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Microfluidic mixing through electrowetting-induced droplet oscillations
Oscillation spectrums and beat phenomenon of a water droplet driven by electrowetting

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Droplet oscillation is an intriguing study that has inspired researches in many fields, and it becomes increasingly important on biochip applications. In this study, amplitude and phase spectrums and beat phenomenon of water droplet oscillation driven by ac electrowetting are studied using frequency scanning method. It is observed by experiment that at resonant frequencies of water droplets phase differences between the driving voltage and the droplet motion are ±90°. In addition, near resonant frequencies beat phenomenon of water droplet oscillation is also observed by experiment. © 2009 American Institute of Physics. [DOI: 10.1063/1.3120563]

Droplet oscillation is a very interesting phenomenon that has been inspiring researchers in many fields. The analytical solution of the oscillation mode of droplet was proposed by Rayleigh in 1879, and later in 1936 in the field of nuclear physics Bohr adopted the concept of droplet oscillation to describe the nuclear fission and explained many behaviors of nucleus. Recently, droplet oscillation again draws people’s attention due to the emerging development of biochip techniques that are related to the transportation, oscillation, and mixture of droplets on the chip. Although the analytical solution of the droplet oscillation mode has been proposed in 1879, there have been few systematic researches on frequency spectrum and eigenmodes of droplet oscillation by experiments. In this paper, water droplet oscillation driven by ac electrowetting is studied. The amplitude spectrum, phase spectrum, and beat phenomenon of water droplet oscillation are investigated using frequency scanning technique, which allows whole frequency scanning on droplet oscillation.

The schematic pictures of the device used in our experiment are shown in Fig. 1(a). There are three electrodes across the droplet, two of them being located at the two sides served as driving electrodes and the other in the central area served as the ground electrode. The driving electric potential is periodic square wave with offset, as shown in Fig. 1(b), and the droplet is actuated/relaxed during the active/inactive period. The side length of each electrode is set to be 1.5 mm and the gaps between electrodes are chosen to be 50 μm, which is about 3.3% of a single electrode, for effective actuation. Figure 1(c) shows the experimental setup of the frequency scanning system in our study. The periodic driving voltage is provided by a function generator that is connected to a high voltage amplifier. The root-mean-square (rms) amplitude for the potential is set to be 150 V. He–Ne laser incidents from one side of droplet as probing beam, and the position sensor detects the movements of interference fringes refracted from the oscillating droplet. The images of oscillating droplets are taken by an optical microscope equipped with a charge-coupled device (CCD) camera.

Figure 2(a) shows the amplitude spectrum of oscillation for a 15 μl de-ionized (DI) water droplet at various driving frequencies. The voltage amplitudes in the vertical axis obtained from the amplified electrical signals of the position sensor are proportional to the oscillating amplitude of the droplet. Resonant frequencies that correspond to maximum vibrating amplitudes appear at 26, 53, 78, 102, 124, 148, 170, and 191 Hz. The frequency scanning system proposed here is utilized in the observation of droplet oscillation that is conventionally investigated using high-speed video camera. This measurement method can realize the observation and analysis of droplet oscillation at very high frequencies in contrast to the limited picture rate of high-speed video camera that fails to observe motions with very high frequency.

Figure 2(b) shows the phase spectrum of oscillation for the cosine value of the phase difference between the driving voltage and the droplet motion. From Fig. 2(b) it can be found that φ varies from −180° to 180° repeatedly with

![Schematic pictures for the device and experimental setup used in our experiment.](https://example.com/schematic.png)

**FIG. 1.** (Color online) Schematic pictures for the device and experimental setup used in our experiment. (a) (Left) Side view of the device. (Right) Top view of the device, two side pads as the driving electrodes and the central pad as the ground electrode. F is the resultant force of electrocapillary forces. (b) Driving electric potential in periodic square waves with offset. The droplet is actuated/relaxed during the active/inactive period. (c) Experimental setup of the frequency scanning system.
increasing frequency, and at resonant frequencies the phase differences $\phi$ are $\pm 90^\circ$ like the behavior of an ideal spring under driving force. To avoid the shift in resonant frequencies due to the evaporation of water droplet, the measured data in Fig. 2 are a combination of five measurements made in frequency ranges from 20 to 60 Hz, 60 to 100 Hz, 100 to 140 Hz, 140 to 180 Hz, and 180 to 200 Hz. Figure 3 shows the side view images of an oscillating droplet taken by CCD camera. The ghosting in these images represents the standing waves formed on the droplet surface. The standing waves with maximum amplitudes also occur at the resonant frequencies.

For an isolated droplet, the resonant frequency of the $n$th mode can be expressed as follows, neglecting the viscous damping:

$$f_n = \sqrt{(n-1)(n+2)\gamma/4\pi^2\rho R^3},$$

(1)

where $R$, $\rho$, and $\gamma$ are the volume averaged droplet radius, droplet density, and surface tension, respectively. The calculated resonant frequencies using Eq. (1) are listed in Table I. The discrepancy between the theoretical prediction and the experimental results mainly comes from the different boundary conditions between the two cases: an isolated droplet used in Eq. (1) and a sessile droplet used in the experiment, where the effective curvature radius $R_{cur}$ of the droplet has to be taken into account in estimating the resonant frequencies, and Eq. (1) can be rewritten as

$$f_n = \sqrt{(n-1)(n+2)\gamma/4\pi^2\rho R_{cur}^3}.$$

(2)

From Table I it can be found that the calculated resonant frequencies using Eq. (2) are closer to the experiment results compared to the ones obtained from Eq. (1). In addition to the measured resonant frequencies that are close to the calculated resonant frequencies using Eqs. (1) and (2), other resonant frequencies are also observed in our experiment. The difference between the resonant frequencies obtained experimentally and theoretically is believed to come from the neglected frictional force between the droplet and the solid plate.

For a driving electric potential that is periodic square wave with offset, the phase difference $\phi$ between the driving potential and the droplet motion can be expressed as:

$$\phi = \tan^{-1}\left[\frac{2\omega}{\omega^2 - \omega_n^2}(n-1)(2n+1) - \frac{\mu}{\rho R^2}\right],$$

(3)

where $\mu$ is the droplet viscosity. From Eq. (3) it can be found that when the driving voltage frequency $\omega$ equals the droplet natural frequency $\omega_n$, the phase difference $\phi$ approximates $\pi/2$. Different from the ideal case calculated above, the droplet in real case has hysteresis when an electric voltage is applied back and forth on the droplet, meanwhile, the electrical stress acting on the three-phase contact line of the droplet depends on the contact area and contact line between the droplet and the electrodes on the device, and the contact area and contact line are varied with time during the droplet oscillation.

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shows the time-dependent amplitude variations of a 15 μl DI water droplet near the resonant frequency of about 78 Hz.

From Fig. 4 it can be found that the beat caused by the interference between the natural resonance of water droplet and the periodic driving voltage has the beat period, or the period of the envelope, of $1/(f_{\text{ext}} - f_0)$, where $f_{\text{ext}}$ is the frequency of the external driving voltage and $f_0$ is the resonant frequency of the water droplet. In summary, amplitude spectrum, phase spectrum, and beats of water droplet oscillation driven by ac electrowetting are investigated using frequency scanning technique. The results of this study can provide important information for the mixing and separating of microdroplets in the applications of emerging biochip technology.

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