rev}{s} = 48\,\frac{rev}{s} \cdot \frac{2\pi\,rad}{1\,rev} = 301.593\,\frac{rad}{s} \cdot \frac{180°}{\pi\,rad} = 17280°/s$

$\therefore$ TO POWER THE PUMP

$\omega_R = 2\,\frac{rev}{s} = 12.6\,\frac{rad}{s}$

$\omega_r = 48\,\frac{rev}{s} = 301.6\,\frac{rad}{s} \Longrightarrow 3V$ OUTPUT FROM EACH GENERATOR

EXPERIMENTAL

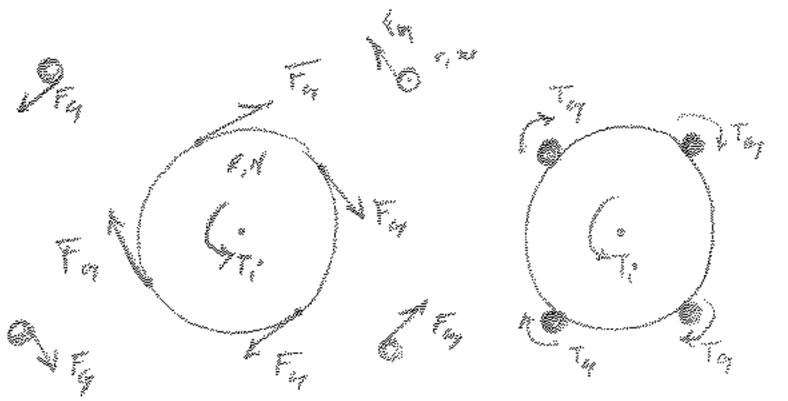
How does this compare to the Theoretical Analysis?

$V = k_a\,\omega$

$k_a = 0.0238$ (from DANIEL)

$\omega = \frac{V}{k_a} = \frac{3V}{0.0238} = 126.05\,\frac{rad}{s}$ $\Big|$ $\therefore\ \eta = \frac{\omega_{THEORETICAL}}{\omega_{EXPERIMENTAL}} = \frac{126.1\,\frac{rad}{s}}{301.6\,\frac{rad}{s}} = 0.4181$ $\boxed{\therefore\ \eta \cong 0.42}$

TO GEAR OR NOT TO GEAR?



$\therefore \sum M_{r} = rF_g = I_r\alpha_r \longrightarrow rF_g = I_r(GR)\alpha_R$

$\sum M_{O_R} = T_i - 4RF_g = I_R\alpha_R \qquad F_g = \frac{I_r(GR)\alpha_R}{r}$

$\theta_r r = \theta_R R$

SUBSTITUTE INTO $\sum M_{O_R}$

$\Rightarrow \theta_r = \frac{R}{r}\theta_R = (GR)\theta_R$

$T_i - 4R\left\{\frac{I_r(GR)\alpha_R}{r}\right\} = I_R\alpha_R$

$\omega_r = (GR)\omega_R$

$\alpha_r = (GR)\alpha_R$

$T_i - 4I_r(GR)^2\alpha_R = I_R\alpha_R$

$T_i = \alpha_R\{I_R + 4I_r(GR)^2\}$

shaft and large gear — small gear (quadrature gear)

$$\boxed{T_i = \alpha_R\{I_R + 2304\,I_r\}}$$

ADD FLYWHEEL TO SHAFT

$I_R \approx 0.0035\ kg\cdot m^2$

$I_r \approx 1.4289\cdot 10^{-7}\ kg\cdot m^2$

$I_T = I_R + 2304 I_r = 0.003829$

$T_i = I\alpha \rightarrow$ assume $\alpha = \frac{\omega_{max} - \omega_{min}}{t_{max} - t_{min}} = \frac{2\frac{rev}{s}\cdot\frac{2\pi\,rad}{1\,rev} - 0}{0.1\,s} = 125.66\ \frac{rad}{s^2}$

Assume constant T_i $\therefore I\alpha = $ CONSTANT

If we want to improve α by 600 x $\rightarrow$ increase I by 600 x

$T_i = (600 I)\left(\frac{\alpha}{600}\right)$

$\alpha = \frac{2\frac{rev}{s}\cdot\frac{2\pi\,rad}{1\,rev} - 0}{600\cdot 0.1\,s} = \underline{0.20944\ rad/s}$

1 min = 60 s

$\therefore I_{new} = 600\cdot I_{curr} = \underline{2.2974\ kg\cdot m^2}$

Try adding flywheel to shaft..

$I_R + 2304(1.4289\cdot 10^{-7}\ kg\cdot m^2) = 2.2974\ kg\cdot m^2$

$I_R = 2.29707\ kg\cdot m^2$

$I_{fly} = 2.29707\ kg\cdot m^2 - .0035\ kg\cdot m^2 = 2.29357\ kg\cdot m^2$

$I_{fly} = m\frac{R^2}{2}$

$R = \sqrt{2 I_{fly}\, m^{-1}} = \sqrt{2(2.29357\ kg\cdot m^2)(58.76 n)^{-1}} = \underline{28.001\ mm}$

Try adding to the generator...

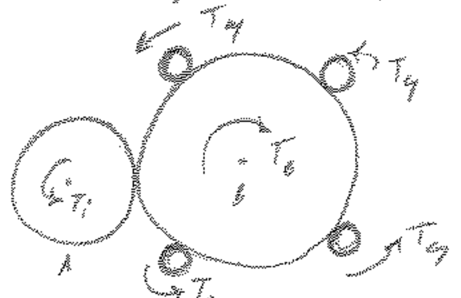
$$I_g + 2304 I_r = 2.2974\ kg\cdot m^2$$

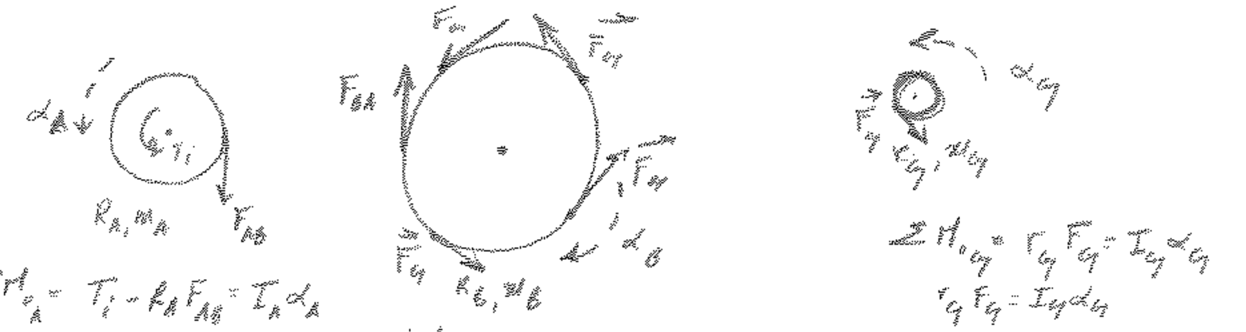
$$I_r = \frac{2.2974\ kg\cdot m^2 - 0.0025\ kg\cdot m^2}{2304} = .000996$$

$$I_{gen\ fly} = .000996\ kg\cdot m^2 = \frac{mr^2}{2}$$

$$r = \sqrt{\frac{2(.000996\ kg\cdot m^2)}{\frac{22.34N}{9.81\ m/s^2}}} = \underline{1.167\ in}$$

What happens if we add another gear to the large gear?





$$\Sigma M_{o_A} = T_i - R_A F_{AB} = I_A \alpha_A$$

$$\Sigma M_{o_{cg}} = r_{cg} F_{cg} = I_{cg}\alpha_{cg}$$

$$r_{cg} F_{cg} = I_{cg}\alpha_{cg}$$

$$\underline{F_{cg} = \frac{I_{cg}}{r_{cg}}\left(\frac{R_B}{r_{cg}}\right)\left(\frac{R_A}{R_B}\right)\alpha_A}$$

$$\Sigma M_{o_B} = -R_B F_{BA} + 4R_B F_{cg} = -I_B \alpha_B$$

SUBSTITUTE

EXAMINE GEARING RELATIONSHIPS

$$\theta_A R_A = \theta_B R_B$$

$$\omega_A R_A = \omega_B R_B$$

$$\alpha_A R_A = \alpha_B R_B \Longrightarrow \alpha_B = \left(\frac{R_A}{R_B}\right)\alpha_A \;\hookrightarrow\; \underline{\alpha_B = (GR_{AB})\alpha_A}$$

$$\theta_B R_B = \theta_{cg} r_{cg}$$

$$\omega_B R_B = \omega_{cg} r_{cg}$$

$$\alpha_B R_B = \alpha_{cg} r_{cg} \Longrightarrow \alpha_{cg} = \left(\frac{R_B}{r_{cg}}\right)\alpha_B \;\hookrightarrow\; \underline{\alpha_{cg} = (GR_{Bcg})\alpha_B}$$

$$\therefore \alpha_{cg} = \left(\frac{R_B}{r_{cg}}\right)\left(\frac{R_A}{R_B}\right)\alpha_A$$

$$\alpha_{cg} = (GR_{Bcg})(GR_{AB})\alpha_A$$

$$-R_B F_{BA} + 4R_B F_{cg} = -I_B \alpha_B$$

$$R_B F_{BA} - 4R_B F_{cg} = I_B \alpha_B$$

SUBSTITUTE F_{cg} & α_B

$$R_B F_{BA} - 4R_B \cdot \frac{I_{cg}}{r_{cg}}\left(\frac{R_B}{r_{cg}}\right)\left(\frac{R_A}{R_B}\right)\alpha_A = I_B\left(\frac{R_A}{R_B}\right)\alpha_A$$

$$R_B F_{BA} - 4 I_{cg}\left(\frac{R_B}{r_{cg}}\right)^2\left(\frac{R_A}{R_B}\right)\alpha_A = I_B\left(\frac{R_A}{R_B}\right)\alpha_A$$

Analysis continued..

$$R_B F_{BA} = I_B\left(\frac{R_A}{R_B}\right)\alpha_A + 4I_{g}\left(\frac{R_B}{r_g}\right)^2\left(\frac{R_A}{R_B}\right)\alpha_A$$

$$R_B F_{BA} = \alpha_A\left\{I_B\left(\frac{R_A}{R_B}\right) + 4I_g\left(\frac{R_B}{r_g}\right)^2\left(\frac{R_A}{R_B}\right)\right\}$$

$$\therefore F_{BA} = \frac{1}{R_B}\left(\frac{R_A}{R_B}\right)\alpha_A\left\{I_B + 4I_g\left(\frac{R_B}{r_g}\right)^2\right\}$$

$$T_i - R_A F_{AB} = I_A\alpha_A$$

SUBSTITUTE F_{AB} $\quad T_i - R_A\left\{\frac{1}{R_B}\left(\frac{R_A}{R_B}\right)\alpha_A\left\{I_B + 4I_g\left(\frac{R_B}{r_g}\right)^2\right\}\right\} = I_A\alpha_A$

$$T_i - \left(\frac{R_A}{R_B}\right)^2\left(I_B + 4I_g\left(\frac{R_B}{r_g}\right)^2\right)\alpha_A = I_A\alpha_A$$

$$\boxed{T_i = \alpha_A\left\{I_A + \left(\frac{R_A}{R_B}\right)^2\left(I_B + 4I_g\left(\frac{R_B}{r_g}\right)^2\right)\right\}}$$

Sane check ⇒ increase I by 600x

$$I_{TOT} = I_A + \left(\frac{R_A}{R_B}\right)^2 \underbrace{\left(I_B + 4I_g\left(\frac{R_B}{r_g}\right)^2\right)}_{\substack{\text{CURRENT } I_{TOT} \\ =.003829\ kg\cdot m^2}}$$

$I_{new} = 2.2974\ kg\ m^2$

$$I_A + \left(\frac{R_A}{R_B}\right)^2(.003829\ kg\cdot m^2) = 2.2974\ kg\cdot m^2$$

You could do this purely by adding a gear ratio

$I_A \cong 0 \Longrightarrow \frac{R_A}{R_B} = 24.5 \cong 73.5\ in$ RADIUS $\cong$ TOO BIG

try a $GR_{AB} = 2$

$$I_A + (2)^2(.003829\ kg\ m^2) = (2.2974\ kg\cdot m^2)$$

$$I_A = 2.28208 = \tfrac{1}{2}mR^2$$

$$R = \sqrt{\left(\frac{2.28208 \cdot 2}{\frac{88.96N}{9.81m/s^2}}\right)} = \underline{\underline{27.93\ in}}$$

Same problem → only decrease by .3in with gearing

try $GR_{AB} = 10$

$$\underline{\underline{R = 25.58\ in}}$$

INITIAL SYSTEM ANALYSIS (GENERIC)

$I\ddot{\theta} + c\dot{\theta} + k\theta = f(t)_{INPUT}$

$\theta(t) = \theta_c(t) + \theta_p(t)$

$I\ddot{\theta} + c\dot{\theta} + k\theta = 0$

$\ddot{\theta} + \frac{c}{I}\dot{\theta} + \frac{k}{I}\theta = 0$

assume $\theta = e^{st}$, $\dot{\theta} = s e^{st}$, $\ddot{\theta} = s^2 e^{st}$ $\Big\}$ $e^{st}\left\{s^2 + \frac{c}{I}s + \frac{k}{I}\right\} = 0$

$$s = \frac{-\frac{c}{I} \pm \sqrt{\frac{c^2}{I^2} - 4(1)\left(\frac{k}{I}\right)}}{2(1)} = \frac{-\frac{c}{I} \pm \sqrt{\frac{c^2}{I^2} - \frac{4k}{I}}}{2}$$

CHARACTERISTIC ROOTS

$s_1 = -\frac{1}{2}\left\{\frac{c}{I} + \sqrt{\frac{c^2}{I^2} - \frac{4k}{I}}\right\}$

$s_2 = \frac{1}{2}\left\{-\frac{c}{I} + \sqrt{\frac{c^2}{I^2} - \frac{4k}{I}}\right\}$

$$\therefore \theta(t) = C_1 e^{-\frac{1}{2}\left\{\frac{c}{I} + \sqrt{\frac{c^2}{I^2} - \frac{4k}{I}}\right\}t} + C_2 e^{\frac{1}{2}\left\{-\frac{c}{I} + \sqrt{\frac{c^2}{I^2} - \frac{4k}{I}}\right\}t} + \theta_p(t)$$

GENERAL FORM OF 2^{nd} ORDER DIFFERENTIAL EQUATION:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 = 0$$

$$\therefore 2\zeta\omega_n = \frac{c}{I} \rightarrow \zeta = \frac{\frac{c}{I}}{2\omega_n} = \frac{c}{I}\cdot\frac{1}{2\omega_n} = \frac{c}{2I\cdot\frac{k^{1/2}}{I^{1/2}}} = \frac{c}{2I^{1/2}k^{1/2}} = \frac{c}{2\sqrt{Ik}}$$

$$\omega_n^2 = \frac{k}{I} \rightarrow \omega_n = \sqrt{\frac{k}{I}}$$

for an overdamped system

$$\zeta > 1 \Rightarrow \frac{c}{2\sqrt{Ik}} > 1$$

$\underline{c > 2\sqrt{Ik}}$ OVERDAMPED SYSTEM

$I\ddot{\theta} + c\dot{\theta} + k\theta = f(t)$

$\mathcal{L}\{I\ddot{\theta} + c\dot{\theta} + k\theta = f(t)\}$

$\mathcal{L}\left\{\ddot{\theta} + \frac{c}{I}\dot{\theta} + \frac{k}{I}\theta = \frac{f(t)}{I}\right\}$

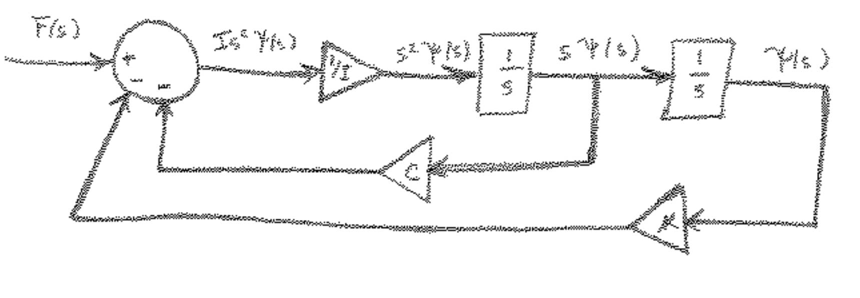
$s^2\Psi(s) - s\cancel{\theta(0)} - \cancel{\dot{\theta}(0)} + \frac{c}{I}(s\Psi(s) - \cancel{\theta(0)}) + \frac{k}{I}\Psi(s) = \frac{F(s)}{I}$

assume $\dot{\theta}(0) = \theta(0) = 0$

$s^2\Psi(s) + \frac{c}{I}s\Psi(s) + \frac{k}{I}\Psi(s) = \frac{F(s)}{I}$

$\Psi(s)\left\{s^2 + \frac{c}{I}s + \frac{k}{I}\right\} = \frac{F(s)}{I}$

$$\frac{\Psi(s)}{F(s)} = \frac{(1/I)}{\left(s^2 + \frac{c}{I}s + \frac{k}{I}\right)} \equiv \text{TRANSFER FUNCTION}$$

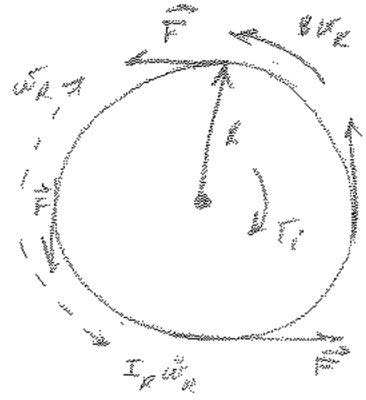


BLOCK DIAGRAM WITH $\underline{\Psi(s) = \theta(t)}$

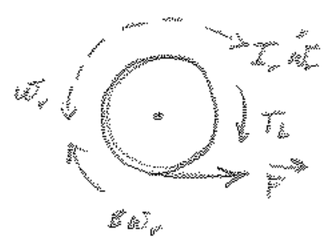
$$\omega_r r = V_T = \omega_R R$$

$$\therefore \omega_r = \left(\frac{R}{r}\right)\omega_R$$

LARGE GEAR



GENERATOR GEAR



$$\circlearrowleft + \Sigma M_{o_R} = 0 = -I_R\dot{\omega}_R + T_i - 4RF - B_R\omega_R = 0 \qquad \circlearrowright + \Sigma M_{o_r} = 0 = rF - T_L - B_r\omega_r - I_r\dot{\omega}_r = 0$$

$$-I_R\dot{\omega}_R + T_i - 4RF - B_R\omega_R = 0 \quad \xleftarrow{\text{SUBSTITUTE}} \quad F = \frac{I_r\dot{\omega}_r + B_r\omega_r + T_L}{r}$$

$$-I_R\dot{\omega}_R + T_i - 4\frac{R}{r}\left\{I_r\dot{\omega}_r + B_r\omega_r + T_L\right\} - B_R\omega_R = 0$$

$$-I_R\dot{\omega}_R + T_i - 4\left(\frac{R}{r}\right)\left\{I_r\left(\frac{R}{r}\right)\dot{\omega}_R + B_r\left(\frac{R}{r}\right)\omega_R + T_L\right\} - B_R\omega_R = 0$$

$$-I_R\dot{\omega}_R + T_i - 4\left(\frac{R}{r}\right)^2 I_r\dot{\omega}_R - 4\left(\frac{R}{r}\right)^2 B_r\omega_R - 4\left(\frac{R}{r}\right)T_L - B_R\omega_R = 0$$

$$-\dot{\omega}_R\left\{I_R - 4\left(\frac{R}{r}\right)^2 I_r\right\} - \omega_R\left\{B_R + 4\left(\frac{R}{r}\right)^2 B_r\right\} - 4\left(\frac{R}{r}\right)T_L + T_i = 0$$

$$\boxed{\therefore \left\{I_R + 4\left(\frac{R}{r}\right)^2 I_r\right\}\dot{\omega}_R + \left\{B_R + 4\left(\frac{R}{r}\right)^2 B_r\right\}\omega_R + 4\left(\frac{R}{r}\right)T_L = T_i}$$ FIRST ORDER SYSTEM MODEL

$I_R + 4\left(\frac{R}{r}\right)^2 I_r$ = Resultant Inertia (I_T)

$B_r + 4\left(\frac{R}{r}\right)^2 I_r$ = Resultant "Frictional" coefficient (damping) (B_T)

$4\left(\frac{R}{r}\right)$ = amplitude of Resistive Torque from the Generator (A)

T_i = Input Torque as a function of time

SIMPLIFIED MODEL w/ I_T, B_T, & A

$$I_T\dot{\omega}_R + B_T\omega_R + AT_L = T_i$$

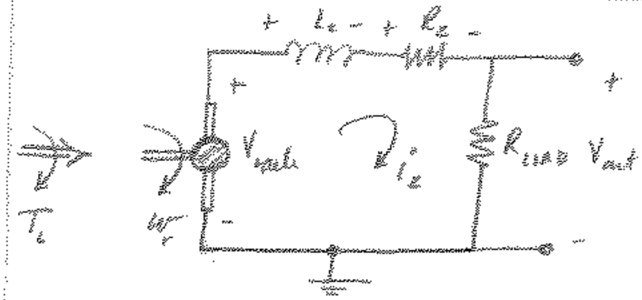
$$\mathcal{L}\left\{\dot{\omega}_R + \frac{B_T}{I_T}\omega_R + \frac{A}{I_T}T_L = \frac{T_i}{I_T}\right\}$$

$$s\,\Omega(s) + \cancel{\omega(0)}^{\,0\,(\text{ASSUME})} + \frac{B_T}{I_T}\Omega(s) + \frac{A}{I_T}T_L(s) = \frac{T_i(s)}{I_T}$$

$$\Omega(s)\left\{s + \frac{B_T}{I_T}\right\} + \frac{A}{I_T}T_L(s) = \frac{T_i(s)}{I_T}$$

Need to define in terms of $T_i(s)$ to determine TF: $\frac{\Omega(s)}{T_i(s)}$

ANALYSIS



R_{LOAD} ≡ EQUIVALENT RESISTANCE OF HE SYSTEM + PUMP

V_{out} ≡ VOLTAGE TO SYSTEM (3V per Generator

LOOP ANALYSIS: $-V_{tach} + V_{L_2} + V_{R_2} + V_{R_{LOAD}} = 0$

$$-K_{tach} w_r + L_2 \frac{di_2}{dt} + i_2 R_2 + i_2 R_{LOAD} = 0$$

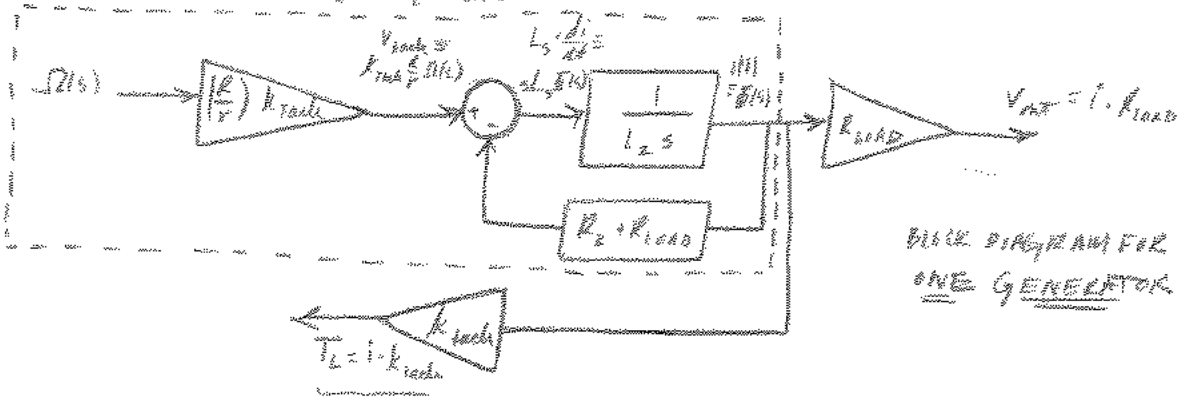
$$\therefore \; L_2 \frac{di_2}{dt} + (R_2 + R_{LOAD}) i_2 = \overbrace{K_{tach} w_r}^{\text{INPUT}}$$

$$\mathcal{L}\left\{ L_2 \frac{di_2}{dt} + (R_2 + R_{LOAD}) i_2 = K_{tach}\left(\frac{R}{r}\right) w_R \right\}$$

$$L_2 s I(s) + \cancel{L_2 i(0)}^{0} + (R_2 + R_{LOAD}) I(s) = K_{tach}\left(\frac{R}{r}\right)\Omega(s)$$

$$I(s)\{L_2 s + (R_2 + R_{LOAD})\} = K_{tach}\left(\frac{R}{r}\right)\Omega(s)$$

$$\frac{I(s)}{\Omega(s)} = \frac{K_{tach}\left(\frac{R}{r}\right)}{L_2 s + R_2 + R_{LOAD}}$$ TF for GENERATOR with $\overline{W}$ INPUT



BLOCK DIAGRAM FOR ONE GENERATOR

$$T_L(s) = I(s) \cdot K_{tach} = \frac{K_{tach}^2\left(\frac{R}{r}\right)\Omega(s)}{L_2 s + R_2 + R_{LOAD}}$$

SUBSTITUTE INTO MECHANICAL MODEL

$$\Omega(s)\left\{s + \frac{B_T}{I_T}\right\} + \frac{A}{I_T} T_L(s) = \frac{T_i(s)}{I_T}$$

$$\Omega(s)\left\{s + \frac{B_T}{I_T}\right\} + \frac{A}{I_T}\left\{\frac{K_{tach}^2\left(\frac{R}{r}\right)\Omega(s)}{L_2 s + R_2 + R_{LOAD}}\right\} = \frac{T_i(s)}{I_T}$$

$$\Omega(s)\left\{ s + \frac{B_T}{I_T} + \frac{A K_{tach}^2}{I_T}\left(\frac{R}{r}\right)\frac{1}{L_2 s + R_2 + R_{LOAD}} \right\} = \frac{T_i(s)}{I_T}$$

$$\Omega(s)\left\{ I_T\left(L_2 s^2 + (R_2 + R_{LOAD})s\right) + B_T(L_2 s + R_2 + R_{LOAD}) + A K_{tach}^2\left(\frac{R}{r}\right)\right\} = T_i(s)(L_2 s + R_2 + R_{LOAD})$$

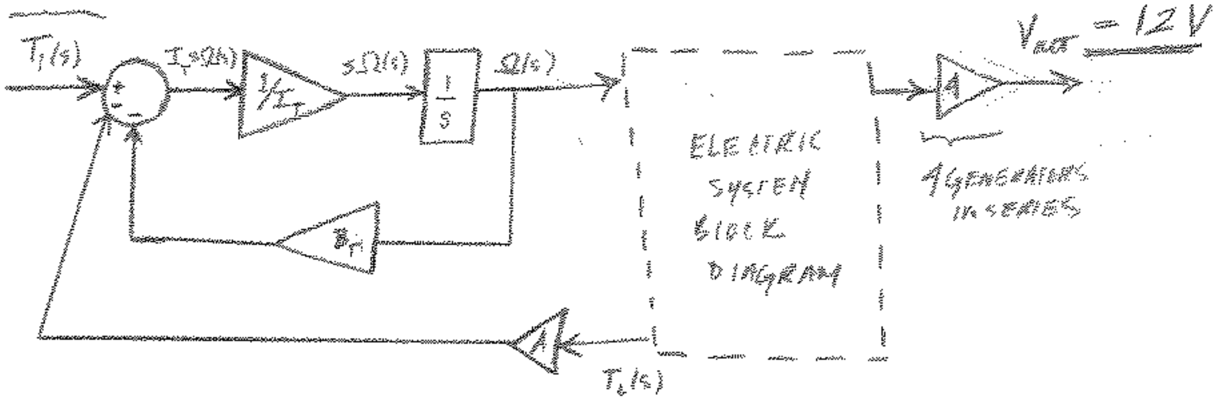
$$\Omega(s)\left\{ I_T L_2 s^2 + I_T(R_2 + R_{LOAD})s + B_T L_2 s + B_T(R_2 + R_{LOAD}) + A K_{tach}^2\left(\frac{R}{r}\right)\right\} = T_i(s)(L_2 s + \ldots)$$

$$\Omega(s)\left\{ I_T L_2 s^2 + \left\{ I_T (R_2 + R_{LOAD}) + B_T L_2 \right\} s + \left\{ B_T (R_2 + R_{LOAD}) + A k_{mech}^2 \left(\frac{k}{\eta}\right) \right\} \right\} = T_1(s)\left(L_2 s + R_2 + R_{LOAD}\right)$$

$$\boxed{\therefore \frac{\Omega(s)}{T_1(s)} = \frac{L_2 s + R_2 + R_{LOAD}}{I_T L_2 s^2 + \left(I_T (R_2 + R_{LOAD}) + B_T L_2\right) s + \left(B_T (R_2 + R_{LOAD}) + A^2 k_{mech} \left(\frac{k}{\eta}\right)\right)}}$$

TRANSFER FUNCTION OF THE ENTIRE SYSTEM (MECHANICAL & ELECTRICAL)

INPUT TORQUE



SEE SIMULINK MODEL