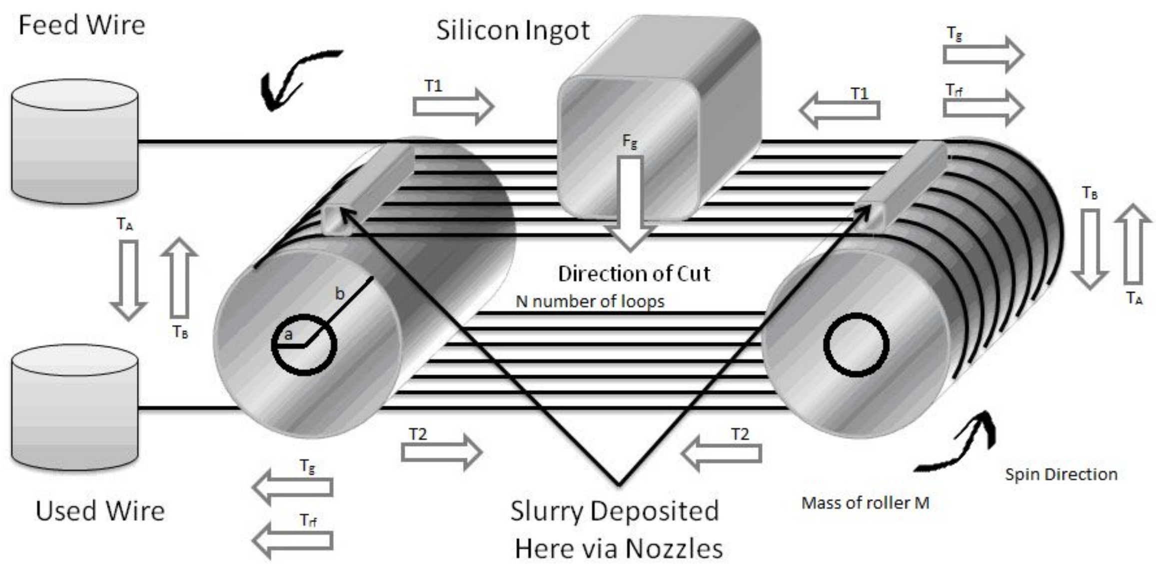
MSD 1 P16601: Guide Roller Team
Torque Analysis/Feasibility
Analysis by Viniamin Tokarchuk

Analysis for one roller (assume standard metric units):



Modified image from:
http://www.crs-reprocessing.com/resources/a-hierarchy-of-slurry-reprocessing-options.php

$$\sum T = I \propto$$

$$\omega = \frac{V}{R} = \frac{V\,wire}{b}$$

$$\alpha = \frac{a}{R}$$

$$I = \tfrac{1}{2} M(a^2 + b^2)$$

$$\sum T = T_A - T_{rf} - T_B - T_g$$

T_A is Applied Motor Torque (Nm)
T_B is torque from bearing friction
T_{rf} is torque from the rolling friction due to wire on rollers
T_g is the torque due to the glass friction from cutting

One rather valid assumption is that both guide rollers accelerate and move exactly together, so that there is no tension differences in the wire. Therefore, rolling friction is used between the wire and rollers. Another not very valid assumption is that when the glass ingot is pressed into the wire, the tension will be adjusted to remain the same and is distributed equally. This is not quite accurate since the wire leading up the ingot will experience slackening, while the wire after cutting will be tauter. Also, putting the ingot into the wire would in theory add to the tension.

Bearing friction can be estimated by the following expression (Appendix A link):

$$T_B = 0.5\mu_b P d_B$$

Where
μ_B is the coefficient of friction given by table in Appendix A
P is the force applied to each bearing (Nm)
d_B is the bearing bore diameter (m)
can say that:

$$P = (T_1 + T_2)N$$

Assuming there are two bearings, it would put a factor of 2 in the equation, but it would also mean each bearing bears half the load, so the equation remains the same.

Rolling friction can be approximated by:

$$T_{rf} = \mu_r F nR = N\mu_r(T_1 + T_2)b$$

μ_r is the rolling friction between the wire and guide rollers
F_n is the normal force which is equivalent to the combined tensions T_1 and T_2
N is the number of wire loops (for 30 cuts N=30)
b is the radius of the guide roller

The friction from the glass was approximated by:

$$T_g = N\mu_g F_g b$$

F_g is the normal force of the glass pushing on one wire segment.
μ_g is the cutting coefficient of friction.
Therefore, the necessary torque needed for the motor can be modeled by:

$$T_A = T_B + T_{rf} + T_g + I\alpha$$

$$T_A = 0.5\mu_b(T_1 + T_2)Nd_B + N\mu_r(T_1 + T_2)b + N\mu_g F_g b + \tfrac{1}{2}M(a^2 + b^2)\alpha$$

$$T_A = \mu_b NTd_B + 2\mu_r NTb + \mu_g NF_g b + \tfrac{1}{2}M(a^2 + b^2)\alpha$$

The rolling friction of train wheels on a track is about 0.001 (Appendix C), but obviously may be way different for the wire on roller case. The three coefficients of friction are really unknown. F_g is also difficult to calculate, but can be done eventually by either direct or indirect measurements from the DS 264. From the project readiness package the force of the ingot on the wires was given as 150-200 Newtons. To get F_g, assume that half the guide roller was 2 mm spacing and half at 6 mm. Therefore, 200 wires on one half and about 67 on the other. Dividing 200 N by 267 loop gives .75 N/loop. The bearing diameter can be approximated to be 0.16 meters, but overall it is clear that the bearings will not be adding much torque with a coefficient of friction around 0.0015 for low loop numbers. Below, the table shows 4 calculations using the above equation with estimated values for each term. It was assumed that the rollers were 3" thick and hollowed out. The mass M for calculation 4 was calculated by taking the ratio of the volume of the DS 264 rollers and smaller 230 mm rollers (to fit 9" diameter piece- assuming two roller design) and multiplying it by 130 kg, the weight of the DS 264 rollers. The acceleration needed was assumed to be 2 m/s^2 for the angular acceleration determination.

Table 1: Calculations/Results from Derived Equation

For calculation 1:	For calculation 2:	For calculation 3:	For calculation 4:
N=8000 loops	N=800 loops	N=30 loops	N=30 loops
μ_b=.0015	μ_b=.0015	μ_b=.0015	μ_b=.0015
T_1=T_2=25 N	T_1=T_2=25 N	T_1=T_2=25 N	T_1=T_2=25 N
d_B=0.16 m	d_B=0.16 m	d_B=0.16 m	d_B=0.1 m
μ_r=.005	μ_r=.005	μ_r=.005	μ_r=.005
b=0.16 m	b=0.16 m	b=0.16 m	b=0.115 m
μ_g=0.1	μ_g=0.1	μ_g=0.1	μ_g=0.1
F_g=0.75 N	F_g=0.75 N	F_g=0.75 N	F_g=0.75 N
M=130 kg	M=130 kg	M=130 kg	M=105.8 kg
a=0.129 m	a=0.129 m	a=0.129 m	a=0.077 m
α=12.5 rad/sec^2	α=12.5 rad/sec^2	α=12.5 rad/sec^2	α=17.4 rad/sec^2
Results:			
T_B=48 Nm	T_B=4.8 Nm	T_B=0.18 Nm	T_B=0.113 Nm
T_{rf}=320 Nm	T_{rf}=32 Nm	T_{rf}=1.2 Nm	T_{rf}=0.863 Nm
T_g=96 Nm	T_g=9.6 Nm	T_g=0.36 Nm	T_g=0.26 Nm
T_I=34.3 Nm	T_I=34.3 Nm	T_I=34.3 Nm	T_I=17.64 Nm
T_A=498.3 Nm	T_A=80.7 Nm	T_A=36.04 Nm	T_A=18.88 Nm

The first calculation is an approximation of the machines maximum torque needed. Each guide roller motor is rated at 500 Nm so surprisingly the equation makes some sense. Calculation 1 is essentially a test of the equations validity and it seems to hold up. However, 8000 loops is impossible when we take into account the thickness of the wire; 4000 loops is more reasonable. Calculation 1 would then be about 266 Nm, which may mean that some of the coefficients of friction are slightly greater in reality. Calculation 2 removes wire loops by a factor of 10 and the rolling friction drops significantly along with the other terms by a factor of 10. Doing 30 cuts with the DS 264 only requires about 36 Nm out of the 500 Nm available according to this derivation, so clearly this is the reason we are designing a smaller machine (calculation 3). Also, bearing friction, rolling resistance, and cutting friction are now negligible in comparison to the inertia of the rollers.

Calculation 4 takes a roller that would fit 9” ingot diameters (same design as DS 264 but scaled down with same length) with 30 cuts and the same maximum wire cutting speed. The result is very small and will be smaller considering the new rollers will not be the same length as the current design. 9" is about 230 mm. However, to maintain the maximum wire speed of 15 m/s, the roller RPM must increase to about 1245. Three-phase motors usually have a standard RPM values such as 3600, 2400, and 1200. Therefore, to take advantage of those motor standards, a 240 mm (for example) guide roller could be built in a two-roller system.

Power rating for a motor as given by MachineDesign.com (Appendix D)

$$\text{hp of motor} = T*N/5250$$

where N is the motor rpm and T is the required torque.
From the above results in calculation 4, the required power of the motor would be approximately 5 hp based on a speed of 1200 rpm. If the rollers were actually half the size (420 mm instead of 840 mm), 2.5 to 3 hp motors would be enough.
Here is a westinghouse motor catalog example, the 5 hp motor has about 22 Nm of torque and costs about 1300 $ (much cheaper on other websites such as witmer motor service).

MAX-PE™ NEMA Premium CC002A

TYPE: AEHH8P, AEHH8PCF - FOOTED C-FACE, NEMA PREMIUM

Effective 06-14-15
Supercedes 05-01-13

HP	RPM	FL EFF (%)	FL PF (%)	FL AMPS (460V)	FOOTED FRAME				FOOTED C-FACE			
					FRAME	CATALOG NO.	SHIPPING WT. (lbs.)	LIST PRICE ($)	FRAME	CATALOG NO.	SHIPPING WT. (lbs.)	LIST PRICE ($)
1	3600	82.5	85.0	1.34	143T	NP0012	83	387	143TC	NP0012C	85	418
1	1800	85.5	73.0	1.50	143T	NP0014(1)	48	394	143TC	NP0014C(1)	50	449
1	1200	82.5	65.5	1.73	145T	NP0016(1)	90	500	145TC	NP0016C(1)	92	577
1.5	3600	84.0	83.5	2.00	143T	NP1/52	85	442	143TC	NP1/52C	87	500
1.5	1800	86.5	78.0	2.08	145T	NP1/54	78	444	145TC	NP1/54C	80	508
1.5	1200	87.5	63.5	2.53	182T	NP1/56(1)	120	526	182TC	NP1/56C(1)	122	600
2	3600	86.5	86.0	2.52	145T	NP0022	62	469	145TC	NP0022C	64	544
2	1800	86.5	78.0	2.78	145T	NP0024	90	476	145TC	NP0024C	92	553
2	1200	88.5	70.5	3.00	184T	NP0026	132	589	184TC	NP0026C	134	680
3	3600	88.5	90.0	3.53	182T	NP0032	130	552	182TC	NP0032C	132	654
3	1800	89.5	84.0	3.74	182T	NP0034	135	537	182TC	NP0034C	137	624
3	1200	89.5	78.0	4.02	213T	NP0036(1)	164	773	213TC	NP0036C(1)	166	987
5	3600	88.5	92.5	5.72	184T	NP0052	135	679	184TC	NP0052C	137	771
5	1800	89.5	85.5	6.12	184T	NP0054	133	618	184TC	NP0054C	135	683
5	1200	91.0	82.5	6.24	215T	NP0056	210	1,109	215TC	NP0056C	212	1,260

https://www.tecowestinghouse.com/PDF/TWMC_price_book.pdf

An allen bradley VFD drive shown below would do the job in controlling the motor. Example part no: 20F11FD011AA0NNNNN

RexelUSA.com has a price of 2085 $ for one so definately not cheap.

An Allen Bradely 847H-DN2C-RE05000 (random choice to approximate cost) encoder costs about 800$.

A 3 hp motor would cost about 1000 $ if we wanted to do a two motor drive design.

Therefore, very approximate cost estimate for a 2 motor design (ignoring PLC costs) would be about 7800$. The one motor design would use one less VFD's therefore would be cheaper 4200$, but would require designing the interlocking mechanical design etc. Table 2 uses prices from witmer motor service.

Table 2 Cost Analysis

	Model No.	Price $	2 Motor Design	1 Motor Design
3 Hp Motor	NP0036	345	x2	
5 Hp Motor	NP0056	497		x1
Encoder	847H-DN2C-RE05000	800	x2	x1
VFD	20F11FD011AA0NNNNN	2085	x2	x1
Totals			6460	3382

VFD image from ab.rockwellautomation.com

motor image from witmermotorservice.com

Roller RPM Analysis

Since the inertia affects the torque necessary the most, it can be changed by changing the roller radius, hollow diameter a, or changing the mass. The thing to keep in mind is that if the inertia becomes too small, the other terms from large equation above will start playing a larger role so the torque estimate will be less accurate.

However, assuming the maximum ingot size is 9", we can make the rollers 230 mm in diameter or 115 mm. Assuming we scale down the current rollers (130 kg) to that size, we can estimate the weight of the smaller rollers. Assuming weight is directly proportional to volume:

$$V = \pi r^2 l$$

$$\frac{V_1}{V_2} = \frac{r_1^2 l_1}{r_2^2 l_2} = \frac{M_1}{M_2}$$

Keeping the length of the rollers constant, but decreasing from 320 mm to 230 mm diameter, should decrease the mass to about 67.2 kg, which is a pretty large drop. A 28% drop in radius leads to a 48% drop in mass. The current guide rollers are not solid so that is why there is an a^2 term in the inertia. Assuming the cylinders were solid, the ratio of inertias would be:

$$\frac{I_1}{I_2} = \frac{M_1 b_1^2}{M_2 b_2^2}$$

So there's like a 73.3% reduction in inertia by reducing the radius 28%. A smaller inertia means a smaller motor, but it also may suggest that we need to size the motor a little larger than expected since the frictional terms will start playing a larger role. Changing the length of the roller will also reduce the inertia.

Since the required speed is 15 m/s, the angular velocity is directly related to guide roller diameter. Decreasing to 230 mm, the rpm of the motor must increase to 1246 rpm from the original 900, so not a bad trade off at all. Basically a 39% increase from a 28% decrease in diameter. 1200 rpm is a standardized value in the industry so the rollers would have to be about 120 mm in radius (240 mm diameter) to take advantage of this fact and maintain approximately a max speed of 15 m/s for the wire speed. Using VFD's, the wire speed can be adjusted from 0 to 15 m/s just like in the DS 264.

Appendix A
http://www.skf.com/us/products/bearings-units-housings/ball-bearings/principles/friction/estimating-frictional-moment/index.html

table 1 - Constant coefficient of friction μ for open bearings

Bearing type	Coefficient of friction μ
Deep groove ball bearings	0,0015
Angular contact ball bearings	
– single row	0,0020
– double row	0,0024
– four-point contact	0,0024
Self-aligning ball bearings	0,0010
Cylindrical roller bearings	
– with a cage, when $F_a \approx 0$	0,0011
– full complement, when $F_a \approx 0$	0,0020
Needle roller bearings with a cage	0,0020
Tapered roller bearings	0,0018
Spherical roller bearings	0,0018
CARB toroidal roller bearings with a cage	0,0016
Thrust ball bearings	0,0013
Cylindrical roller thrust bearings	0,0050
Needle roller thrust bearings	0,0050
Spherical roller thrust bearings	0,0018

Appendix B

Young's Modulus is known to be about 200 GPa for steel so given a length of wire L with no tension on it, the forces from the glass cutting can be estimated. As N becomes very large (such as for the DS 264) the friction starts to be significant. However, from approximating the three coefficients of friction with N=30, it was evident that the biggest concern is really the inertia of the rollers.

Stainless steel wire 195 GPa for 80 µm diameter
chrome-extension://oemmndcbldboiebfnladdacbdfmadadm/http://drum.lib.umd.edu/bitstream/1903/7998/3/tr_2008-9.pdf

for calculating F_g this equation might come in handy

$$\frac{F}{S} = Y\frac{\Delta L}{L}$$

F is the force on the wire
S is the wire cross-sectional area
Y is Young's Modulus
L is the original wire length and ΔL is the change due to the force applied.

https://ccrma.stanford.edu/~jos/pasp/Young_s_Modulus_Spring_Constant.html

Appendix C

Rolling Resistance Coefficient		
c	c_l *(mm)*	
0.001 - 0.002	0.5	railroad steel wheels on steel rails
0.001		bicycle tire on wooden track
0.002 - 0.005		low resistance tubeless tires
0.002		bicycle tire on concrete
0.004		bicycle tire on asphalt road
0.005		dirty tram rails
0.006 - 0.01		truck tire on asphalt
0.008		bicycle tire on rough paved road
0.01 - 0.015		ordinary car tires on concrete
0.03		car tires on tar or asphalt
0.04 - 0.08		car tire on solid sand
0.2 - 0.4		car tire on loose sand

http://www.engineeringtoolbox.com/rolling-friction-resistance-d_1303.html

Appendix D

Horsepower estimation

http://motion.schneider-electric.com/downloads/whitepapers/Electric_Motors_whitepaper.pdf