Take-Up Spool Sizing Analysis Summary

Questions being answered:

- What is the proper size for the take-up spool in the P16602 wire movement fixture?

Assumptions:

- Wire is spooled at a constant tension of 25N
- Wire has a constant diameter of 0.175 mm
- Total wire length held on a full take up spool is 381 km (3 times that of the supply spool)
- Perfect layer wind on the take-up spool
- Compressive strength of cast iron 570-1290 MPa [1]
- Calculations are an overestimation of drum pressure
- State of plane stress in the drum (no stress in the axial direction)
- Modelled as a thick walled pressure vessel

Equations:

1. Drum pressure calculation [2]
   - $P$ is the uniform pressure on the spool drum
   - $T$ is the wire tension during spooling
   - $D$ is the spool diameter
   - $d$ is the wire diameter
   - $x$ is the radial increase per layer (see figure 1)
   - $N$ is the number of layers
   - $i$ is the current layer (zero indicating the first layer)

   $$ P_{total} = \sum_{i=0}^{N} \frac{2T}{(D+ix)d} $$

   EQN (1)

Figure 1: Schematic showing a section of a perfect wire layer wind on a spool used to calculate the radial increase per layer, $x$

$$ x = d \times \sin(60^\circ) $$

EQN (2)
2. Stress calculation in a thick wall cylinder using polar coordinates
   - $\sigma_\theta$ is the stress in the angular direction
   - $\sigma_r$ is the stress in the radial direction
   - $\sigma_z$ is the stress in the axial direction
   - $P_0$ is the external pressure calculated using EQN (1)
   - $r_0$ is the outer radius of the spool drum
   - $r_i$ is the inner radius of the spool drum

![Stress equations](image)

**EQN (3-5)**

**EQN (6-8)**

**Figure 2:** Stress equations taken from [3]


$$\left[\frac{(\sigma_\theta - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2}{2}\right] = \sigma_{Von Mises}$$  

**EQN (9)**

**Analysis:**

1. Set up an excel spreadsheet to calculate drum pressure for various number of layers, this requires an initial estimate for the outer diameter and length of the spool drum.
2. Change parameters (outer diameter and length) until reasonable pressures are found.
3. Use the pressure found for the number of layers needed to store 381 km of wire in the stress calculations.
4. Set up a second excel spreadsheet to calculate principal stresses and the Von Mises stress for different inner radii ranging from 0 – 0.1 meters.
5. Select the largest possible inner radii while still maintaining stresses less than the compressive strength of cast iron.
6. Find the diameter of a full spool and add 4 cm to find the flange diameter (this ensures that the flange extends 2 cm past the last wire layer).

**Recommendation:**

After completing all the calculations and analysis explained above, the take up spool drum must be 0.2 m in outer diameter, OD, 0.13 m in inner diameter, ID, and 0.4 meters in length, L, in order to achieve best results (low pressure, low weight, low cost, and reasonable diameter). This results in a drum pressure of about 280 MPa and a max stress of 965 MPa which is well within the range of compressive strength for cast iron. In order to maintain a relatively constant cast
thickness the flanges will be 2 cm thick, Ft, and 0.31 m in diameter, FD. This recommended spool is over designed given the pressure calculation approach. With these spool dimensions, the cast would be about 0.01 m³ in volume and about 76 kg (~170 lbs) in mass.

Next Steps:

1. Design spool mounting and locating solutions
2. Complete cast drawing
3. Contact casting companies for quotes and lead times
4. Order spool
5. Receive spool
6. Complete post-casting machining to finish off design features and surface finish
References: