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Action of low frequency vibration on liquid droplets and particles

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Abstract

Liquids handling is an important issue in biomedical analysis. Two different devices for acoustic manipulation of droplets have already been tested. The first one, more classical, uses a high frequency travelling wave and acoustic streaming. The second one uses low frequency flexural standing waves in a plate. This means of liquid handling is original and easy to implement but the physical principle is not obvious. In order to understand more precisely the phenomena involved we present new observations on droplet displacement between two planes and on the behaviour of a droplet on an inclined vibrating plane with this method. The physical principle involved is discussed. The common acoustic radiation pressure formulation is expressed via the non-linear theory of sound propagation, but in our case the acoustic wavelength is much smaller than the height of a water droplet. To get a better understanding of the phenomenon, further experiments on the internal liquid flow and behaviour of particles in the droplet have been performed. These will be compared with results obtained with particles in a thin water-filled vibrating glass tube. The general conclusion is that the phenomenon is practical to use for droplet displacement even if its complex mechanism is not completely understood.

Keywords: Vibrating beam; Droplets displacement; Particles; Tilted beam

1. Introduction

We present here the low frequency (about 30 kHz) action of liquid droplets. For example, on a vibrating beam it is possible at a sufficiently high level of vibration to move a droplet toward the nearest anti-node of vibration [1]. By changing the vibration mode it is possible to move the droplet step by step. The force on the droplet can be used differently. We can stop a descending droplet on a tilt beam by vibration. It is also possible to move a droplet between a fixed plane and a vibrating one. The method described here can be compared with another purely acoustic method using a surface acoustic wave at a frequency of some tens of megahertz produced by interdigital transducers on a piezo-electric crystal. But in fact the mechanism is completely different. At high frequency the wave absorption produces a gradient that induces a force: so-called acoustic streaming. This mechanism is negligible at low frequency. We discuss the physical mechanism in a later paragraph.

2. Experiment

2.1. Droplet displacement

A drop on a flexural mode-vibrating beam moves toward an anti-node of vibration if the vibration amplitude is high enough

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Notations

A
c
c_l f
F_p amplitude of vibration of the structure sound
F_c velocity
~F_v sound velocity in the liquid part
F_v frequency
g h projection on the x-axis of the droplet weight
k_l projection on the x-axis of the interfacial force
k_v vibration force
L projection on the x-axis of the vibration force
L_{eq} gravitational acceleration thickness
of the liquid layer wavenumber in
the liquid wavenumber for the
vibrated beam
L_P drop perimeter
P_R R equivalent perimeter of drop taking into account receding/advancing angle and drop perimeter
r_p characteristic length acoustic radiation pressure
T characteristic particles trajectories curvature radius
spherical particles radius time

u_0
u_L
u_v vibration amplitude
v vibration elongation in the liquid vibration
V elongation in the beam
a particle speed
a_c volume of drop
b angle of inclination of tilt critical
angle of inclination of tilt
c local angle between normal of the contact line and the
c_w opposed direction of the motion
g liquid to gas surface tension non-linear
k_l parameter for water
h water viscosity wavelength
in the liquid
h_R contact angle, variable along contact line receding
contact angle, assumed constant along receding
h_A contact line
advancing contact angle, assumed constant
q along advancing contact line
q_p density of drop particle
x density angular frequency

(see Fig. 1). Experiments show that the amplitude must be greater than 1 lm. It is better to suppress the disturbing modes, that is to say modes with a node line parallel to the beam length. This can be obtained with a beam including periodic teeth (see Fig. 2). Two piezo-electric actuators excite the experimental device.

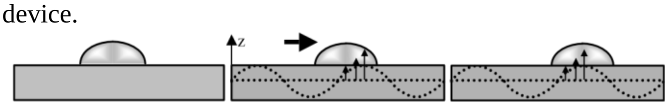


Fig. 1. Droplet displacement on a vibrating beam.

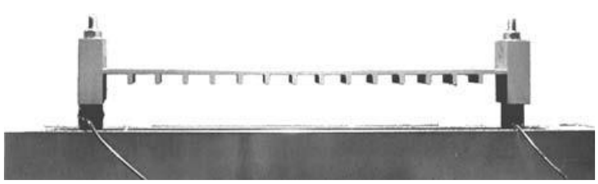


Fig. 2. Experimental device.

To achieve a large displacement we can use a set of modes with suitable anti-node positions but with different frequencies. A controlled and reversible displacement has been achieved [2] using modes 16, 17, 18, 19, 20 and 21 with frequencies between 22.9 and 35.2 kHz.

Droplet displacement is also possible using a thin plastic sheet deposited on the surface of the vibrating beam. This property can be important for bio-chemical applications where the removal of the plastic sheet is used to avoid a cleaning process. It is also possible to move a drop between two beams with only one vibrating. In this case also the drop moves toward the anti-node.

2.2. Force measurement on a droplet

In this part, the beam is inclined until the droplet moves with gravity [3]. Then the droplet is stopped at an anti-node by the effect of vibration. Two forces act on the drop, F_P force causing the drop to move steadily down the surface, F_c causing the drop to adhere on the surface (see Fig. 3). The aim of this part is to evaluate the force F_c. When the drop is at rest, the force balance will verify

$$F_P \quad F_c = 0 \qquad (1)$$

First, without exciting the beam, the tilt angle is measured when the drop starts to move. This angle is called critical 'sticking' angle (a_c). At this equilibrium point it is considered that the force due to gravity F_P is equal to the interfacial force Fc. Obviously, the vibration force F_v is null. The force due to gravity F_P is evaluated by projection of the droplet weight on the plane of the displacement:

$$F_P = qVg \sin(a_c) \qquad (2)$$

The critical 'sticking' angle a_c is plotted for different droplet volumes (see Fig. 4) on. These measurements give us an approximation of F_P.

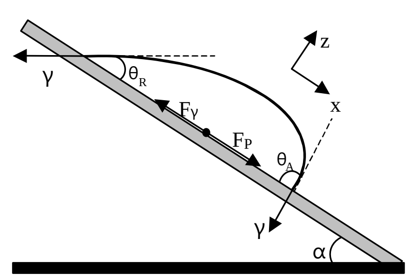


Fig. 3. Schematic of a droplet on an inclined plane.

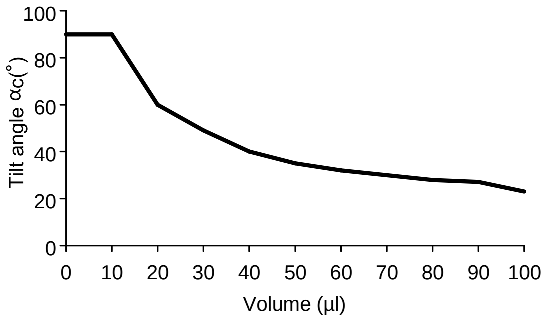


Fig. 4. Critical angle value of tilt until the drop fall.

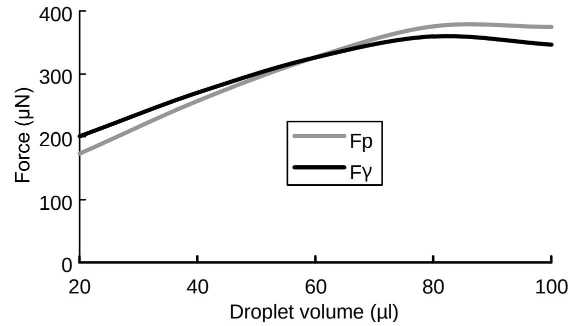


Fig. 5. Interfacial tension on the contact line.

Fig. 6. Evaluation of F_p and F_c as a function of droplet volume at the critical 'sticking' angle.

Then, according to the interfacial stress (see Fig. 5), Podgorski approximates the interfacial force [4]

$$F_c = \gamma L_{eq} \quad (3)$$ with

$$L_{eq} = \int_L \cos\theta \cos\beta \, dL \quad (4)$$

An approximation of L_{eq} is given by

$$L_{eq} \approx \frac{L}{2}(\cos\theta_R - \cos\theta_A) \quad (5)$$

The drop perimeter L is assumed constant along the length of the drop. Indeed, it is closer to a rectangle shape than a spherical shape due to the spreading of the droplet on the anti-node vibration, which is why it is possible to approximate L_{eq} by the above equation.

With measurements of h_R, h_A and L, it is possible to evaluate F_c and compare it to F_P when the drop is at rest at the critical static equilibrium (see Fig. 6).

Fig. 6 shows that without vibration and taking into account the Podgorski approximations, F_P and F_c evolve in the same direction and force values are very close (error lower than 10%). The evaluation of F_c has been verified. When the structure is vibrating the same experiments are carried out when the droplet is on a flexural anti-node of the beam. The critical 'sticking' angle (a_c) is measured for different voltages corresponding to different amplitudes of vibration of the structure (see Fig. 7).

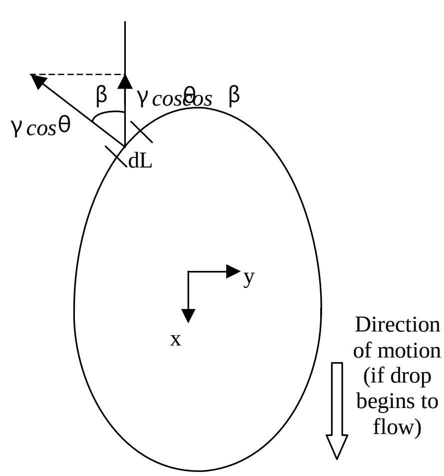
The vibration force exerts a third force on the droplet. This force called vibration force $\vec{F}_v$ is a volume force that changes the droplet shape. The h_A value decreases, h_R value increases and the droplet perimeter L increases too (see Fig. 8).

The direction of the projection force F_v seems to have the same direction of F_c. Indeed F_v tends to retain the droplet on the surface. Before the droplet starts to move, the critical 'sticking' angle (a_c) must be increased when the vibration force acts on the droplet. When the drop is at rest with the vibration force, the force balance will be

$$F_P - F_c - F_v = 0 \quad (6)$$

Knowing F_P and Fc, the vibration force F_v can be deduced from the measurements (see Fig. 9).

This method can give an approximation of the force acting on the droplet. It should be noticed that the force is saturated for a vibration amplitude value greater than 2 lm. The droplet was spread out on the anti-node of vibration taking the largest perimeter. If the vibration amplitude is increased beyond 2.5 lm, the droplet will be sprayed. This method of drop displacement shows large application opportunities. The main advantage compared with the high frequency method is the fact that it is not necessary to use a piezo-electric crystal such as Lithium niobate. Furthermore,



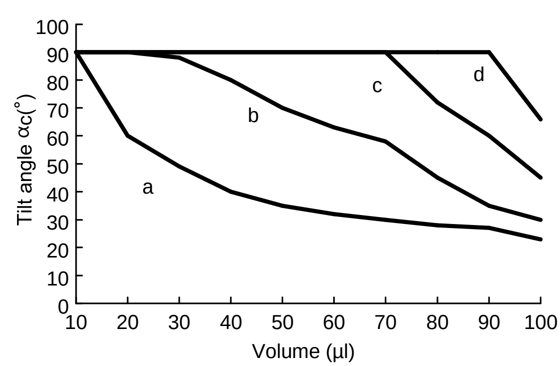


Fig. 7. Critical 'sticking' angle (a_c), function of drop volume for different amplitudes of vibration of structure (A): (a) no vibration; (b) A = 355 nm; (c) A = 700 nm; (d) A = 1 lm.

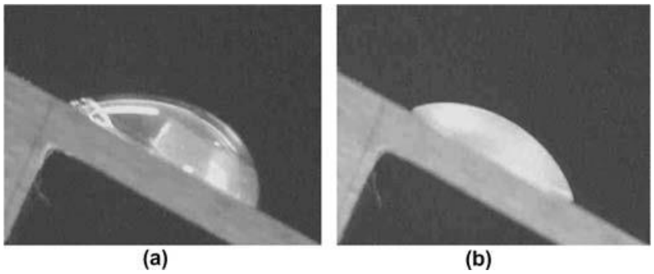
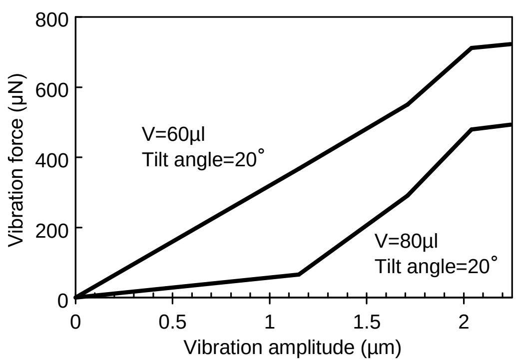


Fig. 8. (a) 60 ll droplet without vibration and (b) 60 ll with 2.24 lm vibration amplitude.



we can use a vibrating beam wrapped with thin plastic film, which is useful for bio-chemical applications.

2.3. Particle displacement in a fluid

We performed experiments on particle movements inside a liquid. These particles are made of PMMA (Young modulus E = 2.4–3.3 GPa and density q = 1200 kg/m^3) with diameters of 1 lm, 8 lm and 100 lm. The experiments show different behaviour according to the particle dimension. Small particles move along a closed curve at a speed of about 1 cm s^1 and radius of curvature R = 2 mm; large particles move slowly (1 mm s^1) toward the drop center and stop (see Fig. 10).

Let us analyze the particle behaviour in the fluid. The Reynolds number Re in the fluid is given by

$$Re ¼ \frac{qL_Pv}{g} ¼ 10^6 L_Pv \qquad ð7Þ$$

With q = 10^3 kg m^3, L_P = 1 cm, v = 0.1 m/s and g = 10^3 Pa s, we get Re = 1000 < 2000; therefore we are in laminar flow. At the scale of 100 lm particles the Reynolds number is less than 10. The expression for the friction force is $6pgr_pv$. If we compare that with the inertial force then we can evaluate acceleration using a characteristic time s (we choose s = v/R). We get

$$q_p \frac{4}{3} \bar{p} r_P^3 \frac{v^2}{R} \quad 6pgr_pv \qquad ð8Þ$$

Then

Fig. 9. Value of the vibration force (F_v) function of the vibration amplitude of the beam.

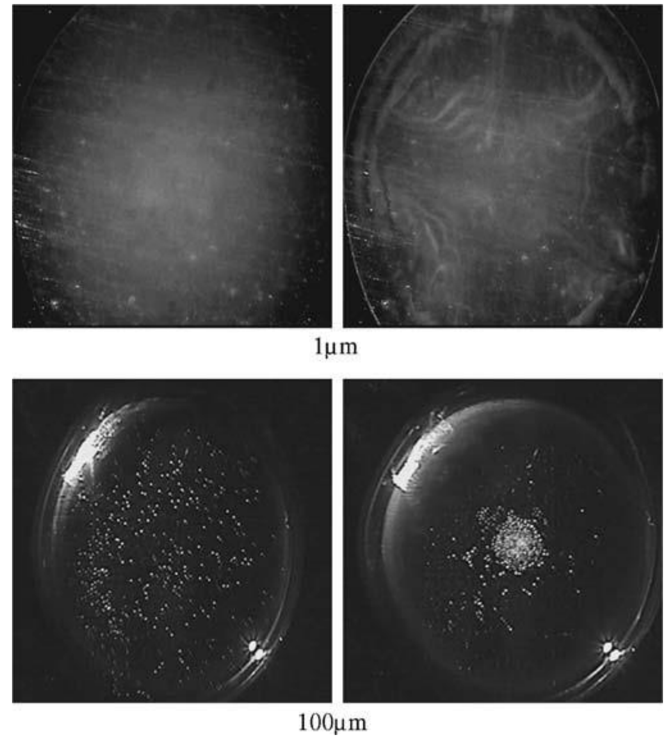


Fig. 10. 1 lm and 100 lm particles displacement; left: before, right: after action of vibration.

$$r_P^2 \frac{9gR}{2q_pv} \qquad ð9Þ$$

With R = 10^3 m, v = 10^2 m s^1, q = 1200 kg m^3, we get r_P^2 4 · 10^7 or r_P 6 · 10^4 m. Therefore we can understand why the small particles follow the internal movement of fluid but not the large (100 lm) one. The large particles are mainly submitted to inertial forces that are proportional to r^3 and their behaviour is not under the control of internal fluid movement. Let us remark that the observed phenomenon is not the same as that observed for particles in a tube filled with water and vibrating on an eigenmode. For this last case, particles move toward a node or anti-node according to the rigidity and the density of particles. It is the node for the particles used here.

3. Discussion

To explain the drop displacement, it is necessary to consider a second order phenomenon creating a time constant force and a force gradient. The force produced by the pressure gradient due to the wave attenuation is significant only for a high frequency device (called acoustic streaming) but is negligible for the frequency used here (30 kHz). Usually the phenomenon put forward for this frequency is the acoustic radiation pressure. This acoustic radiation pressure is given for a plane wave by ð1 þ c_wÞ*du$_{L}$2+

$$P_R ¼ qc^2 \qquad ð10Þ$$

The vibration elongation of the beam is

$$u_v = A\sin(k_v x)\cos(\omega t) \quad (11)$$

Two different cases were considered to estimate the amplitude of the wave in the fluid. For the first case an infinite layer of liquid was considered and for the second case a small layer of liquid was considered, which leads to a stationary wave in the liquid. In the case of an infinite layer of liquid, the elongation wave in the fluid could be written: $u_l(x,z,t) = u_0\cos(\omega t - k_l z)\sin(k_v x)$ (12)

That gives

$$\left\langle \frac{du_L^2}{dz} \right\rangle = \frac{1}{2} k_l u_0^2 \sin^2(k_v x) \quad (13)$$

One can deduce the acoustic radiation pressure:

$$P_R = \frac{1}{4}\rho c_l k_l u_0^2 \frac{1+\gamma}{2} \sin^2(k_v x) \quad (14)$$

With c_w = 6.2 (for water), f = 30 kHz and a vibration amplitude u_0 = 1 lm, we get P_R = 120 Pa which easily allows the drop to be displaced. Let us emphasize the fact that the previous amplitude is chosen to be equal to the amplitude of the vibrating beam. This value has been experimentally obtained by heterodyne interferometry. This equality is correct if the liquid thickness is large compared to the acoustic wavelength k in the fluid. If we consider the case of a small layer of liquid, this approach uses two longitudinal waves propagating in opposite directions (see Fig. 11).

The acoustic field is given by

$$u_l(x,z,t) = \frac{1}{2}\left(u_A(x)e^{ik_l z} + u_B(x)e^{-ik_l z}\right)e^{i\omega t}\sin(k_v x) \quad (15)$$

Taking into account the boundary conditions, at the above liquid layer the free surface implies p = 0 and $u_A = u_B$ and at the liquid-beam interface $u_l(h) = u_{v0}e^{i\omega t}$. The elongation wave in the fluid becomes

$$u_l(x,z,t) = u_0 \frac{\cos(k_l z)}{\cos(k_l h)}\cos(\omega t)\sin(k_v x) \quad (16)$$

For this case k is about 50 mm and the thickness h of fluid is about 3 mm. That is to say $k_l h \ll 1$ that gives

$$\left\langle \frac{du_L^2}{dz} \right\rangle = \frac{1}{2} k_l^4 u_0^2 h^2 \sin^2(k_v x) \quad (17)$$

Therefore, the ratio between thin layer and infinite layer radiation pressure is

$$\frac{\text{Thin layer}}{\text{Infinite layer}} = \frac{\frac{1}{2}k_l^4 u_0^2 h^2 \sin^2(k_v x)}{\frac{1}{2}k_l^2 u_0^2 \sin^2(k_v x)} = k_l h \quad (18)$$

The ratio is about 0.14. For a small layer of fluid if the acoustic amplitude is 0.14 times lower than the infinite layer then the acoustic radiation pressure seems to be too weak to displace the droplets. The fluid compressibility does not seem to be a relevant parameter. In their papers, Qun Wan and Kutznetsov [5,6] study a flow in a channel limited by a vibrating wall. A numerical example is given for a 10 lm vibration amplitude, a frequency f = 21 kHz and the wall vibration wavelength k = 25.4 mm. They find a horizontal speed at the limit of the boundary layer of about 1.3 m/s.

In their case the fluid and the boundary conditions are very different but the vibrating beam conditions are similar to our setup. Let us note that the authors, to solve the problem, use the mass and the momentum conservation equations (Navier–Stokes) for a non-compressible fluid. We conclude that it is possible to create a flow similar to the effect of the acoustic streaming in a non-compressible fluid. Let us remark that the fluid mechanics and acoustics deal with the same problem at the beginning of the formulation but in practice both fields use different approximations. The second order acoustics approach is closer to fluidic than the linear one, and we see that streaming can occur with or without compressibility. Even if we do not have a model for the non-linear vibration of the drop itself, we are in a domain common to hydrodynamics and acoustics. In that case, we write the following equations (mass and momentum conservation equations):

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0 \quad (19)$$

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right) + \nabla p = \vec{F}_{EXT} + \vec{F}_{VISC} \quad (20)$$

According to the case under consideration, we can neglect different terms. For example in non-linear acoustics we neglect the term vÆ$v. The present case is unusual case because the compressibility and the propagation are neglected but not the non-linearity. The main difficulty is the boundary condition on

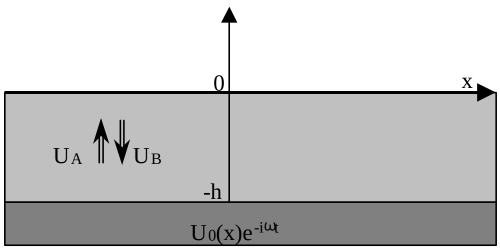


Fig. 11. Acoustic field in a liquid.

the water surface with capillarity.

Let us say that the acoustics theory with an amplitude the acoustic wave in the liquid taken equal to the beam vibrati amplitude gives an acoustic radiation pressure value compatible wi the experimental one. In fact, it is not coincidence because even if t vibration amplitude of the beam is not the amplitude of the acous wave in the liquid, it still is the amplitude of fluid mechanic oscillation. Consider also a paper of Daniel et al. [7] that describes t horizontal displacement of the drop by low frequency (about 100 H They give an explanation based on hysteresis of the lin drop/air/liquid. Obviously, t

conditions are very different but it shows the effect of nonlinearity of vibration on fluids.

4. Conclusion

The described experiment shows a practical result for droplet displacement at low frequency vibrations (30 kHz). In fact the droplet can be displaced on a metallic beam with (or without) a thin plastic sheet. We showed that the vibration also has an effect on particles within the liquid droplets. This last allows the generation of concentration or mixing in the droplet. It is also of theoretical interest that non-linear phenomena in the ‘near field acoustic’ domain can be used to explain droplet displacement.

Acknowledgement

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