Design of a Test Loop for Performance Testing of Steam Turbines Under a Variety of Operating Conditions

by

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Abstract

The steam turbine is one of the most widely used energy conversion devices in the world, providing shaft power for electricity production, chemical processing, and HVAC systems. There are new opportunities in growing renewable and combined cycle applications. End-users are asking for energy efficiency improvements that require manufacturers to renew their experimentally verified design methods.

A structured design approach was carried out along three integrated research thrusts. The first two thrusts, Turbine Performance Prediction and Measurement Planning, were carried out with the aim of supporting the theoretical modeling required for the third thrust, System Modeling. The primary use of the steam turbine test loop will be to improve performance prediction techniques. Thus the primary focus of the first thrust was to describe empirical loss correlations found in the literature. For the second thrust, a preliminary review of measurement codes and standards was carried out to determine their impact on overall test loop design. For the third thrust, quasi-steady theoretical models were derived from first principles for the turbine, condenser, pump, boiler, and pipe components using control volume analyses. The theoretical models were implemented in a new open source simulation environment that carries out the calculation process over a range of up-to three turbine model inputs.

A parametric study was undertaken with the goal of defining preliminary design specifications for the test loop components. The test loop was simulated across a wide range of steady states for three different turbine blade configurations, each at three different values of the blade row enthalpy-loss coefficient. The parametric study demonstrates full coverage of possible turbine operating conditions. The results of the simulations were analyzed to narrow the required operating range of the test loop to a series of turbine test paths. The final operational envelope yielded a set of test loop component requirements for the condenser, pump, boiler, and dynamometer. These requirements were used to recommend off-the-shelf options available from manufacturers of each component type.
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Nomenclature

\[
\begin{align*}
A & \quad \text{area} \\
AF & \quad \text{air to fuel ratio} \\
C & \quad \text{chord length} \\
CP & \quad \text{specific heat capacity at constant pressure} \\
Cr & \quad \text{heat capacity ratio} \\
CV & \quad \text{specific heat capacity at constant specific volume} \\
D & \quad \text{diameter} \\
f & \quad \text{friction factor} \\
gc & \quad \text{gravitational constant} \\
h & \quad \text{enthalpy} \\
H & \quad \text{head} \\
L & \quad \text{length} \\
M & \quad \text{speed of sound} \\
m & \quad \text{mass flow rate} \\
N & \quad \text{shaft speed} \\
NTU & \quad \text{number of transfer units} \\
P & \quad \text{pressure} \\
Q & \quad \text{volume flow rate} \\
\dot{Q} & \quad \text{heat transfer rate} \\
r & \quad \text{radius} \\
R & \quad \text{reaction} \\
Re & \quad \text{Reynold’s number} \\
s & \quad \text{entropy} \\
S & \quad \text{blade spacing} \\
t & \quad \text{blade thickness} \\
T & \quad \text{temperature} \\
U & \quad \text{blade speed} \\
UA & \quad \text{heat exchanger characteristic value} \\
v & \quad \text{specific volume} \\
V & \quad \text{absolute velocity} \\
W & \quad \text{relative velocity} \\
\dot{W} & \quad \text{power} \\
Y & \quad \text{pressure loss coefficient}
\end{align*}
\]
Greek
\[\alpha\] absolute flow angle
\[\alpha'\] relative flow angle
\[\beta\] blade angle
\[\delta W\] stage work
\[\Delta W\] turbine work
\[\epsilon\] heat exchanger effectiveness
\[\eta\] efficiency
\[\gamma\] ratio of specific heats
\[\mu\] dynamic viscosity
\[\omega\] shaft frequency
\[\Phi\] flow coefficient
\[\Psi\] work coefficient
\[\rho\] density
\[\sigma\] velocity ratio
\[\tau\] torque
\[\zeta\] enthalpy loss coefficient
\[\zeta_s\] entropy loss coefficient

Subscripts
\[0\] stagnation property
\[A\] station at inlet to turbine inlet CV
\[B\] station at exit from turbine inlet CV
\[C\] cold side, station at inlet to turbine diffuser CV
\[D\] station at exit from turbine diffuser CV
\[e\] exit
\[h\] enthalpy-based
\[H\] hot side
\[i\] inlet
\[m\] mean
\[MLV\] mixed liquid-vapor
\[N\] nozzle
\[P\] pressure-based
\[R\] rotor
\[s\] entropy-based or isentropic
\[S\] stator
\[SC\] subcooled liquid
\[SH\] superheated
\[ss\] stage isentropic
\[tot\] total quantity for a stage
\[ts\] total-to-static
\[tt\] total-to-total
\[x\] axial component
\[y\] tangential component
Chapter 1

Introduction

1.1 Motivation

Growing markets for solar thermal and combined cycle power plants, which produce low to medium temperature working fluids, present future opportunities for steam turbines that perform efficiently over a broad range of operating conditions. Additional opportunities for new turbines and energy efficiency upgrades exist in shaft-driven applications like HVAC and fossil fuel extraction. A key issue for turbine end-users is that the turbine will operate at off-design conditions for a large portion of its life. New design and manufacturing techniques for axial turbines show promise for improved efficiency at design-point and off-design operations. Capitalizing on the current and future states of the steam turbine marketplace will require performance prediction methods that provide more accurate and higher resolution results, so that manufacturers can make confident predictions for their customers.

The steam turbine designer has several options in the performance prediction toolbox, including empirical correlations, numerical techniques such as computational fluid dynamics, and theoretical analysis, all of which are supported by experimentation. Many prominent research institutions that study axial turbine performance are equipped with industry-funded gas turbine laboratories. This is partly because high temperature gas turbines offer a higher Carnot efficiency than steam turbines; it is primarily because the gas turbine’s high power density elicits financial backing from the aircraft industry, where research dollars are more readily available than in the stationary power industry. As a result, there are few novel methods aimed at steam turbine performance prediction in the open literature.

Conventional empirical correlation methods cited in recent literature were developed by
gas turbine researchers, including Ainley and Mathieson [6], Dunham and Came [14], Craig and Cox [15], and Kacker and Okapuu [7]. Their results can be applied directly to noncondensing steam turbines, with some modification made for state equations. The extension to condensing steam turbines requires a correction factor which is derived from theory and verified empirically. In their 1970 paper, Craig and Cox [15] state that numerous methods exist for applying a correction factor to account for two-phase flow, none of which exhibits more merit than the others. A review of the recent literature in steam turbine research reveals a lack of correlation development. In 2002, Bassel [16] used the simple method recommended by Craig and Cox, that is to assume a loss of 1% in stage efficiency per every 1% of mean stage wetness. If accurate and high resolution steam-specific turbine performance prediction is to evolve to quantitative analysis, new and sustained steam turbine experimentation is needed.

1.2 Industry Practice

Understanding the business model of potential commercial users is the first step in the design of the steam turbine test loop. It is anticipated that Dresser-Rand will be its first user. Dresser-Rand operates within a specific market niche for steam turbines that produce 100MW or less. According to Fred Woehr, Vice President of Research and Development for the Steam Turbine Business Unit, Dresser-Rand’s business focus is to provide products (turbines, pumps, compressors, and others) that give the best performance. A customer specifies a set of requirements based on any combination of inlet and outlet flow conditions, power input or output, and rotational speed. Dresser-Rand’s goal is to match the customer’s application, as opposed to getting orders for as many of the same product as possible, because their end-users see the value of high efficiency over the lifetime of the turbine.

In order to select and design the most energy efficient turbine for a specific customer application, Dresser-Rand customer engineers require specifications for the turbine operating conditions from the end-user. In order to illustrate their process of thermodynamic performance prediction, let us consider two common customer specification scenarios. One scenario is when a customer specifies the steam flow rate at a design point (temperature
and pressure), possibly with variations in availability for seasonal, weekly, daily, or hourly changes. In this case, the customer wants Dresser-Rand to predict how much electric power can be produced by a given generator. Another scenario is when a customer specifies a horsepower for a driven product, such as a centrifugal pump, and a temperature and pressure for the steam source. In this case, the customer wants Dresser-Rand to predict how much steam is needed to drive the device. Once the customer specifications are established, Dresser-Rand can carry out their predictions in order to select or design the right turbine.

In order to carry out their predictions, Dresser-Rand has a set of software-based selection and design tools organized around three segments of turbine design and selection. First, for turbines that produce 25MW or less, Dresser-Rand uses a parametric selection tool to choose a design from standard off-the-shelf turbines. Second, for turbines that produce between 25MW and 100MW, or for turbines that are operated in remote locations or under severe operating conditions, Dresser-Rand performs a custom design using another set of software tools. Third, for turbines that require new or derivative product development, Dresser-Rand generates custom prediction methods based on literature reviews, experimentation, and externally purchased design tools. It is this third segment of performance prediction that has inspired the present thesis.

1.3 Objectives

The sharp contrast in academic research between steam and gas turbines and the business opportunities expressed by emerging combined cycle power plants necessitate the building of a steam turbine research facility. The purpose of this thesis is to create a first-order design for a steam turbine test loop that will be installed at the future Rochester Energy-systems Experiment Station (REES) on the campus of the Rochester Institute of Technology (RIT). The first aspect of the design, and the primary focus of this thesis, is the selection of components that have an appreciable impact on the thermodynamic performance of the test loop cycle, including, at a minimum, those labeled in Fig. 1.1. The second aspect of the design, which will be discussed only as it affects component selection, is the selection and
placement specifications for the sensors necessary to deduce the desired performance characteristics of the steam turbine being tested. For example, temperature, pressure, and flow measurements in the test loop piping will be required to characterize overall steam turbine efficiency, while internal turbine pressure and velocity sweeps would be required to determine stage efficiencies. The system design will be analyzed using a simulation environment that can model the behavior of the test loop under a range of operating conditions.
Chapter 2

Statement of Work

The end product of this thesis will be a quasi-steady open source open architecture system modeling tool and a resulting preliminary system design for a steam turbine test loop. The primary function of the test loop will be to develop performance prediction schemes for turbine selection and design tools. The progression from commercial R&D testing needs to a preliminary system design will employ a structured approach, with the project segmented into three separate thrusts, as depicted in Fig. 2.1. The first thrust, *Turbine Performance Prediction*, is an investigative stage that will shed light on how the results of the steam turbine testing will be used in industry. A review of axial turbine performance prediction will ensure the final test loop design serves the needs of the modern steam turbine community. The second thrust, *Measurement Planning*, includes an investigation of system-level and turbine-level measurement practices and a sensor scheme design for the test loop. The final

Figure 2.1: Thrust areas for this project.
thrust, *Detailed System Modeling*, will use a custom simulation environment to generate a final design for the test loop, including component selection. The specific deliverables for each of these thrusts are described in the following paragraphs.

1. **Turbine Performance Prediction** - Perform a critical review of axial turbine performance prediction methods, focused primarily on empirical correlations. Review these models to determine how the general turbine community would likely use the test loop. Consult with a commercial steam turbine design firm to get an accurate picture of how they derive prediction methods that are used in turbine selection and design.

2. **Measurement Planning** - Perform an initial review of axial turbine testing methods, especially those contained in relevant codes and standards. Determine the requirements placed on overall test loop layout and component selection by the measurement systems. Provide sources of information for future test loop detail designers to consult when selecting sensors for the test loop.

3. **System Modeling** - Create theoretical models of critical steam turbine test loop components from first principles. Create a test loop simulation tool that will predict the quasi-steady response of the test loop to available control inputs. Create a preliminary system design of the test loop that can be used to define the simulation environment. Conduct a parametric study of the critical design parameters to arrive at an acceptable test loop design solution.

The scope of this thesis is *not* to develop a new performance prediction method, or to build an actual steam turbine test loop. The scope of this project ends with the completion of a preliminary design for a test loop.
Chapter 3

Literature Review

3.1 Introduction

The literature review has been conducted with the focus of predicting the performance of a steam turbine under a variety of operating conditions. The prediction methods contained in the literature review are of an empirical nature. Section 3.2 defines some terms that are common to the axial turbine industry and also the variables of interest to the present investigation. Section 3.3 gives a detailed description of empirical turbine loss models focused on single blade row loss mechanisms. Section 3.4 gives a brief overview of analytical and numerical turbine analysis methods, which will not be used in this thesis, but could serve future researchers. Section 3.5 covers an investigation into measurement systems, including those used in academia and those required by relevant codes and standards.

3.2 Definitions

3.2.1 Total Properties

A total property is that which would result from an isentropic deceleration to zero velocity. It is equal to the sum of the static and dynamic components. Common to turbine analyses are the use of total enthalpy, total pressure, and total temperature. They are defined, respectively, as follows:

\[ h_0 \equiv h + \frac{v^2}{2g_c}, \]
\[ P_0 \equiv P + \frac{\rho v^2}{2g_c}, \]
\[ T_0 \equiv T + \frac{v^2}{2c_p g_c}. \] (3.1)

The expression for total temperature is derived from the one for total enthalpy assuming constant specific heat at constant pressure. Total temperature can also be expressed in terms of Mach number, \( M \), and the specific heat ratio, \( \gamma \), as

\[ \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2. \] (3.2)

### 3.2.2 Stage Geometry

Each stage of an axial turbine has a stationary stator row followed by a rotating rotor row. These alternating rows can be seen in Fig. 3.1 which depicts a research turbine with the upper housing removed.

![Figure 3.1: Photograph of the internals of a 4-stage research turbine (Courtesy of Dresser-Rand [1])](image)

The stator blades are sometimes referred to as nozzles, guide vanes, or nozzle guide vanes (NGV), because they accelerate the flow much like a nozzle and also because traditional single stage turbines used conical shaped nozzles. The stator row is mechanically attached to the turbine casing, leaving a gap between the stator ring and the shaft, which is minimized by a diaphragm seal. The rotor blades, often referred to as buckets, are directly attached
to the shaft by the rotor wheel and are usually connected at their tips by a shroud ring. The gap between the blade and casing or the shroud and casing is controlled by tip leakage seals. These seals and the primary flow passage are depicted in Fig. 3.2.

![Figure 3.2: Approximate section view of the flow passages of a multistage turbine](image)

The stator and rotor blades can take on very complex shapes. With the advent of high precision four axis milling machines, manufacturers can produce practically any shape they desire. Of course there is a tradeoff between performance and cost. For typical blade dimensions that will be used in this thesis, refer to Fig. 3.3.

When the fluid radial velocity is assumed to be zero, referred to as a 2-D analysis, the fluid flowpaths of each stage are described with a single velocity diagram as depicted in Fig. 3.4. The velocity diagram of Fig. 3.4 represents the flow conditions at the “mean line”, or the “pitchline”, of the stage. This refers to the plane at a radius equal to the average of the hub and tip radii. The view is radial, outward away from the shaft center. The axial direction is defined by the x-axis, and the tangential direction is defined by the y-axis. All angles are taken as positive as they are drawn in the schematic. It was common in traditional steam turbine practice to measure fluid angles from the rotational plane; however, the axial datum convention shown in the above diagram is used in most modern
axial turbine analyses, and will be used in the analysis to follow.

Dixon [3] gives the following description of the axial turbine flow path:

Fluid enters the stator at absolute velocity $V_1$ at angle $\alpha_1$ and accelerates to an absolute velocity $V_{2S}$ at angle $\alpha_{2S}$. All angles are measured from the axial direction. ...the rotor inlet relative velocity $w_{2R}$ at an angle $\alpha'_{2R}$, is found by subtracting, vectorially, the blade speed $U_2$ from the absolute velocity $V_{2R}$. The relative flow within the rotor accelerates to velocity $w_3$ at an angle $\alpha'_3$ at rotor outlet; the corresponding absolute flow $(V_3, \alpha_3)$ is obtained by adding, vectorially, the blade speed $U_3$ to the relative velocity $w_3$. 
The velocity diagram of Fig. 3.4 applies when the turbine is operating at design conditions, but additional terms are necessary to analyze off-design operation. When a turbine operates at off-design rotational speed or mass flow rate, the flow incidence on the leading edge of the blade $\alpha_{in}$ becomes nonzero. Fig. 3.5 illustrates the nomenclature used to describe the variation of incidence relative to the blade inlet metal angle, $\beta_{in}$, the design inlet flow angle, $\alpha_{in,des}$, and the actual inlet flow angle, $\alpha_{in}$. The subscript $in$ is used to denote the blade inlet. For the stator row the inlet would be at station 1 of Fig. 3.4 and for the rotor, the inlet would be at station 2$R$. For the moving rotor, the relative velocity angle $\alpha'_2$ is substituted for the effective incidence angle $i_{eff}$.

### 3.2.3 Efficiency and Losses

The isentropic efficiency of a steam turbine is expressed as

$$\eta_s = \frac{h_{in} - h_{outs}}{h_{in} - h_{outs}},$$

(3.3)
where $h_{outs}$ is the enthalpy resulting from an isentropic expansion through the turbine to the same exit pressure as the actual expansion. Isentropic efficiency accounts for the aerodynamic losses of a turbine, and finding its value is a major goal of performance testing in a steam turbine test loop. Aerodynamic losses include skin friction, separation, vorticity generation, and others. Static enthalpies are used in Equation 3.3 because the fluid kinetic energy change across the entire turbine is assumed to be negligible. Another important efficiency is the turbine thermal efficiency,

$$\eta_{th} = \frac{\dot{W}_{sh}}{h_{in} - h_{out}},$$

(3.4)

which accounts for rotordynamic losses in the turbine. Rotordynamic losses include bearing friction and leakage over external seals (not tip or diaphragm seals), which prevent steam from escaping to open air. The product of thermal and isentropic efficiencies is the overall turbine efficiency,

$$\eta_{oa} = \frac{\dot{W}_{sh}}{h_{in} - h_{out,s}} = \eta_s \cdot \eta_{th},$$

(3.5)
which is the ratio of the actual shaft output power, $\dot{W}_{sh}$, to the ideal maximum enthalpy drop across the turbine. One method of determining thermal efficiency is to make the necessary fluid and shaft measurements to find isentropic and overall efficiencies, and then divide $\eta_{oa}$ by $\eta_s$.

Often the greatest concern of a turbine end user is to know how the three turbine efficiencies vary over the operating range of their particular steam power loop. In response, manufacturers plot the efficiency performance maps against pressure ratio and flow rate. They do not produce these maps by running every turbine through an extensive series of tests. Instead, they use engineering analysis to interpolate the results of tests on a research turbine to the appropriate off-the-shelf product. In addition to the aerodynamic and rotordynamic loss breakdown, each of these is separated into multiple sources of loss.

A reasonable breakdown of aerodynamic losses, given by Mathis [17], separates the losses into different control volumes: inlet, stator, rotor, diffuser, and exit. The boundaries for each of these volumes depends on the particular turbine being considered. All areas of loss are of interest to predicting overall aerodynamic performance, but the current focus will be on stator and rotor losses. Each stage being comprised of a single stator and rotor pair, these losses will impact the stage efficiency. Stage efficiencies are defined as the ratio of the real enthalpy drop in the stage to the ideal maximum enthalpy drop, similar to the turbine isentropic efficiency. Total enthalpy is used in single stage analysis, because velocity changes are drastic across each stage row and kinetic energy change cannot be assumed negligible.

Two stage efficiencies are of interest to the present research, the first being total-to-total efficiency,\

$$\eta_{tt} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}}.$$  \hspace{1cm} (3.6)\

The appropriate states are illustrated in the Mollier diagram of Fig. 3.6. Total-to-total efficiency sets the minimum possible exit energy content as the ideal stage exit total enthalpy, $h_{03ss}$, because it is assumed that the exit kinetic energy is used either in a subsequent stage, or for thurst in the case of an aircraft gas turbine. The second efficiency of interest is total-to-static, expressed as\

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}}.$$  \hspace{1cm} (3.7)
Total-to-static efficiency is used for single stage stationary turbines or for the final stage of a multiple stage stationary turbine, when the exit kinetic energy is not used. The total-to-static efficiency will always be less than the total-to-total efficiency. Both of these efficiencies are of the isentropic variety, because they use an isentropic process to determine the ideal exit states. Determining the true sources of loss in a particular stage requires consideration of the blade row performances. There are several types of blade row loss parameters in use when losses are accounted for on a row-by-row basis in addition to the stage basis. One type of loss parameter is the enthalpy loss coefficient. \( \zeta_N \) and \( \zeta_R \) are the ratios of enthalpy loss to relative exit kinetic energy in the stator and rotor, respectively. They are expressed as

\[
\zeta_N = \frac{h_2 - h_{2s}}{2V_2^2},
\]  

(3.8)
and

\[ \zeta_R = \frac{h_{3} - h_{3s}}{2w_3^2}. \quad (3.9) \]

The enthalpy loss is the deficit between the ideal row exit enthalpy for an isentropic process and the actual row exit enthalpy. The relative exit velocity for the stator is equal to the absolute velocity. The variation of these loss coefficients vs. blade geometries and other non-dimensional parameters is found by fitting experimental data to a preconceived loss model. The result enables axial turbine designers to predict row-by-row performance in the early turbine design phase, without much computational cost. A modern loss correlation method is described in Section 3.3.

### 3.2.4 Non-dimensional Parameters

**Velocity Ratio, \( \sigma \)**

The variable most often used as the independent variable in non-dimensional plots for steam turbines is the velocity ratio,

\[ \sigma = \frac{U_m}{V_{2s}}. \quad (3.10) \]

One of the major benefits of velocity ratio is that it must be between 0 and 1. If the blade speed exceeds the stator exit velocity, then the flow will actually extract energy from the rotor. The only time this would occur is for severe off-design performance. In order to represent a real operating case, other stages would need to first extract energy from the flow.

**Flow coefficient, \( \Phi \)**

An alternative independent variable used in gas turbine analyses, and in most textbooks, is the flow coefficient. It starts as the ratio of mass flow rate to rotational speed, and simplifies to

\[ \Phi = \frac{V_x}{U}. \quad (3.11) \]

The value of \( \Phi \) varies from rotor inlet to exit. Schobeiri [18] uses the technique of considering the flow coefficient at both locations, using \( \Phi_2 \) and \( \Phi_3 \). For the purposes of turbomachinery design, this is a robust approach, because it accounts for changes in axial velocity and
blade speed. Dixon [3] takes the alternate approach of assuming constant axial velocity and uniform blade speed, so that the flow coefficient is constant across the rotor row. For the purposes of the present analysis, the variation of axial velocity and blade speed will be accounted for by using their average values. Thus the average axial velocity is

\[ V_{xm} = \frac{V_{x1} + V_{x2} + V_{x3}}{3}, \]  

(3.12)

and the average blade speed is

\[ U_m = \omega \frac{r_2 R_m + r_3 m}{2}. \]  

(3.13)

**Loading coefficient, \( \Psi \)**

The stage loading, or work, coefficient used in this thesis is

\[ \Psi = \frac{\delta W}{U_m^2}. \]  

(3.14)

Comparing the fluid work input across the rotor to the angular momentum of the rotor enables turbine designers to gauge the effectiveness of a particular stage design. One might think that higher work coefficients are best; however, losses due to flow friction and separation increase as well. There is a tradeoff between high work coefficient and high efficiency that requires knowledge of the specific application area. Schobeiri [18] once again uses an alternative approach. The loading coefficient above is expressed by \( \lambda \). He then expresses yet another parameter, the stage enthalpy coefficient, by \( \Phi \). The stage enthalpy coefficient is the ratio of the isentropic stage mechanical energy to the rotor blade kinetic energy. Schobeiri uses the blade exit speed, \( U_3 \), for both of his stage work coefficients, instead of the mean value used above. Schobeiri’s differing conventions for flow and work coefficients surely make his analysis more complex, but they also more fully define the turbine stage behavior than conventional analyses. This fact should be considered when deciding on conventions for future work.

**Reaction, \( R \)**

Another parameter of interest is the stage degree of reaction, which is the ratio of the specific energy extracted from the fluid across the rotor to the specific energy extracted across the
entire stage, excluding kinetic energy. Two turbine stages that have similar reaction values at similar non-dimensional design points will operate similarly over their entire operating range. Reaction is represented by one of two expressions, depending on the source. The pressure-based reaction is

\[ R_P = \frac{P_2 - P_3}{P_1 - P_3}, \tag{3.15} \]

and the enthalpy-based reaction is

\[ R_h = \frac{h_2 - h_3}{h_1 - h_3}. \tag{3.16} \]

The relevant static enthalpies and pressures are illustrated in the P-h diagram of Fig. 3.7. When describing turbine design methodologies, textbooks (e.g. [10], [3], [18]) use the enthalpy-based reaction, because it can be expressed in terms of \( \Phi, \Psi, \) and the blade metal angles when additional assumptions are made. However, many turbine experimentalists use pressure-based reaction, because the verifying experimental data is easier to collect. The theoretical error between the two reaction types is small when the blade row losses are negligible, but that is an unrealistic limiting case.

Stages are often referred to as being of impulse type. The impulse condition occurs when the static pressure drop across the rotor is zero, thus \( R_P = 0. \) For a stage with losses greater than zero (all realistic stages), this condition requires \( R_h < 0. \) An impulse stage is one which satisfies the impulse condition at its design point. Dixon [3] states that the enthalpy-based blade row reaction should be greater than or equal to zero in order to avoid diffusion of the flow within a blade row. “This is because the adverse pressure gradient (arising from the flow diffusion) coupled with large amounts of fluid deflection” drastically increases the likelihood of boundary-layer separation and large losses. The reactions defined above are the stage reactions, whereas Dixon defines blade row reactions for the stator and rotor as

\[ R_{h,S} = 1 + \frac{V_x}{2U_m} (\tan \alpha_3 - \tan \alpha_2), \tag{3.17} \]

and

\[ R_{h,R} = \frac{V_x}{2U_m} (\tan \beta_3 - \tan \beta_2), \tag{3.18} \]

respectively. However, Dixon derived both of these expressions assuming constant axial velocity, uniform blade speed, and for a stage with no change in kinetic energy. The turbine
theory developed in Chapter 4 accounts for variations in all three of those values, so Dixon’s expressions aren’t applicable. Without further knowledge or literature on the subject, one might think the most applicable limit for the present analysis would be that the enthalpy-based stage reaction must be greater than or equal to zero. However, this limitation would severely hinder the operating envelope of an impulse stage, which has a design point at $R_P = 0$ and $R_h < 0$.

### 3.2.5 Miscellaneous Terms

**Axial turbine vs. radial in-flow turbine**

The steam turbine test loop in question will be designed to operate axial turbines, as most steam turbines fall into this category. Axial turbines generally have an axial inlet flow to the first stage, and an axial/tangential exit velocity from the final stage. Radial turbines can be of the in-flow or the out-flow type. In the first case, the inlet flow is directed radially inward toward the rotational axis, and the exit is directed axially away from the rotor disc.
Radial out-flow turbines simply reverse the flow direction.

**Steam vs. gas turbine**

Steam turbines use gaseous and/or mixed liquid-vapor $H_2O$ as a working fluid, and have only a fluid expansion section. Gas turbines use a non-$H_2O$ gaseous working fluid, with nearly zero condensation. The phrase “gas turbine” is commonly used in reference to either the expansion section (turbine) or the entire rotor system composed of fan, compressor, combustor, and turbine sections. Herein, the term “gas turbine” will refer to only the expansion section. The first principles approach for analyzing steam and gas turbines is identical. Conservation of mass, momentum, and energy are used ad infinitum. In general, the energy extracted across a turbine stage is proportional to the pressure drop across that stage. Additional loss factors are used to account for losses due to steam condensation. Less energy will be converted to shaft momentum for every unit of pressure drop across that stage with condensation than for a stage without it. The energy conversion deficit depends on many factors, including the average water droplet size and the droplet concentration. The steam turbine test loop could be used in the future to investigate these phenomena, but the details are far beyond the scope of this thesis.

**Condensing vs. non-condensing steam turbines**

A non-condensing or back pressure steam turbine is similar to a gas turbine in that there is zero, or nearly zero, condensation. Both condensing and non-condensing turbines can be analyzed using methods similar to those used for gas turbines, so long as steam state equations are utilized and the ideal gas law is not. However, performance analysis of condensing turbines requires an additional condensation loss accounting method, since condensed water droplets do not effectively transfer momentum to steam turbine rotor blades. Condensing turbines have condensation in the later stages and generally operate at lower exit pressures than non-condensing turbines. Non-condensing turbines tend to have exit gauge pressures greater than zero, which is controlled by a regulating valve at the turbine exit. Condensing turbines have no exit regulating valve, and often have exit gauge pressures less than zero. The actual exit pressure depends on the rate at which the condenser and pump can remove...
steam from the turbine. By operating at low exit pressures, a condensing turbine absorbs more of the energy stored in the steam than a comparable sized non-condensing turbine. So if the user’s goal is to make their power cycle as efficient as possible, they would be interested in using a condensing turbine.

A turbine can have condensation above atmospheric pressure. Steam turbines used in the nuclear power industry almost exclusively operate near the condensation boundary. After traversing several turbine stages (when the steam quality is predicted to be below some threshold) the steam is sent back to the reactor for reheating, and then returns to the turbine for recondensation in later stages. This type of turbine is also being used in recent solar thermal power applications. The hot temperature generated by concentrating the sun’s energy is much lower than for a natural gas or coal combustion process. So the high temperature required for superheating steam is out of the question. Condensing turbines also could be used in combined heat and power or waste steam power recovery, where high efficiency can be the deciding factor in terms of system payback periods. Condensing might not be used in all applications, because the turbine exit stream is often used for some other process. This could even be the case in a combined heat and power application if the steam power cycle is the top cycle instead of the bottom cycle. In order to meet the testing requirements of potential applications for both types of steam turbines, the test loop designed herein will provide both condensing and non-condensing operational capabilities.

Aerodynamic vs. rotordynamic performance

Aerodynamic losses refer to those accounted for by isentropic efficiency, whereas rotordynamic losses are those accounted for by thermal efficiency. Aerodynamic losses include skin friction, separation, vorticity generation, and others. Rotordynamic losses include bearing friction and leakage over external seals, which prevent steam from escaping to open air.

3.3 Empirical Turbine Performance Prediction

It is important to understand the state of the art of turbine performance prediction, which has evolved over the past century from qualitative guess and check to complex systems of
empirical, numerical, and analytical equations. Empirical correlation results often serve as inputs to the more complex numerical and analytical methods. One common purpose of an empirical correlation is to simplify the analysis of a three-dimensional, unsteady, turbulent or transitioning, compressible flow problem by accounting for complex flow behaviors with coefficients and correction factors. Although misuse of simplified empirical correlations can lead to poor turbine designs, Benini et al. [19] state the following in their 2008 conference paper:

In spite of the remarkable advances in the field of Computational Fluid Dynamics, algebraic models built upon empirical loss and deviation correlations are still one of the most reliable and effective tools to predict the performance of gas turbine stages with reasonable accuracy, especially when low-reaction, multi-stage architectures are considered.

The fact that correlations developed a half-century ago are still relevant in current performance prediction is possible because the correlations are restructured or retuned when a new design paradigm is believed to impact turbine performance. The relative improvements between empiricists come from introducing new variables to the correlation, which are speculated from witnessing experimental or numerical flow patterns. Published correlations are optimized to fit the available measurement data, while particular manufacturers may have proprietary correlations that apply to their specific line of products. While the algebraic expressions for each correlation differ, some common assumptions apply to many of them.

Most empirical prediction methods fit into the category of 2-D or “mean line” analysis, because they assume zero radial gas velocity across a turbine blade, and they assume the tangential and axial velocity vectors at the mean blade radius are representative of conditions at all other radii. This is illustrated in the velocity diagram of Fig. 3.4, which for a mean line analysis would represent the conditions for the entire blade length. If, in addition, a method assumes axisymmetric flow about the rotor axis then it follows a 1-D analysis, since flow conditions vary only in the axial direction. A 1-D analysis assumes that the losses through a turbine stage depend solely on the mass-averaged inlet and exit conditions and the physical geometry of the stage. Often in the literature, one author referring to 2-D theory and another to 1-D theory are discussing the same correlation method. The empirical
methods discussed herein are 1-D, mean line, axisymmetric analyses. They are also limited to aerodynamic performance prediction as defined in Section 3.2. Rotordynamic losses are not accounted for in the present analysis.

There are several published approaches to developing an empirical correlation, which adhere to the limitations imposed by the preceding assumptions. It is useful to organize these approaches in order to assess their utility for the present research. Conventional approaches will be divided according to their complexity and the abundance of their citations in recent publications. An additional novel approach given by Denton [20] will also be presented. Although the conventional approaches are more prevalent in the literature, Denton states his method will be more accurate once the necessary experimental and numerical data has been collected.

3.3.1 Conventional Approaches

In 1985, Sieverding [21] separates conventional empirical axial turbine performance prediction methods into two groups. The first group, here termed Overall Parameter Models, bases turbine stage performance on overall turbine parameters and is used in the initial design phase for the selection of the turbine design parameters. Sieverding says the second group, here termed Detailed Parameter Models, achieves a deep understanding of the flow, and takes into account details of the blading and of the meridional flow channel. These two groups of empirical prediction methods will be reviewed for their relative utility in a steam turbine test loop system model, and for their potential improvement by actual performance testing. As described in the Motivation for this thesis the most recent literature for both groups focuses on gas turbines.

Overall Parameter Models

These models are among the oldest in the turbine community and they are still used in the early design stages to define the turbine flowpaths. The most-cited model of this type in textbooks is Soderberg’s correlation, which can be found in the works of Horlock [10], Dixon [3], and Mathis [17]. Dixon and Horlock both declare that Soderberg’s method gives turbine efficiencies with errors less than 3% over a wide range of Reynolds number and aspect ratio
when additional corrections are included to allow for tip leakage and disc friction. Soderberg uses the enthalpy loss coefficients given in Equations 3.8 and 3.9.

The coefficients are correlated against turbine overall parameters, such as flow coefficient and loading coefficient, for a range of space-chord ratios and flow turning angles. Unfortunately, Soderberg’s correlation, like other Overall Parameter Models, is unable to predict the off-design performance of a turbine, because he assumed stator and rotor incidence angles to be zero at the mean line. This is simply not the case when the non-dimensional stage characteristics differ from their design values. In today’s modern designs a non-zero incidence at the mean line is possible at the design-point, especially if the blade has twist. The twist angle refers to variation of the inlet and exit metal angles along the blade span. Horlock [10] cites several other Overall Parameter Models, including those of Hawthorne [22], Emmert [23], and Vavra [24]; however, since their approach is similar to Soderberg’s and none have been updated by recent tests, their specific differences will not be discussed further.

Overall Parameter Models represent the most basic performance prediction schemes. Their strength lies in their simplicity, but their weakness lies in an integrated architecture that complicates recalibration to modern design. Because they are unable to predict off-design turbine performance, Overall Parameter Models cannot be used for a full spectrum steam turbine test loop model; however, their continued use in the turbine industry presents an opportunity for improvement. The next section will illustrate the benefit of the modular Detailed Parameter Models, and also the increased complexity required to meet the requirements of off-design prediction.

Detailed Parameter Models

These models have a modular architecture that has enabled continued improvement over the last 50 years. The model most often cited in the literature is the pressure-loss model by Ainley and Mathieson (AM), published in 1951 [5, 6]. It is a prime candidate for modeling the steam turbine in a research test loop, because it is a complete loss model for predicting design-point and off-design losses. The AM[5, 6] correlation has been modified and improved by Dunham and Came (AMDC) [14], Kacker and Okapuu (KO) [7], Moustapha et al.
(MKT) [25], and most recently by Benner et al. (BSM) [26], [27], [8], [28]. Each researcher has improved the AM [5, 6] model by introducing new experimental data or reformulation to extend the applicability of the original correlation. The AM [5, 6] model and its extensions are designed to predict gas turbine performance, but their results apply to all axial turbines with a gaseous working fluid, including steam turbines. Additional losses occur due to condensation. The prevalence of the AM [5, 6] model extensions in recent literature for gas and steam turbines warrants the following in depth description.

Ainley and Mathieson [5, 6] state that whenever convenient, pressure loss components should be expressed in terms of a loss coefficient, as

\[
Y = \frac{\text{inlet total pressure} - \text{exit total pressure}}{\text{exit total pressure} - \text{exit static pressure}}.
\]

(3.19)

One of the major advantages of pressure-loss methods is that the pressure terms can be measured directly in experiments, whereas enthalpy can’t be measured directly and requires measurement of two independent state properties. In terms of the loss coefficient defined in Equation (3.19) and the stator and rotor sections defined in the velocity diagram of Fig. 3.4, their respective total loss coefficients are

\[
Y_{N,\text{total}} = \frac{P_{01} - P_{02}}{P_{02} - P_2}
\]

\[
= \frac{P_{01} - P_{02}}{\frac{1}{2} \rho_2 V_2^2},
\]

(3.20)

and

\[
Y_{R,\text{total}} = \frac{P_{02rel} - P_{03rel}}{P_{03rel} - P_3}
\]

\[
= \frac{P_{02rel} - P_{03rel}}{\frac{1}{2} \rho_3 w_3^2}.
\]

(3.21)

Stator losses are defined in terms of the absolute velocities while rotor losses are defined in terms of the relative velocities. AM [5, 6] separate the total pressure-loss coefficient for both the rotor and stator stage rows into several loss components:

\[
Y_{\text{total}} = Y_P + Y_S + Y_{TC},
\]

(3.22)
where \( Y_P \), \( Y_S \), and \( Y_{TC} \) are the profile, secondary flow, and tip clearance losses respectively. Kacker and Okapuu modified the formulation as

\[
Y_{total} = Y_P f(Re) + Y_S + Y_{TC} + Y_{TE},
\]

(3.23)

where \( Y_{TE} \) is a trailing edge loss component related to wake formation. Although the literature separates loss components along clear lines of distinction, they aren’t physically separate. Summed component loss models simplify the flow so that correlations are made simpler. After all, the point of using an empirical correlation is to simplify the overall turbine analysis.

The analysis is made simpler by the fact that pressure-loss correlations are most often derived from experiments performed on rectilinear, not radial, stages that employ a cascade of 2-D blade profiles extending from a planar wall in a wind tunnel as shown in Fig. 3.8. The experiments adhere strongly to the assumption of 2-D flow at the mean line, since there are no centrifugal forces to accelerate the flow toward either the “tip” or the “hub” of the blades. The working fluid is compressed air, and calculations are performed using nondimensional parameters in order to generalize the results. Refer to [3] or [10] for a more complete description of cascade testing.

The cascade pressure-loss coefficient can be converted to an enthalpy-loss coefficient using an expression given by Horlock [10]:

\[
Y = \zeta \left[ 1 + \frac{\gamma - 1}{2} M_2^2 \right]^{\gamma/(\gamma-1)},
\]

(3.24)

where \( M_2 \) is the blade row exit Mach number and \( \gamma \) is the ratio of specific heats. Horlock states that the derivation for a similar relationship for the rotor blades is simple, but he assumes a perfect gas in his derivation, eliminating its applicability to steam turbines. Even more limiting is that direct use of cascade pressure losses in complete turbine analyses might not be appropriate. Dixon [3] states “the aerodynamic efficiency of an axial-flow turbine is significantly less than that predicted from measurements made on equivalent cascades operating under steady flow conditions.” Additionally, Benner [28] states,

The loss correlations derived from cascade data are scaled or ‘calibrated’ to reproduce stage efficiencies derived from rig or engine data...there are evidently
additional and significant loss generating mechanisms in the engine environment that are not captured in cascade testing.

In order to understand the correlation between pressure-loss correlations gleaned from cascade tests and actual losses in full turbine rigs, an experimental comparison would be required. Because designing a test facility that would enable such a comparison is the object of this thesis, the comparison is not possible prior to its completion. To summarize, the cascade experiments are important, as they allow inexpensive testing of many blade geometries and flow boundary conditions. They also enable reconfiguration of instruments and isolation of each loss component (i.e. profile, secondary, tip clearance), without dismantling a complex turbine assembly. However, the resulting correlations should not be taken as accurate for all cascades, or turbines, under all operating conditions. The scope of this thesis does not require an accurate loss prediction method, but it does require a turbine performance model that is accurate enough to draw conclusions about the test loop design.
A conventional detailed parameter loss model

In order to stress the motivation behind this thesis and give direction for future research, the pressure-loss components of the most modern version of the Ainley and Mathieson [5, 6] loss breakdown scheme are described. As has already been described, the losses are broken down into several components. Each component applies to a particular region of the blade row. Fig. 3.9 shows some of the loss mechanisms in action on a rectilinear blade row. The loss model is to applied to a single blade row, whether it is a stator or a rotor. A subscript of “1” refers to the blade row inlet and “2” refers to the blade row exit. These numbers should not be interchanged with the stage numbering presented in Section 3.2. Also, the velocities and Mach numbers for the rotor row should be the relative values with respect to the moving blade row.

![Diagram of various loss mechanisms in a blade row](image)

Figure 3.9: Illustration of various loss mechanisms in a blade row [4].

Profile Losses, $Y_P$ - This is the primary loss mechanism due to skin friction and flow separation in the center section of the blade. The profile loss coefficient is heavily dependent
on flow incidence, which becomes important for off-design performance prediction. According to their method (and those that followed), profile loss is determined initially at zero incidence (design-point), as a function of inlet and exit blade angles. The zero-incidence

![Figure 3.10: Profile loss coefficient for thickness-to-chord ratio $t/c = 0.2$ for “nozzle blades” (a) and “impulse blades” (b) by Ainley and Mathieson [5, 6].](image)

profile loss is found as an interpolation between a “nozzle blade” (Fig. 3.10a) and an “impulse blade” (Fig. 3.10b) for a thickness-to-chord ratio of $t/C = 0.2$. When the Ainley and Mathieson correlation was first derived, nozzle referred to a blade with an inlet gas angle of $\alpha_{in} = 0$ and impulse referred to a blade with equal inlet and outlet gas angles. Kacker and Okapuu [7] adjusted the original form of the Ainley and Mathieson [5, 6] correlation to this one

$$Y'_P = \left\{ Y_{P(\beta_1=0)} + \left| \frac{\beta_1}{\alpha_2} \right| \left( \frac{\beta_1}{\alpha_2} \right)^2 \left( Y_{P(\beta_1=\alpha_2)} - Y_{P(\beta_1=0)} \right) \right\} \left( \frac{t_{max}/c}{0.2} \right)^{\frac{\beta_1}{\alpha_2}}. \quad (3.25)$$

They introduced the term $\left| \frac{\beta_1}{\alpha_2} \right| \left( \frac{\beta_1}{\alpha_2} \right)^2$ to allow for negative inlet angles. The original Ainley and Mathieson [5, 6] interpolation had $\left( \frac{\beta_1}{\alpha_2} \right)^2$. The multiplying factor at the end of the expression is used to account for variations in thickness to chord ratio away from the nominal value of 0.2 in Fig. 3.10. Kacker and Okapuu defined the design-point profile losses as

$$Y_P = 0.914 \left( \frac{2}{3} Y'_P K_P + Y_{SHOCK} \right), \quad (3.26)$$
where
\[
K_P = \begin{cases} 
1 - \left( \frac{M_1}{M_2} \right)^2 [1.25(M_2 - 0.2)] & \text{for } M_2 > 0.2 \\
1 & \text{for } M_2 \leq 0.2 
\end{cases}
\]
and
\[
Y_{SHOCK} = 0.75(M_{1,HUB} - 0.1)^{1.75} \left( \frac{R_{HUB}}{R_{TIP}} \right) \left( \frac{p_1}{p_2} \right) \left[ 1 - \left( 1 + \frac{\gamma - 1}{2} M_1^2 \right) \right]^{\frac{\gamma}{\gamma - 1}}. \tag{3.27}
\]

\(Y_{SHOCK}\) accounts for losses due to shocks at the blade leading edge and \(K_P\) is a correction to the Ainley and Mathieson \([5, 6]\) profile losses for varying Mach number. Ainley and Mathieson \([5, 6]\) conducted their cascade tests at low subsonic velocities, not accounting for Mach number effects. According to KO \([7]\), “In an accelerating flow passage, operation closer to sonic exit velocities will tend to cause suppression of local separations and the thinning of boundary layers. This effect is most pronounced where inlet Mach numbers are only slightly lower than exit Mach numbers”. The ratio \(M_{1,HUB}/M_{1,MEAN}\) is given as an empirical function of hub-to-tip ratio for rotors and nozzles (stators) in Fig. 3.11. \(M_{1,MEAN}\) is the Mach number at the midspan of the blade. The Reynold’s number correction factor

![Figure 3.11: Inlet Mach number ratio for nonfree-vortex turbine blades from Kacker and Okapuu \([7]\).](image)

\[
\begin{align*}
f_{Re} &= \left( \frac{Re}{2x10^5} \right)^{-0.4} & \text{for } Re \leq 2x10^5 \\
&= 1.0 & \text{for } 2x10^5 < Re < 10^6 \\
&= \left( \frac{Re}{10^6} \right)^{-0.2} & \text{for } Re \geq 10^6.
\end{align*}
\]
The KO [7] extension to the Ainley and Mathieson [5, 6] correlation is the most recent development for design-point profile loss calculation. The most recent correlation for off-design incidence cascade profile losses was by Benner et al. [8] in 1997. This work is a follow-up to Moustapha et al. [25]. Moustapha determined that the leading edge diameter, \( d \), was a key parameter in predicting losses for high incidence flow. Benner subsequently found that \( d \) was much less critical than previously thought and that actually the leading edge wedge angle was playing a role on off-design incidence losses. Benner defines his incidence parameter as

\[
\chi = \left( \frac{d}{s} \right)^{-0.05} We^{0.2} \left( \frac{\cos \beta_1}{\cos \beta_2} \right)^{-1.4} [\alpha_1 - \alpha_{1,\text{des}}],
\]

where \( \alpha_{1,\text{des}} \) is the design inlet gas angle, \( \alpha_1 \) is the actual gas inlet angle, \( \beta_1 \) is the cascade inlet blade angle, \( \beta_2 \) is the cascade exit blade angle, \( We \) is the leading-edge wedge angle, and \( d/s \) is the leading edge diameter to spacing ratio. Benner defines his loss in terms of a change to the kinetic-energy coefficient, \( \phi \), because it varies more weakly with Mach number than does the pressure loss coefficient, \( Y \). The kinetic-energy coefficient is defined as the ratio of the actual exit kinetic energy to the kinetic energy obtained in an isentropic expansion through the blade row, \( i.e. \)

\[
\phi^2 = \frac{V_2^2}{V_{2,is}^2}.
\]

Benner’s loss correlation is

\[
\Delta \phi_P^2 = a_8 \chi^8 + a_7 \chi^7 + a_6 \chi^6 + a_5 \chi^5 + a_4 \chi^4 + a_3 \chi^3 + a_2 \chi^2 + a_1 \chi,
\]

where

\[
a_8 = 3.711 \times 10^{-7}, \quad a_7 = -5.318 \times 10^{-6},
\]
\[
a_6 = 1.106 \times 10^{-5}, \quad a_5 = 9.017 \times 10^{-5},
\]
\[
a_4 = -1.542 \times 10^{-4}, \quad a_3 = -2.506 \times 10^{-4},
\]
\[
a_2 = 1.327 \times 10^{-3}, \quad a_1 = -6.149 \times 10^{-5},
\]

for \( \chi \geq 0 \), and

\[
\Delta \phi_P^2 = 1.358 \times 10^{-4} \chi^2 - 8.720 \times 10^{-4} \chi
\]
for $\chi < 0$. Moustapha et al. [25] and Benner et al. [8] give the identical formulae for conversion between the kinetic energy loss coefficient and pressure loss coefficient as

$$Y = \left[ 1 - \frac{\gamma - 1}{2} M_2^2 \left( \frac{1}{\varrho_2} - 1 \right) \right]^{\frac{1}{\gamma - 1}} - 1 \cdot \frac{1}{1 - \left( 1 + \frac{\gamma - 1}{2} M_2^2 \right)^{-\frac{1}{\gamma - 1}}}. \quad (3.32)$$

**Secondary (Endwall) Losses, $Y_S$** - According to Dixon [3],

Secondary losses arise from complex three-dimensional flows set up as a result of the end wall boundary layers.... There is substantial evidence that the end wall boundary layers are convected inwards along the suction surface of the blades as the main flow passes through the blade row, resulting in a serious maldistribution of the flow, with losses in stagnation pressure often a significant fraction of the total loss.

Multiple correlations exist for end wall losses. One of the most recent methods was presented by Benner et al. in a two part paper. In the first part [8] Benner redefines the breakdown of profile and secondary losses. In order to use his correlation for secondary losses, one must adhere to this new loss breakdown scheme. He divides the blade surface into a “Primary” region and a “Secondary” region as shown in Fig. 3.12, which shows a schematic picture of a typical blade suction surface oil film pattern. The profile loss, $Y_P$, described in the previous paragraphs is redesignated $Y_{mid}$, the loss encountered at the midspan of the blade. The new profile loss is then

$$Y_P = Y_{mid} \left( \frac{A_{prim}}{A_s} \right) \quad (3.33)$$

where $A_{prim}$ is the area of the primary region projected onto the plane shown in Fig. 3.12.

This is equal to

$$A_{prim} = hC_x - 2 \left( \frac{1}{2} Z_{TE} C_x \right). \quad (3.34)$$

$A_s$ is the area where the flow exhibits three-dimensional character and is equal to

$$A_s = \frac{1}{2} C_x Z_{TE}. \quad (3.35)$$
Figure 3.12: Suction surface definition for the Benner et al. [8] loss breakdown scheme.

$Z_{TE}$ is called the spanwise penetration depth, denoting the distance from the endwall at which the flow is more or less two-dimensional. The final result for profile losses is

$$Y_P = Y_{mid} \left( 1 - \frac{Z_{TE}}{h} \right).$$  \hfill (3.36)

Benner also gives a correlation for penetration depth, $Z_{TE}$, as

$$\frac{Z_{TE}}{h} = 0.10(F_t)^{0.79} \left( \frac{CR}{h^2} \right)^{0.55} + 32.70 \left( \frac{\delta^*}{h} \right)^2,$$  \hfill (3.37)

where $CR$ is the convergence ratio. It accounts for acceleration of the flow through the blade channel and is expressed in terms of the inlet and exit gas angles as

$$CR = \frac{\cos \alpha_1}{\cos \alpha_2}.$$  \hfill (3.38)

$F_t$ is the tangential loading parameter. It represents the tangential force per unit length nondimensionalized by dynamic pressure based on the vector mean velocity. $F_t$ is given by Benner as

$$F_t = 2 \frac{s}{C_x} \cos^2 \alpha_m (\tan \alpha_1 - \tan \alpha_2),$$  \hfill (3.39)

where $\alpha_m$ is the mean vector angle through the airfoil row. It is expressed as

$$\tan \alpha_m = \frac{1}{2} (\tan \alpha_1 + \tan \alpha_2).$$  \hfill (3.40)
The last variable in the penetration depth equation that might be difficult to determine is $\delta^*$, the endwall boundary layer displacement thickness. If all of these parameters can be found, then the secondary losses are determined by a method from the second part [28] of Benner’s paper. The secondary loss expression is

$$Y_S = \begin{cases} 
0.038 + 0.41 \tanh(1.20 \frac{\delta^*}{h}) & \text{for } h/C \leq 2.0 \\
0.052 + 0.56 \tanh(1.20 \frac{\delta^*}{h}) & \text{for } h/C > 2.0 
\end{cases} \sqrt[0.55]{\cos \gamma CR(h/c)} \left( \frac{C_L s}{c} \right)^{2 \cos^2 \alpha_m} \cos \alpha_m, \quad (3.41)$$

**Tip Clearance (Leakage) Losses, $Y_{TC}$** - These losses occur at the tips of stator blades at the hub-blade interface or the tips of rotor blades at the casing interface. The loss coefficient depends on the size and nature of the interface (plain or shrouded). The unshrouded relationship will not be given here for the sake of brevity, but for shrouded blades, the KO [7] expression is

$$Y_{TC} = 0.37 \frac{c}{h} \left( \frac{k'}{c} \right)^{0.78} \left( \frac{C_L s}{c} \right) \frac{2 \cos^2 \alpha_m}{\cos \alpha_m}, \quad (3.42)$$

where

$$k' = \frac{k}{N_{seals}^{0.42}}. \quad (3.43)$$

$C_L$ is the airfoil lift coefficient and $h$ is the blade height. The reason for the use of $C_L$ is that traditional cascade testing used lift and drag force indicators to determine stage losses. The KO [7] expression is the same as the DC [14] expression except for the ability to account for multiple seals with $k'$. According to Yaras and Sjolander (YS) [29], the AMDC [14] model substantially overestimates the losses. They classify the Ainley and Mathieson [5, 6], DC [14], and KO [7] methods as “models based on momentum considerations” that were essentially inviscid. YS [29] state that viscous losses, which were assumed to be accounted for in profile losses by the previous authors, actually increase with clearance height. YS [29] present a more complex system of tip clearance loss based on energy considerations. If tip clearance loss prediction is of interest to future research, the YS [29] would be a good starting point. There is a great deal of literature since the 1992 YS [29] article that could be reviewed in order to get a modern view of tip clearance loss measurement and prediction. A study of modern techniques to measure tip clearance losses would be beneficial to future steam turbine testing as well.
**Trailing Edge Losses,** \(Y_{TE}\) - These losses arise due to the final thickness of the trailing edge being nonzero. The pressure and suction surfaces cannot converge to a point on a real blade, because the stresses generated by the high pressure gradient, high temperature, high speed flow are too great. The losses are associated with a wake generated by flow separation due to a low pressure pocket behind the trailing edge. In addition to generating pressure losses, the nonuniform flow impacts the performance of the following blade row. The original Ainley and Mathieson [5, 6] correlation included trailing edge losses within the profile loss component, but the newer KO [7] model which is presented now gives an empirical relationship for \(Y_{TE}\). In their scheme, trailing edge losses are found in terms of the kinetic energy coefficient and are interpolated in a similar manner to profile losses are in Equation (3.25), i.e.

\[
\Delta \phi_{TE}^2 = \Delta \phi_{TE}^2(\beta_1=0) + \left| \frac{\beta_1}{\alpha_2} \right| \left[ \Delta \phi_{TE}^2(\beta_1=\alpha_2) - \Delta \phi_{TE}^2(\beta_1=0) \right].
\]  

(3.44)

\(\Delta \phi_{TE}^2(\beta_1=\alpha_2)\) and \(\Delta \phi_{TE}^2(\beta_1=0)\) are correlated against \(t/o\) in Fig. 3.13. KO [7] gave a slightly different conversion from the kinetic energy coefficient, \(\phi_2\), to the pressure-loss coefficient, \(Y\), than did Benner. KO [7] gave \(Y = f(\Delta \phi^2)\), instead of Benner’s \(Y = f(\phi^2)\), as follows:

\[
Y = \left[ 1 - \frac{\gamma - 1}{2} M_2^2 \left( \frac{1}{1-\Delta \phi^2} - 1 \right) \right]^{-\frac{\gamma - 1}{\gamma - 1}} - 1.
\]  

(3.45)

In order to understand the difference between the two conversions and how to use the loss correlations given by both groups, the conversion expression will need to be rederived over
the course of the thesis. An important development for trailing edge losses occurred in 1990 when Denton [30] attempted to verify an analytical relationship for $Y_{TE}$ with numerical results. This development will be investigated to determine its utility in the present thesis project. Additionally, a study will be performed to identify modern trailing edge loss measurement techniques and the results will be presented in the Measurement Planning section of the final thesis.

Summary of Conventional Approaches

The combined Ainley and Mathieson [5][6], KO [7], MKT [25], BSM [26][27][8][28] method presented above is the most recent available in the open literature, but it is not necessarily the best candidate for use in a steam turbine model. At the 2008 IMECE conference, Benini et al. presented a comparison of six different Detailed Parameter Models, based on their accuracies in design-point and off-design prediction of the performance of a low reaction, two-stage gas turbine. The most recent method considered in Benini et al. ’s quantitative comparison is that of Moustapha et al. Benini et al. reference some additional pressure-loss models that should be addressed in future research, including those of Craig & Cox [15], Traupel [31], and Balje & Binsley [32]. Of particular interest is the relatively high accuracy of the Craig & Cox method in predicting total-to-total efficiency and stator and rotor total pressure-loss coefficients in the first stage. Of course, the ideal Detailed Parameter Model will also accurately predict the performance of subsequent stages.

The comprehensive applicability of Detailed Parameter Models to design and off-design operating conditions comes at the cost of complexity. If accuracy and an adherence to first principals is the true goal of performance prediction, then Denton [20] presents the next step in loss correlations.

3.3.2 Denton’s Novel Approach

The major shortcoming of conventional methods, like Ainley and Mathieson [5][6] and its extensions, is the lack of first principles used in their derivation. The philosophy of the pressure-loss model presented previously is that additional correction factors, correlation factors, or even loss components can be added to account for new hypotheses on loss sources,
or for new flow regimes of Mach or Reynolds numbers. Continued development in this
direction will lead to an overly complex model for losses based on speculation, not on sound
theory. No correlation derived under such a philosophy can be general for every turbine since
the overall method (AM [5, 6], AMDC [14], KO [7], etc.) is tuned by each manufacturer to
match their product line. In his 1993 International Gas Turbine Institute Scholar Lecture,
Denton[20] charges that the conventional methods are not based on a sound understanding
of the flow physics. He also explains the upshot of adhering to this philosophy in the turbine
design world:

There have been many instances where a designer was unwilling to try out a
new idea because a 30-year-old loss correlation predicted that it would give no
improvement.

Such correlations can tell us nothing about new design features that were not
available at the time the correlation was developed.... It is the author's view that
a good physical understanding of the flow, and particularly of the the origins
of loss, is more important to the designer than is the availability of a good but
oversimplified correlation.

Denton proposes a novel approach, to study the detailed results of experiments and
numerical calculations to understand the fundamental physical reasons for loss in a turbine
stage. He says losses can still be divided between regions of “profile”, “endwall”, and
“leakage” components. In addition, each of these components should be divided into the
real sources of irreversibility, e.g.

1. Viscous friction in boundary layers or free shear layers.

2. Heat transfer across finite temperature differences.

3. Nonequilibrium processes such as occur in very rapid expansions or in shock waves.

In order to quantify the results, he expresses stator and rotor losses in terms of entropy
coefficients for the nozzle,

\[ \zeta_{Ns} = \frac{T_2 \Delta s}{h_{02} - h_2}, \]  

(3.46)

36
and for the rotor,
\[
\zeta_{Rs} = \frac{T_3 \Delta s}{h_{03,rel} - h_3}.
\] (3.47)

These loss coefficients are similar to the enthalpy loss coefficients in Equations 3.8 and 3.9, their only difference being in the numerator. This difference is easily resolved by looking at a Mollier diagram, such as in Fig. 3.6. The slope of the constant pressure lines is approximately equal to static temperature for small changes in enthalpy. That is, \( \left( \frac{\partial h}{\partial s} \right)_p = T \).

Denton derived analytical expressions for the loss coefficients in terms of the real sources of irreversibility.

Denton withholds a definitive analytical expression for endwall losses, which represent on the order of 1/3 of total losses. If a particular loss component cannot be quantified, then neither can the total loss coefficient. This is the greatest barrier to implementing Denton’s approach in the present steam turbine system model. On the breakdown of endwall losses into its real flow components, he had this to say, “In all the situation is too complex and too dependent on details of the flow and geometry for simple quantitative predictions to be made. The main hope in the near future is that the loss can be quantified by three-dimensional Navier-Stokes solutions, which already give good qualitative predictions of the flow.”

### 3.3.3 Summary of Empirical Loss Prediction

As the most widely used loss type in the literature, pressure-loss models represent a primary candidate for a conventional performance prediction method to be empirically tuned by performance testing. The contemporary improvement of the 50-year-old Ainley and Mathieson correlation by Benner et al. is a testament to its continued versatility; however, supposed limits to its accuracy and resolution necessitate exploration of other approaches as well. In 1993, Denton proposed to advance his empirical loss theory by numerical solutions of the Navier-Stokes equations, a not-so empirical process. As was discussed at the outset to Section 3.3, turbine performance prediction requires a combination of empirical, numerical, and analytical methods. A brief description of these additional methods are described in the following section.
3.4 Analytical and Numerical Turbine Performance Prediction

3.4.1 Analytical Methods

The most prevalent loss prediction methods in the literature depend on 1-D analytical approaches, which assume radial and circumferential flow uniformity. A theoretical turbine model derived from first principles under these assumptions is presented in Section 4.1. More complex analytical approaches are also presented in the literature, but they require a more thorough understanding of turbomachinery flow physics.

One of the most widely used 2-D analytical turbine design methods is radial equilibrium. According to Dixon [3], “The radial equilibrium method is based on the assumption that any radial flow which may occur is completed within a blade row, the flow outside the row then being in radial equilibrium.” Such an assumption is most accurate for turbines with large hub-to-tip ratios, where the blades are too short for much radial momentum to appreciate. Dixon describes several radial equilibrium approaches, which are divided into “indirect” and “direct” problems. The free-vortex approach assumes that the product of radius and tangential velocity remains constant at all radii \( rV(y) = K \) and thus the axial vorticity component is zero. The forced vortex approach, sometimes called solid body rotation assumes that tangential velocity varies directly with radius \( V_1y = K_1r \) and \( V_2y = K_2r \). For both of these indirect methods, the flow angle variation over the blade span is then found through the analysis. This differs from the direct approach where the flow angle variation is specified, and the solution of \( V_x \) and \( V_y \) are found. Dixon also presents an analysis for predicting the off-design performance of a free-vortex turbine stage.

When a turbine of low hub-tip ratio is considered, Dixon recommends the more accurate actuator disc approach, which was first used in the propeller theory. The method takes the view that the axial width of each blade row is shrunk while the space-chord ratio, the blade angles and overall length of the machine are maintained constant. For future research, it would be useful to investigate this method further.
3.4.2 Numerical Methods

Dixon [3] gives a brief overview of numerical approaches for solving the momentum, energy, and state equations. The first group of approaches is the “through-flow” category, which includes the streamline curvature, matrix through-flow or finite difference, time-marching, and stream function methods. Through-flow methods assume that the flow is steady in both the absolute and relative frames of reference, and that the flow is axisymmetric outside the blade rows, ignoring the effects of blade wakes from upstream blade rows. Dixon lists several authors of importance for detailed descriptions of through-flow methods: Macchi (1985) and Smith (1966) for streamline curvature; Marsh (1968) for matrix through-flow and Denton (1985) for time-marching. Contrary to its name, the time-marching method is not used to solve the unsteady flow problem, but rather is marched in time toward a converged steady state solution.

The second group of approaches is the Computational Fluid Dynamics (CFD) category, which are able to solve, accurately or not, the three dimensional unsteady flow equations. According to Dixon [3], the use of CFD grew in the 1990’s when there was demand for a faster solution method that costs less and achieves greater resolution than the traditional axisymmetric through-flow methods. Although CFD results are useful for investigating flow phenomena around particular stage geometries, Horlock and Denton [33] indicate that loss predictions by CFD are still not accurate. Dixon states, “There are many reported examples of the successful use of CFD to improve designs but, it is suspected, many unreported failures.” CFD is used heavily in the steam and gas turbine industries. Its use tends to be limited to more expensive projects or product development, due to the computational time.

3.5 Measurement Planning

The primary motivation of a steam turbine test loop is the development of performance prediction methods for turbine design and selection. A review of current methods of turbine performance prediction has been presented, with a focus on empirical correlations. In order to verify, update, and create these prediction methods, a test loop will need to capture specific experimental data.
Two groups of literature present relevant examples for test loop measurement planning. The first group, Codes and Standards, give a description of overall turbine performance measurement, as opposed to stage performance. They detail proper placement of temperature, pressure, flow, torque, and speed sensors in the test loop, but not within the turbine itself. The second group, Existing Research Facilities, includes industrial test loops and academic steam or gas turbine research facilities that have been built previously and can guide the design of the present test loop. Industrial researchers follow the ASME Performance Test Codes (PTC) for overall test loop designs; however, their stage performance measurement methods are not published. The stage performance testing methods are not investigated for the present thesis, but this area will be critical in the final test loop design stages in order to meet the needs of potential customers. Academic research facilities rarely use full rig test stands, and are usually limited by cost to cascade testing. The measurement techniques used in cascade testing do have potential for conversion to full rig setups. A comprehensive review of the two groups of literature has not been carried out, but they are presented in enough detail here to aid the steam turbine test loop design.

3.5.1 Codes and Standards

The most applicable codes to the steam turbine test loop design are the ASME Performance Test Codes (PTC). Of those, PTC 6 for Steam Turbines holds the most relevant information for overall system design and sensor placement. That code also refers to the detailed senser codes, such as PTC 19.2 for Pressure Measurement, PTC 19.3 for Temperature Measurement, PTC 19.5 for Flow Measurement, and PTC 19.7 for Measurement of Shaft Power. With the exception of PTC 19.7, all of these codes are American National Standards. The detailed sensor codes are targeted at selection and calibration of each type of property measurement. They also provide information on the measurement errors of different types of sensors. For instance, PTC 19.3 discusses several types of thermocouples and RTD’s, each of which is most accurate across a different temperature range. PTC 6 selects several sensor types that provide the best accuracy for each property measurement and also states rules for their placement.

For temperature measurement, PTC 6 requires accuracy of $\pm 1^\circ F$ (0.5K). It recommends
use of one of the following:

- platinum RTD’s with random-bridge measuring instruments (0.03% accuracy)
- suitable thermocouple with continuous thermocouple wires and integral cold junctions calibrated and used with a random high-quality digital voltmeter (±0.03% uncertainty)
- calibrated thermocouples or random thermometers with an uncertainty not exceeding ±0.5 °F (0.3K) for cold junction ambient temperature reference measurements.

All temperature measurements for fluids inside a pipe must have the sensor inside a thermowell. Location of sensors for enthalpy determination must be as close as possible to, and downstream of, the corresponding pressure measurement device. Critical temperature measurements must have at least two sensors in separate wells, spaced at least 2 pipe diameters apart axially or at least 45 ° apart circumferentially. The temperature difference between the two measurements must not exceed 1 °F. The requirements for pressure measurement are much more detailed and do not impact the test loop design herein, so they will not be discussed in detail. The most important pressure measurement requirement of PTC 6 is that calibrated pressure transducers of the 0.10% accuracy class are the primary candidate for steam pressure measurement.

Pressure drop is the key state property used to determine primary system flow. PTC 6 recommends measurement of water flow in the feedwater cycle, either between the condenser and pump or between the pump and boiler. The pipe length between the condenser and pump will probably be the best choice, because the pump will generate downstream unsteady phenomena. The code recommends primary water flow be measured using a low-beta-ratio throat-tap nozzle, as they provide the most accurate results. The most important requirement to this thesis is that a minimum of 20 pipe diameters of length upstream and 10 pipe diameters of length downstream must be straight pipe free from obstructions, including thermowells. Pressure is measured in taps one pipe diameter upstream and at the throat of the nozzle to determine the pressure drop. A flow straightener (2 pipe diameters long) must be placed at least 16 pipe diameters upstream of the nozzle entrance. There are more specific requirements for the throat tap nozzle design and its connection to the flow.
measurement section in which it is contained. These details should be used in the final test loop design, which will not be carried out herein. For complete details, refer to PTC 6.

Determination of overall turbine efficiency for different stator and rotor bladesets is the minimum use of the test loop. For a turbine without condensation at the exit, overall efficiency determination requires knowledge of the shaft power output, inlet enthalpy and velocity, and exit enthalpy and velocity. The actual measured properties are the inlet pressure and temperature, exit temperature and pressure, shaft speed, and torque. For a turbine with condensation at the exit, the steam quality will need to be determined in order to find the exit enthalpy. PTC 6 lists several available methods: there are radioactive and nonradioactive tracers; a heat balance can be performed around the condenser; a throttling calorimeter can be used. Tracers are the most accurate option, but radioactive tracers present additional regulatory barriers. The heat balance has inherent uncertainties associated with heat transfer to the surrounding environment. The accuracy of the throttling calorimeter is limited by the principle that “there can never be assurance that the sample is representative of the average conditions of the steam flowing in the pipe” [34]. The calorimeter method seems like the most likely to be used in the present endeavor if condensing performance is pursued. More information on steam sampling is provided in ASTM D-1066 “Method of Sampling Steam” and ASME PTC 19.11. Both should be consulted if calorimeters are used.

PTC 6 has three addendums that should be used when final loop design is carried out. They are PTC 6S, Procedures for Routine Performance Tests of Steam Turbines; PTC 6A, Appendix A to PTC 6 The Test Code for Steam Turbines; and PTC 6 Report, Guidance for Evaluation of Measurement Uncertainty in Performance Tests of Steam Turbines. The ASME PTC’s are crucial to making the test loop design useful to potential customers. Satisfying their requirements is the bare minimum for test loop measurement practices. Doing so will enable determination of gross turbine efficiency within industry accepted accuracy limits.
3.5.2 Existing Research Facilities

Fig. 3.14 shows the high speed cascade wind tunnel at Carleton University in Ottawa. The compressed air tanks enable test run durations around 40 seconds, and blade row exit mach numbers between 0.5 and 1.2. As mentioned in Section 3.3, Cascades are widely used by gas turbine empiricists to isolate specific loss components and flow phenomena. Such measurements are difficult to obtain in a full-rig turbine, which has a rotating shaft and very little extra spacing for measurement devices to be inserted.

The Isentropic Compression Tube Annular Cascade Facility [35] at the Von Karman Institute (VKI) represents a hybrid between a rotating full-rig and a stationary cascade. It supplies compressed air to a 1.5 stage turbine. A full stage with stator and rotor is followed by a single stator blade row. One of the major uses of the Von Karman annular cascade is to investigate the effects of wakes caused by an upstream stage on stator performance. It can model up to 1.5 hp turbine stages for test durations between 0.2 and 0.4 seconds. The
measurement instrumentation methods used would be directly usable on a full rig turbine, barring any space or extreme temperature and pressure limitations.

The measurements required to determine row-by-row losses are very technical. The detail design is not necessary for completion of this thesis, but it should be considered for future work. For full rig turbine testing, the row-by-row measurements for a single stage and the gross measurements of the entire turbine might give enough data to predict performance for all blade rows. If a desire for row-by-row measurements arises from customer needs statements, then literature related to particle imaging velocimetry (PIV) and laser anemometry should be considered. A final option for determining fluid velocity without inhibiting the flow is slip-ring telemetry and/or radio communications for data transfer between the rotating frame of reference and stationary computers.
Chapter 4

Theoretical Models

The test loop design that is modelled in this Chapter is illustrated in Fig. 4.1. Quasi-steady theoretical models are derived for the turbine, condenser, pump, boiler, and pipe components. Theses models are intended to aid in the selection of each of those components for the steam turbine test loop. The component models will are not implemented in a feedback loop or control system, so a quasi-steady model of the dynamometer would simply follow the speed and torque performance of the steam turbine. Therefore, it was deemed unnecessary to model the dynamometer, but results relevent to its selection, and to all other components’ selection, are presented in Chapter 6.

Over the course of deriving the theoretical models, the governing first principles are conservation of mass, momentum, and energy applied to a control volume (CV). All CV’s in this chapter have one inlet and one exit and all analyses are carried out assuming steady state operating conditions. The initial forms for these equations are given here. Mass conservation is started from the form

\[ 0 = \int_{CS} \rho \vec{V} \cdot d\vec{A}. \]  

(4.1)

Momentum conservation is started from the form

\[ \sum_{CV} \vec{F} = \int_{CS} \vec{V} \rho \vec{V} \cdot d\vec{A}. \]  

(4.2)

For a single inlet, single exit CV operating at steady state, the conservation of energy (first law of thermodynamics) is expressed as

\[ \dot{Q}_{CV} - \dot{W}_{CV} = \dot{m} \left[ \left( h_e + \frac{V_e^2}{2g_c} + gz_e \right) - \left( h_i + \frac{V_i^2}{2g_c} + gz_i \right) \right]. \]  

(4.3)
\( \dot{Q} \) is positive for heat input to a CV and \( \dot{W} \) is positive for work output from a CV. The entire cycle model has been divided into a series of CV’s, some of which repeat, and some which do not. The inlet thermodynamic state of one CV is equal to the exit thermodynamic state of the previous CV. All of the theoretical models have been derived to work in any unit set. However, the computer implementation of the models has been carried out in US Customary units. The program can be modified for other unit sets by placing a conversion on the front end at the inputs and on the back end at the outputs.

4.1 Turbine Component

4.1.1 CV Description

The turbine model is derived to accept four variables as inputs. They are as follows:

1. \( \dot{m} \), inlet mass flow rate (lbm/s)

Figure 4.1: Schematic of the theoretical model of the test loop
2. $\omega$, shaft speed ($rad/s$)

3. $T_1$, inlet temperature ($^\circ F$)

4. $P_1$, inlet pressure ($psia$)

As is shown in Appendix [A], the turbine is chosen as the first component in the computer model for the cycle. Its input variables were taken as the inputs for the entire test loop system model, because their required operating ranges are given by manufacturers or designers who want to use the test loop. For example, a customer might say, “The dynamometer should be able to handle turbine shaft speeds between 2,000 and 15,000 RPM, and the turbine inlet temperature should be able to fluctuate between 300 and 500 $^\circ F$.”

The turbine is subdivided into several CV’s, each of which impacts the turbine performance. The first of these is the inlet CV, followed by any number of single stage CV’s, and finally the diffuser CV. The single stage CV is broken down further into the stator, interblade, and rotor CV’s. A theoretical model is developed for each CV separately according to first principles. The single stage CV’s are assembled by adjoining their boundary surfaces as depicted in Fig. 4.2. The state numbering for a stage goes from 1 to 2S to 2R to 3. Subscript ‘2S’ refers to the real stator exit state, which differs from state ‘2R’ that refers to the real rotor inlet state. The subscript ‘2Ss’ refers to the ideal conditions at stator exit for an isentropic process between states 1 and 2S.

### 4.1.2 Single Stage - Stator CV

The stator causes the fluid to have tangential or rotational momentum which will be imparted on the rotor. It does so by converting pressure energy to kinetic energy. Both static pressure and static enthalpy decrease across the stator, while velocity increases.

**Conservation of Mass**

Conserving mass across the stator results in

$$\dot{\bar{m}} = \rho_1 V_{x1} A_1 = \rho_{2S} V_{2S} A_{2S}.$$
Thus the axial velocity at stator exit is

\[ V_{x2S} = \frac{\dot{m}}{\rho_{2S} A_{2S}}. \]  \hspace{1cm} (4.4)

**Conservation of Momentum**

Assuming that the gas exit angle from the stator is equal to the blade exit angle,

\[ V_{2S} = \frac{V_{x2S}}{\cos \alpha_2}. \]  \hspace{1cm} (4.5)

This assumption is very common in 1-D axial turbine analyses and both Horlock [10] and Dixon [3] say it is a reasonable estimate.
Conservation of Energy

Since it is assumed that there is no work or heat flux across the stator, energy conservation simplifies to $h_{02S} = h_{01}$, thus

$$h_{2S} = h_{02S} - \frac{V_{2S}^2}{2g_c} = h_{01} - \frac{V_{2S}^2}{2g_c}.$$ \hspace{1cm} (4.6)

Losses are accounted for with the enthalpy loss coefficient $\zeta_S$ by

$$h_{2Ss} = h_{2S} - \frac{\zeta_S V_{2S}^2}{2g_c}.$$ \hspace{1cm} (4.7)

Combined Equation Set

Equations of state complete the equation set, \textit{i.e.}

$$\rho_{2S} = EOS (P_{2S}, h_{2S}),$$ \hspace{1cm} (4.8)

and

$$P_{2S} = EOS (h_{2Ss}, s_1).$$ \hspace{1cm} (4.9)

So long as $\zeta_S$ can be found, there are 6 equations (4.4-4.9) with 6 unknowns. Since the equations of state are implicit, the set must be solved iteratively. The details of this solution are shown in Fig. A.4 of Appendix A. The remaining state properties and stagnation properties are found using equations of state of the form

$$SP_{2S} = EOS (P_{2S}, h_{2S}, V_{2S}),$$

so that a complete thermodynamic state is passed to the interblade CV. The practice of fully defining the exit state is carried out for every CV, so that the inlet state to the following CV is always completely known. Therefore the inlet state to every CV is known, so long as the inlet state to the first CV in the test loop, \textit{i.e.} the turbine component, is fully defined. There are some caviates for the pump and boiler CV’s in order to close the cycle.

4.1.3 Single Stage - Interblade CV

The interblade CV analysis is applied between the stator exit and rotor inlet of every stage. It is also applied between the rotor of one stage and the stator of the next stage, if one exists.
Most 1-D axial turbine analyses are carried out using non-dimensional coefficients that cover the bulk stator and rotor rows. They make no mention of the interblade region except as the area in which leakage flows leave and rejoin the primary flow. The interblade CV of this analysis accounts for area changes between blade row exits and inlets that change the state of the working fluid. The state numbering used in this section refers to the interblade CV within a single stage. Thus the inlet state is ‘2S’ and the exit state is ‘2R’. If the same analysis were applied to the region between two different stages, it would be between state 3 of the preceding stage and state 1 of the following stage.

**Conservation of Mass**

Conserving mass in the axial direction yields

\[
\dot{m} = \rho_{2S} V_{x2S} A_{2S} = \rho_{2R} V_{x2R} A_{2R}.
\]  

(4.10)

The leakage flows associated with the interblade CV are accounted for with tip leakage loss components in the stator and rotor row. These loss components are included in the blade row loss coefficients. Thus the interblade CV process is assumed to be isentropic with only a single inlet and exit.

**Conservation of Momentum**

Conserving momentum on the interblade region requires an accounting of the radial velocities associated with leakage flows in and out of the CV. The magnitude of the leakage flow is linked to the leakage component of the stator and rotor loss coefficients. The theoretical analysis for this thesis assumes some external algorithm is used to determine all loss coefficient components. Thus no information is known about the magnitude of the leakage flow or the radial velocity.

An alternative approach used to generate a solvable set of equations across the interblade CV is to assume an isentropic process. The full conservation of momentum cannot be carried out since the leakage flows are unknown. The mass-averaged tangential velocity is assumed constant from inlet to exit, i.e.

\[
V_{y2R} = V_{y2S}.
\]  

(4.11)
The mass-averaged axial velocity will fall out from the combined equation set, which includes the mass conservation.

**Conservation of Energy**

Similar to the stator, there is assumed to be no heat or work flux out of the interblade CV, so

\[ h_{02S} = h_{02R} = h_{2R} + \frac{V_{2R}^2}{2g_c}. \]

Solving for the exit enthalpy yields

\[
\begin{align*}
    h_{2R} &= h_{02S} - \frac{V_{2R}^2}{2g_c}, \\
    h_{2R} &= h_{02S} - \frac{V_{x2R}^2 + V_{y2S}^2}{2g_c}. \quad (4.12)
\end{align*}
\]

**Combined Equation Set**

Equation [4.11] is substituted into Equation [4.12] to give

\[ h_{2R} = h_{02S} - \frac{V_{x2R}^2 + V_{y2S}^2}{2g_c}. \] \quad (4.13)

Equations [4.10] can be rearranged to solve for the exit axial velocity as

\[ V_{x2R} = \frac{\dot{m}}{\rho_{2R} A_{2R}}. \] \quad (4.14)

The assumption of constant entropy enables the exit pressure and density to be found from equations of state as

\[ P_{2R} = EOS (h_{2R}, s_{2S}), \] \quad (4.15)

and

\[ \rho_{2R} = EOS (P_{2R}, h_{2R}), \] \quad (4.16)

respectively. Thus Equations [4.13], [4.14], [4.15], and [4.16] are four equations with four unknowns, \( h_{2R}, V_{x2R}, P_{2R}, \) and \( \rho_{2R} \). The use of an implicit equation of state necessitates an iterative solution, as described in Fig. A.5 of Appendix A. Once the solution is found, the thermodynamic state at the CV exit is fully defined for passing to the rotor CV.
4.1.4 Single Stage - Rotor CV

The final process for the stage, and the most important to work extraction, is across the rotor blade row. The inlet state is ‘2R’ and the exit state is ‘3’. The rotor absorbs the tangential momentum of the flow, and converts it to shaft work. The rotor absorbs the most momentum when the exit tangential velocity is zero. If it is nonzero, then the fluid kinetic energy has not been reduced to its minimum possible value, and there remains energy to be absorbed or dissipated. For the final stage in the turbine, it is assumed that all excess tangential kinetic energy is dissipated or lost.

**Conservation of Mass**

Similar to the stator and interblade CV’s, conservation of mass yields

\[ \dot{m} = \rho_{2R} V_{x2R} A_{2R} = \rho_3 V_{x3} A_3. \] (4.17)

Equation 4.17 is combined with velocity triangles to express the tangential gas velocities as

\[ V_{y2} = V_{x2S} \tan \alpha_2 \]
\[ V_{y2} = \frac{\dot{m}}{\rho_{2S} A_{2S}} \tan \alpha_2, \] (4.18)

and

\[ V_{y3} = W_{y3} - U_3 \]
\[ V_{y3} = V_{x3} \tan \beta_3 - U_3 \]
\[ V_{y3} = \frac{\dot{m}}{\rho_3 A_3} \tan \beta_3 - U_3. \] (4.19)

This is assuming that \( \beta_3 \approx \alpha'_3 \), which both Horlock [10] and Dixon [3] say is a reasonable estimate.

**Conservation of Momentum**

From conservation of momentum, Euler’s turbine equation shows that

\[ h_{02} - h_{03} = U_2 V_{y2} + U_3 V_{y3}. \] (4.20)
In some textbooks, the “+” on the right hand side of Equation 4.20 is switched to a “−” sign. The difference arises from the convention for identifying the positive tangential direction. Textbooks that show a “−” sign here identify the positive tangential direction as the one in which the rotor blades turn for all blade rows. In the present study, positive blade exit angles are associated with the direction the fluid is most likely to rotate upon exiting a blade row, i.e. the direction in which the blade directs the fluid. This direction alternates between stators and rotors, because rotors direct fluid in a direction opposite to the direction of their rotation, so as to absorb its momentum. Positive blade inlet angles are taken as the opposite tangential direction as the exit angle, since that is the direction from which the fluid is arriving on the blade row.

**Conservation of Energy**

From conservation of energy across the entire stage, it is shown that the stage specific work is

\[ \delta W = \frac{\dot{W}}{\dot{m}} = h_{01} - h_{03} = h_{02} - h_{03}. \]

After substituting Equation 4.20 the stage specific work is

\[ \delta W = U_2 V_{y2} + U_3 V_{y3}, \]  \hspace{1cm} (4.21)

where the blade speeds are given by

\[ U_2 = \omega \cdot r_{m2R}, \]
\[ U_3 = \omega \cdot r_{m3}. \]  \hspace{1cm} (4.22)

\( r_{m2R} \) and \( r_{m3} \) are the mean rotor inlet and exit radii. Energy losses across the rotor are accounted for with the enthalpy loss coefficient,

\[ \zeta_R = \frac{h_{3} - h_{3ss}}{W_3^2}. \]  \hspace{1cm} (4.23)

**Combined Equation Set**

Upon combining Equations 4.18, 4.19 and 4.21 the final expression for the stage specific work is

\[ \delta W = \left[ \dot{m} \frac{U_2 \tan \alpha_2}{\rho_2 S_{2b}} + U_3 \left( m \frac{\tan \beta_3}{\rho_3 A_3} - U_3 \right) \right]. \]

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Upon rearrangement to group the independent variables $\dot{m}$ and $\omega$, this becomes

$$\delta W = \dot{m} \omega \left( r_{m2R} \frac{\tan \alpha_2}{\rho_2 S A_2 S} + r_{m3} \frac{\tan \beta_3}{\rho_3 A_3} \right) - \omega^2 r_{m3}^2. \quad (4.24)$$

The only unknown in the final expression for $\delta W$ is the stage exit density, $\rho_3$. Through a combination of velocity triangles, mass and energy equations, and equations of state $\rho_3$ is found from

$$\alpha_3 = \arctan \left( \frac{\tan \beta_3 - \omega r_{m3} \dot{V}_x}{V_3} \right),$$

$$V_3 = \frac{V_3}{\cos \alpha_3},$$

$$W_3 = \frac{W_3}{\cos \beta_3},$$

$$h_3 = h_{03} - \frac{V_3^2}{2g_c},$$

$$h_{3s} = h_3 - \frac{\zeta \omega W_3^2}{2g_c},$$

$$P_3 = EOS \left( h_{3s}, s_{2R} \right),$$

$$\rho_3 = EOS \left( P_3, h_3 \right),$$

$$V_{x3} = \frac{\dot{m}}{\rho_3 A_3}.$$  \quad (4.25)

This equation set requires the exit stagnation enthalpy which is found from

$$h_{03} = h_{02} - \delta W. \quad (4.26)$$

An initial guess for $\rho_3$ enables $\delta W$ and $h_{03}$ to be found. Then an iterative solution of Equations 4.25 is carried out to find a new value of $\rho_3$. Equation 4.24 and Equation 4.26 are evaluated at this new value of $\rho_3$ to find a new value of $\delta W$ and $h_{03}$, respectively. The iterative process to find $\rho_3$ is repeated, and so on. Therefore the system of equations requires two levels of iteration, as illustrated in Fig. A.6 of Appendix A. Once the specific work has been determined, the power output and torque are found from

$$\dot{W} = \dot{m} \cdot \delta W,$$

$$\tau = \frac{\dot{W}}{\omega}. \quad (4.27)$$

When a multi-stage turbine is considered the total specific work, power, and torque are equal to the sum of the values from each stage, i.e.

$$\Delta W = \sum \delta W_i,$$

$$\dot{W}_{tot} = \sum \dot{W}_i,$$
\[ \tau_{\text{tot}} = \sum r_i. \quad (4.28) \]

The stage exit stagnation pressure, which is a critical model output for multistage analyses, is found from

\[ P_{03} = P_3 + \frac{\rho_3 V_3^2}{2 g_c}. \quad (4.29) \]

**Design Point Determination**

One way of estimating the design point of the stage is when the flow incidence angle at the rotor inlet is zero, i.e.

\[ \alpha'_{2R} = B_{2R}. \]

From the rotor inlet velocity triangle, the relative rotor inlet tangential velocity component is equal to the difference between the absolute inlet tangential velocity and the inlet blade speed:

\[ w_{y2} = V_{y2} - U_2. \]

The velocity \( V_{y2} \) was assumed constant across the interblade region between the stator and rotor. The tangential velocities can be expressed in terms of the axial velocity and flow angles, resulting in

\[ V_{x2R} \tan \alpha'_{2S} = V_{x2R} \tan \alpha_{2S} - U_2. \]

Separating velocities and angles results in

\[ \frac{U_2}{V_{x2R}} = \tan \alpha_{2S} - \tan \alpha'_{2S}. \]

So at the design point,

\[ \frac{U_2}{V_{x2R}} = \tan \alpha_{2S} - \tan \beta_{2R}. \quad (4.30) \]

Recall that the velocity ratio and rotor inlet flow coefficient are expressed as

\[ \sigma = \frac{U_2}{V_2}, \]

and

\[ \Phi_2 = \frac{V_{x2R}}{U_2}, \]
respectively. The expression on the lefthand side of Equation 4.30 is the reciprocal of the rotor inlet flow coefficient, thus

\[ \frac{1}{\Phi_{2,des}} = \tan \alpha_{2S} - \tan \beta_{2R}. \] (4.31)

Keep in mind that this is the rotor inlet design flow coefficient and is not the same as the average stage flow coefficient design value, which was described in Subsection 3.2.4. The rotor inlet flow coefficient can be converted to velocity ratio by relating the axial and absolute velocities as

\[ V_{x2S} = V_2 \cos \alpha_{2S}. \]

Rearranging and substituting into the expression for velocity ratio yields

\[ \sigma = \frac{U_2}{V_2} = \frac{U_2}{V_{x2S}} \cos \alpha_{2S} = \frac{\cos \alpha_{2S}}{\Phi_2}. \]

Thus the design velocity ratio is equal to

\[ \sigma_{des} = (\tan \alpha_{2S} - \tan \beta_{2R}) \cos \alpha_{2S}. \] (4.32)

Satisfying the rotor inlet design condition does not guarantee that stage efficiency is maximized; however, it does approximate that operational point.

### 4.1.5 Turbine Inlet CV

**Conservation of Mass, Momentum, and Energy**

The conservations of mass, momentum, and energy for the turbine inlet are very similar to those for the interblade CV. The exception is that there are no leakage flows and both inlet and exit tangential velocities are negligible. It is still assumed that entropy is constant. The inlet state is ‘A’ and the exit state is ‘B’.

**Combined Equation Set**

With the exception of the kinetic energy term change of Equation 4.34 compared to that of Equation 4.13 the resulting equation set looks nearly identical:

\[ P_B = EOS (h_B, s_A), \] (4.33)

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\[ h_B = h_{0A} - \frac{V_{xB}^2}{2g_c}, \]  
\[ V_{xB} = \frac{\dot{m}}{\rho_B A_B}, \]  
and
\[ \rho_B = \text{EOS}(P_B, h_B). \]

The inlet CV is used to convert kinetic energy to pressure energy by increasing the flow cross-sectional area. If the area increases too quickly, there could be flow separation and large pressure losses. These are in addition to the pressure losses associated with wall friction. So the theoretical model above is inadequate to predict the behavior of inlet CV’s with substantial pressure losses. The solution method for Equations 4.33 to 4.36 are similar to the equation set for the single stage interblade CV.

### 4.1.6 Turbine Diffuser CV

**Conservation of Mass, Momentum, and Energy**

The conservations of mass, momentum, and energy for the turbine diffuser are very similar to those for the interblade CV. It differs from the inlet CV, because there could be some inlet whirl, as a result of an imperfect momentum transfer in the final rotor row. The associated momentum must be absorbed by a tangential friction force, and the associated kinetic energy must be dissipated in order to conserve energy. Since the magnitude of the dissipative force isn’t of interest to this analysis, conservation of momentum does not need to be repeated from the integral form. The inlet state is ‘C’ and the exit state is ‘D’.

**Combined Equation Set**

The resulting equation set is:
\[ P_D = \text{EOS}(h_D, s_C), \]  
\[ h_D = h_{0C} - \frac{V_{xD}^2}{2g_c}, \]
where
\[ h_{0C} = h_C + \frac{V_{xC}^2 + V_yC^2}{2g_c}. \]
The extra energy associated with tangential velocity at the CV inlet is absorbed into the enthalpy term at the CV exit. The exit axial velocity and density are found from
\[ V_{xD} = \frac{\dot{m}}{\rho_D A_D}, \]  
and
\[ \rho_D = EOS (P_D, h_D). \]
The diffuser is used to recover pressure by similar means to the inlet CV. So this equation set suffers the same weakness as that of the inlet CV, the lack of a pressure or other loss model. The solution method for Equations 4.37 to 4.40 are similar to the equation set for the single stage interblade CV.

4.1.7 Overall Turbine Parameters

So far the axial turbine analysis presented has been for smaller control volumes within the turbine, which, when assembled, achieve the purposes of a complete turbine. The turbine performance is described by a combination of dimensional and non-dimensional parameters. The steps for calculating the dimensional characteristics, such as total power output, at stage and overall turbine levels was described in Subsection 4.1.4. With the exception of efficiencies, the non-dimensional parameters are found at a stage level from the expressions in Subsection 3.2.4. The stage total-to-total and total-to-static efficiencies are calculated using Equations 3.6 and 3.7 where
\[ h_{03ss} = h_{3ss} + \frac{V_{3ss}^2}{2g_c}, \]
and
\[ h_{3ss} = EOS (P_3, s_1), \]
assuming \( V_{3ss} = V_3 \). Remember that the only stage for which \( \eta_{ts} \) is an appropriate measure of performance is the last stage, where there are no following stages to absorb residual kinetic energy.
In order to get a handle on the overall performance of a multistage turbine, the non-dimensional parameters need to be analysed for all stages. For efficiency, it is most appropriate to calculate the turbine total-to-static efficiency as

\[ \eta_{ts} = \frac{h_{0,in} - h_{0,out}}{h_{0,in} - h_{out,ss}} = \frac{\Delta W}{h_{0,in} - h_{out,ss}}, \]

where \( h_{out,ss} = EOS(P_{out}, s_{in}) \). For the purposes of this thesis, the other non-dimensional parameters, such as \( R_h, \sigma, \) and \( \Psi \), are averaged across the stages. Recall that the expressions for these non-dimensional stage parameters were given in Section 3.2. An average stage reaction of 0 does not necessarily mean that all stages are operating at the impulse condition. It would most likely mean that some stages have \( R_P < 0 \) and others have \( R_P > 0 \).

Results for both dimensional and non-dimensional overall turbine parameters are described in Chapter 6.

4.2 Condenser Component

4.2.1 CV Description

The condenser is the first primary component following the turbine. It consists of the hot side and the cold side. The steam test loop working fluid flows through the hot (H) side and liquid coolant flows through the cold (C) side. The associated state naming conventions are shown in Fig. 4.3. The theoretical model must be capable of accepting either superheated vapor or a mixed liquid-vapor fluid on the hot side. Additionally, the condenser must

![Figure 4.3: State nomenclature for the condenser analysis](image_url)
subcool the working fluid in order to avoid cavitation in the pump inlet pipe. The cooling fluid is assumed to be water from an open air pond. Thus the exit temperature must be below boiling for an atmospheric vapor pressure. Another limitation is the maximum operating temperature, which is specified as 375 °F for most off-the-shelf shell and tube heat exchangers. All of these requirements are accounted for in the theoretical model, or in the computer implementation of the model.

4.2.2 Conservation of Mass

Each side of the condenser is a single inlet, single exit CV with heat transfer. So conservation of mass gives

\[ \dot{m}_H = \rho_{H_i} V_{H_i} A_{H_i} = \rho_{H_e} V_{H_e} A_{H_e} \] (4.42)

for the cold side, and

\[ \dot{m}_C = \rho_{C_i} V_{C_i} A_{C_i} = \rho_{C_e} V_{C_e} A_{C_e} \] (4.43)

for the hot side. Because the coolant is subcooled liquid, it is assumed to be incompressible. Also, the inlet and exit flow areas are equal. Therefore, from Equation 4.43,

\[ V_{C_i} = V_{C_e} = V_C. \] (4.44)

So the kinetic energy change on the cold side is effectively zero.

4.2.3 Conservation of Momentum

Conserving momentum would introduce three force balance equations. It would also introduce three unknown reaction forces, none of which are useful to predicting the thermal performance of the test loop. So this portion of the analysis is unnecessary, and has not been carried out.

4.2.4 Conservation of Energy

There is no work and the change in potential energy between the inlet and exit for both hot and cold sides is assumed negligible. Thus the energy equation for the hot side simplifies to

\[ \dot{Q}_H = \dot{m}_H \left( h_{0H_e} - h_{0H_i} \right), \] (4.45)

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where \( \dot{Q}_H < 0 \) indicates heat flux out of the CV. Because the kinetic energy change is negligible on the cold side, the energy equation is

\[
\dot{Q}_C = \dot{m}_C (h_{Ce} - h_{Ci}) .
\]

Assuming constant specific heat, the heat rate can be expressed in terms of the temperature change as

\[
\dot{Q}_C = \dot{m}_C C_p (T_{Ce} - T_{Ci}) .
\]

(4.46)

4.2.5 Combined Equation Set

Assuming the pressure drop on the hot side is negligible, the exit temperature is expressed as

\[
T_{He} = T_{H, \text{sat}} (P_{Hi}) - \Delta T_{\text{subcool}},
\]

(4.47)

where \( \Delta T_{\text{subcool}} \) is the amount of subcool. The exit enthalpy is

\[
h_{He} = EOS (P_{Hi}, T_{He}) .
\]

(4.48)

Because \( \Delta T_{\text{subcool}} > 0 \), the exit state can not be within the vapor dome and the enthalpy can be defined in terms of pressure and temperature. The exit stagnation enthalpy is then

\[
h_{0He} = h_{He} + \frac{V_{He}^2}{2g_c}.
\]

(4.49)

This requires the exit velocity, which can is found by rearranging the mass equation as

\[
V_{He} = \frac{\dot{m}_H}{\rho_{He} A_{He}} ,
\]

(4.50)

where the hot side exit density is

\[
\rho_{He} = EOS (P_{Hi}, h_{He}) .
\]

(4.51)

If \( \Delta T_{\text{subcool}} \) is specified, then there are no unknowns in Equation 4.45. Assuming there is no heat transferred to the environment, \( |\dot{Q}_H| = |\dot{Q}_C| \). If \( T_{C1} \) is specified, the only unknowns remaining are \( \dot{m}_C \) and \( T_{Ce} \). The relation between these two values is unfixed without information about a specific condenser, such as heat transfer area and convective heat transfer coefficients.
4.2.6 Effectiveness - NTU Method

Because it is the goal of this thesis to select an appropriate condenser, it is important to investigate the entire range of possible condensers. The heat transfer coefficient changes substantially between different heat exchanger designs and over the range of operating conditions. The theoretical model accepts a $UA$ value as an input. $U$ is the overall heat transfer coefficient for the condenser, and $A$ is the effective heat transfer surface area. In order to investigate a range of possible condensers, it is useful to use non-dimensional analysis. In the realm of heat exchangers, the most common non-dimensional analysis is the effectiveness-NTU method.

Heat exchanger effectiveness is defined as

$$\epsilon = \frac{\dot{Q}}{\dot{Q}_{MAX}},$$

where $\dot{Q}$ is the actual heat transfer rate. $\dot{Q}_{MAX}$ is the maximum heat transfer rate attainable by an infinitely long counterflow heat exchanger with the same inlet temperatures as those of the actual heat exchanger. The number of transfer units (NTU) is expressed as

$$NTU = \frac{UA}{C_{min}},$$

where $C_{min}$ is the minimum heat capacity of the heat exchanger, i.e.

$$C_{min} = MIN (\dot{m}_C C_{PC}, \dot{m}_H C_{PH}),$$

$$C_{max} = MAX (\dot{m}_C C_{PC}, \dot{m}_H C_{PH}).$$

(4.54)

Another parameter of interest is the ratio of heat capacities,

$$C_r = \frac{C_{min}}{C_{max}}.$$  

(4.55)

For a shell and tube heat exchanger with a single shell pass, the $\epsilon - NTU$ relations are given by Incropera et al. [36] as

$$\epsilon = 2 \left\{ 1 + C_r + (1 + C_r^2)^{0.5} \frac{1 + \exp \left[ -NTU \left( 1 + C_r^2 \right)^{0.5} \right]}{1 - \exp \left[ -NTU \left( 1 + C_r^2 \right)^{0.5} \right]} \right\}^{-1},$$

(4.56)

and

$$NTU = -(1 + C_r^2)^{-0.5} \ln \frac{E - 1}{E + 1},$$

where $E = \frac{\dot{Q}}{\dot{Q}_{MAX}}$. 

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\[ E = \frac{2/\epsilon - 1 - C_r}{(1 + C_r^2)^{0.5}}. \]  \hspace{1cm} (4.57)

Traditionally, \( \epsilon - NTU \) analyses apply to heat exchangers where the fluid on either side is liquid, mixed liquid-vapor, or vapor for the whole heat transfer process. When one of the fluids is in the mixed liquid-vapor region, it is automatically taken as the \( C_{\text{max}} \) fluid and \( C_r = 0.0 \). For the application herein, the hot side fluid will probably encounter the vapor phase and will definitely encounter the mixed phase condition and liquid phase. The traditional heat capacity treatment isn’t sufficient to describe the situation. A weighted heat capacity ratio is introduced as

\[ C_r = X_{\text{SH}} C_{\text{r,SH}} + X_{\text{MLV}} C_{\text{r,MLV}} + X_{\text{SC}} C_{\text{r,SC}}, \]  \hspace{1cm} (4.58)

where ‘SH’ refers to the superheated region, ‘MLV’ to the mixed liquid-vapor region, and ‘SC’ to the subcooled region of the hot side fluid heat transfer. The various X’s are the weights for each of these regions, expressed as

\[
X_{\text{SH}} = \frac{h_{H_i} - h_{H_g}}{h_{H_i} - h_{H_e}},
\]
\[
X_{\text{MLV}} = \frac{h_{H_g} - h_{H_i}}{h_{H_i} - h_{H_e}},
\]
\[
X_{\text{SC}} = \frac{h_{H_l} - h_{H_e}}{h_{H_i} - h_{H_e}}.
\]  \hspace{1cm} (4.59)

The saturation enthalpies can be found using equations of state

\[
h_{H_g} = h_g \left[ T_{H_{\text{sat}}}(P_{H_i}) \right],
\]
\[
h_{H_l} = h_l \left[ T_{H_{\text{sat}}}(P_{H_i}) \right].
\]  \hspace{1cm} (4.60)

The \( C_r \)’s are the heat capacity ratios for each region, expressed as

\[
C_{\text{r,SH}} = \frac{C_{\text{min,SH}}}{C_{\text{max,SH}}},
\]
\[
C_{\text{r,MLV}} = 0.0,
\]
\[
C_{\text{r,SC}} = \frac{C_{\text{min,SC}}}{C_{\text{max,SC}}}. \]  \hspace{1cm} (4.61)

The heat capacities are then

\[
C_{\text{min,SH}} = \text{MIN} \left( \dot{m}_C C_{pC}, \dot{m}_H C_{pH,SH} \right),
\]
\[
C_{\text{max,SH}} = \text{MAX} \left( \dot{m}_C C_{pC}, \dot{m}_H C_{pH,SH} \right),
\]

63
\[ C_{\text{min,MLV}} = \dot{m}_C C_{pC}, \]
\[ C_{\text{min,SC}} = \text{MIN} (\dot{m}_C C_{pC}, \dot{m}_H C_{pH,SC}), \]
\[ C_{\text{max,SC}} = \text{MAX} (\dot{m}_C C_{pC}, \dot{m}_H C_{pH,SC}), \] (4.62)

where
\[ C_{pH,SH} = \frac{C_{ph1} + C_{ph2}}{2}, \] (4.63)
and
\[ C_{pH,SC} = \frac{C_{ph1} + C_{ph2}}{2}. \] (4.64)

In traditional \( \epsilon - NTU \) analyses, the maximum possible heat transfer rate is expressed in terms of \( C_{\text{min}} \) and the inlet temperature difference as
\[ \dot{Q}_{\text{MAX}} = C_{\text{min}} (T_{Hi} - T_{Ci}). \] (4.65)

A logical weighted extension using the heat capacities just introduced would be
\[ \dot{Q}_{\text{MAX}} = X_{SH} C_{\text{min,SH}} (T_{Hi} - T_{Ci}) + X_{MLV} C_{\text{min,MLV}} (T_{Hg} - T_{Ci}) + X_{SC} C_{\text{min,SC}} (T_{Hl} - T_{Ci}). \] (4.66)

Rearranging Equation 4.65 gives an expression for the effective \( C_{\text{min}} \) of any heat exchanger,
\[ C_{\text{min}} = \frac{\dot{Q}_{\text{MAX}}}{(T_{Hi} - T_{Ci})}. \]

Substituting this expression into Equation 4.53 gives a new expression for NTU,
\[ NTU = \frac{UA}{\dot{Q}_{\text{MAX}}} \cdot \frac{(T_{Hi} - T_{Ci})}{(T_{Hi} - T_{Ci})}. \] (4.67)

The cold side mass flow rate, \( \dot{m}_C \), in Equation 4.66 is the primary unknown of interest in the \( \epsilon - NTU \) analysis. The actual value of \( \dot{m}_C \) can be extracted once \( \dot{Q}_{\text{MAX}} \) is known. Once \( \dot{m}_C \) has been found, \( T_{Ce} \) falls out from Equation 4.46. \( \dot{Q}_{\text{MAX}} \) can only be found by iteratively solving the \( \epsilon - NTU \) equation set, which requires an initial guess for \( \dot{m}_C \).

The entire Condenser analysis has relied on the user specifying \( \Delta T_{\text{subcool}} \) as an input to the model. The values for \( \dot{m}_C \) and \( T_{Ce} \) are then the outputs. Locking a condenser onto a single value of \( \Delta T_{\text{subcool}}, \dot{m}_C, \) or \( T_{Ce} \) severely limits the range of achievable operating points.
Still the condenser model needs some realistic input in order to be useful. One solution is to provide a range of values for $T_{Ce}$. The cooling water is assumed to be circulating to a private pond, for which the EPA upper limit on the temperature of outgoing water isn’t readily available. A logical absolute maximum value for $T_{Ce}$ is the boiling point of water at atmospheric pressure. The minimum value must be larger than $T_{Ci}$ in order to satisfy the energy conservation. As a starting point, the minimum value for $T_{Ce}$ is set at $T_{Ci} + 10^\circ F$.

A 1-D search is placed around the entire condenser model described in this section to determine the $\Delta T_{subcool}$ that corresponds to a specified value of $T_{Ce}$. Beginning with $T_{Ce_{max}}$, if the 1-D search can’t converge on a solution, then a lower value of $T_{Ce}$ is used, and so on until $T_{Ce_{min}}$ is reached. If the condenser model does not find a solution within the $T_{Ce}$ limits, then that particular operating point is not achievable with the selected value of UA. The complete condenser computation is described by Figures A.7 to A.9.

### 4.3 Pump Component

#### 4.3.1 CV Description

The pump provides the only means of pressure increase in the test loop. At steady state, the pump must make up for all forms of pressure drop between its exit and its inlet. For the purposes of the complete cycle analysis, it is assumed that the pressure at the exit of the pump is known. Although an initial guess must be provided, an iterative solution method will be used to determine the actual pump exit pressure. The iterative solution includes the models for the boiler and connecting pipes leading up to the turbine, and is illustrated in Fig. A.2 of Appendix A. This solution technique means the required pump performance is dictated by the performances of the other loop components. The head and flow rate are the values required by the system, rather than that achieved by turning an artificial control knob. The test loop system model ends up setting the position of the pump speed control. Subsection 4.3.6 shows how the opposite cause-and-effect relationship governs the actual system. The pump state definitions are relatively trivial compared to the other components, since there is only one inlet and one outlet, as illustrated in Fig. 4.4.
4.3.2 Conservation of Mass

The pump is a single-input/single-output CV. Therefore application of conservation of mass yields equal inlet and exit mass flow rates, \( i.e. \)

\[ \dot{m} = \rho_i V_{xi} A_i = \rho_e V_{xe} A_e \]  \hspace{1cm} (4.68)

The working fluid is a compressed liquid and the pressure increase across the pump is moderate. So the liquid is assumed to be incompressible and the mass equation becomes a volume equation,

\[ \frac{\dot{m}}{\rho} = Q = V_{xi} A_i = V_{xe} A_e, \]  \hspace{1cm} (4.69)

where \( Q \) is the volumetric flow rate.

4.3.3 Conservation of Momentum

If more information was known about the internal geometries of the pump, conservation of momentum would give an expression for the exit tangential velocity in terms of the shaft speed and the flow rate. However this is not the case, so conserving momentum would give no additional useful information relative to the test loop model.
### 4.3.4 Conservation of Energy

It is assumed there is no heat flux from the pump, and that the change in potential energy is negligible from inlet to exit. Therefore, conservation of energy yields

$$\dot{W} = \dot{m} (h_{0i} - h_{0e}), \quad (4.70)$$

where

$$h_{0e} = h_e + \frac{V_e^2}{2g_c} \approx h_e + \frac{V_{xe}^2}{2g_c}. \quad (4.71)$$

The work input to the pump is negative, which is consistent with thermodynamic practice. The ideal (minimum) power input to the pump is the isentropic power $\dot{W}_s$. The pump isentropic efficiency, $\eta_s$, relates the ideal fluid energy increase to the actual increase as

$$\eta_s = \frac{\dot{W}_s}{\dot{W}} = \frac{(h_{0i} - h_{0es})}{(h_{0i} - h_{0e})}, \quad (4.72)$$

where the isentropic stagnation enthalpy is

$$h_{0es} = h_{es}(P_e, s_i) + \frac{V_{es}^2}{2g_c} \approx h_{es} + \frac{V_{xe}^2}{2g_c}. \quad (4.73)$$

This expression requires $P_e$, which is a steam loop requirement. So it must be an input to the model. This differs from the turbine where the pressure drop is calculated as a function of the other operating conditions. The mechanical efficiency, $\eta_m$, relates the shaft power to the actual fluid energy increase as

$$\eta_m = \frac{\dot{W}}{\dot{W}_{SHAFT}} = \frac{\dot{m} (h_{0i} - h_{0e})}{\dot{W}_{SHAFT}}. \quad (4.74)$$

The theoretical pump model requires both efficiencies as inputs. For the purposes of this thesis, they are both set equal to unity.

### 4.3.5 Combined Equation Set

After rearranging Equation [4.69] the exit velocity from the pump is

$$V_{xe} = V_{ei} \frac{A_i}{A_e}. \quad (4.75)$$

Notice that only the axial velocity is accounted for in this equation. Because the pump imparts rotation on fluid, there could also be a tangential velocity component at the exit.
It is assumed that the user has no knowledge of the internal geometries of the pump which would be necessary to predict this component. Therefore the resulting dissipation of energy must be accounted for in the pump efficiency. Unless a detailed analysis of pump power requirement is necessary, this should be sufficient. Rearranging Equation 4.72 gives

\[ h_e = h_{0i} + \frac{h_{0es} - h_{0i}}{\eta_s} - \frac{V_x e^2}{2g_c}. \]  

(4.76)

The shaft power is

\[ \dot{W}_{SHAFT} = \frac{\dot{m}(h_{0i} - h_{0e})}{\eta_m}. \]  

(4.77)

where negative values of \( \dot{W}_{SHAFT} \) indicate power into the system.

4.3.6 Head, Flow, and Shaft Speed

It is assumed that the pump has a variable speed motor, allowing it to cover a wider range of operating points. It is common industry practice to give pump performance maps in terms of head, flow, and speed like the one shown in Fig. 4.5. The head is given by

\[ H = \frac{(P_{0e} - P_{0i})}{\rho}. \]  

(4.78)

The flow used is the volumetric flow from Equation 4.69 expressed in gallons per minute (GPM) or liters per minute (LPM). Generally, the head vs. flow curve for a constant speed fits a quadratic function well. If several points are known on the curve, a second order polynomial least squares fit can be used to find the coefficients of a constant speed equation,

\[ H_{N_i} = A_i Q^2 + B_i Q + C_i. \]  

(4.79)

If several of these curves are known, then the coefficients of a similar function relating head and shaft speed can be found, i.e.

\[ N_{Q_j} = D_j H^2 + E_j H + F_j. \]  

(4.80)

Thus the pump speed can be predicted if \( H \) and \( Q \) are already known. The polynomial least squares fitting technique will not be discussed in more detail, because it is a simple extension of linear regression methods found in many textbooks, including [37] and [38]. There is a flow chart of the pump model implementation in Fig. A.10 of Appendix A.
4.4 Boiler Component

4.4.1 CV Description

Like the condenser, the boiler has a hot and a cold side. In this case, the cold side is where liquid water is converted to superheated steam. The hot side of the heat exchange process follows a combustion chamber, where natural gas is combusted with air. A schematic of the component model is given in Fig. 4.6. The only process between the boiler and the turbine is a pipe length. The boiler is the only means of adding heat to the system, much like the pump in terms of pressure or head. The boiler model follows a similar approach as the pump model, where the exit temperature and pressure are specified externally, and are solved for by iteration. The inlet temperature to the turbine is a good first guess as the exit temperature from the boiler. It is assumed there is negligible pressure loss on the cold side, so the turbine inlet pressure is used as a first guess as the boiler exit pressure. The difference between the boiler exit state and the turbine inlet state, caused by the connecting pipe, is handled by the iterative overall loop algorithm. The boiler model itself is used to determine the internal performance of the boiler at the specified operating point. The major variables
of interest are the boiler fuel rate and heat rate.

4.4.2 Conservation of Mass

Each side of the boiler is a single inlet, single exit CV with heat transfer in or out. Therefore application of conservation of mass simply yields equal inlet and exit mass flow rates. Assuming complete combustion of methane with 100% theoretical air, the combustion equation is

\[
CH_4 + a (O_2 + 3.76N_2) \rightarrow bCO_2 + cH_2O + dN_2.
\]
Conservation of mass yields

\[
\begin{align*}
\ a &= 2, \\
\ b &= 1, \\
\ c &= 2, \\
\ d &= 7.52.
\end{align*}
\]

In order to make the combustion more realistic, 20% excess air is used. The resulting combustion equation is

\[
CH_4 + 2.4 (O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 9.024N_2 + 0.4O_2.
\]

The air to fuel ratio on a molar basis is

\[
AF = \frac{2.4 \times 4.76}{1} = 11.424 \frac{lb_{mol \ Air}}{lb_{mol \ Fuel}}.
\]

The air to fuel ratio on a mass basis is

\[
AF = \frac{M_{Air}}{M_{Fuel}} = 11.424 \frac{28.97}{12.01 + 2 \times 2.016} = 20.63 \frac{lb_{m \ Air}}{lb_{m \ Fuel}}.
\]

So the air mass flow rate is

\[
\dot{m}_{Air} = 20.63 \dot{m}_{Fuel}.
\]

Since the sum of the reactant mass flow rates is equal to the sum of the product mass flow rates, the fuel flow rate is expressed in terms of the hot side mass flow rate as

\[
\dot{m}_{Fuel} = \frac{\dot{m}_H}{21.63} = 0.046 \dot{m}_H. \tag{4.81}
\]

The air flow rate is

\[
\dot{m}_{Air} = \frac{20.63 \dot{m}_H}{21.63} = 0.954 \dot{m}_H. \tag{4.82}
\]

### 4.4.3 Conservation of Momentum

Similar to the condenser, conservation of momentum for the boiler would introduce three force balance equations and three unknown reaction forces, none of which are useful to predicting the thermal performance of the test loop. So this portion of the analysis is unnecessary, and has not been carried out.
4.4.4 Conservation of Energy

Compared with the large energy input from combustion, the change in kinetic and potential energies across the boiler are negligible, for both the hot and cold sides. Conservation of energy yields

\[ \dot{Q}_C = \dot{m} (h_{Ce} - h_{Ci}) \] (4.83)

for the cold side, and

\[ \dot{Q}_H = \dot{m}_H (h_{He} - h_{Hi}) = \dot{m}_H C_{pH} (T_{He} - T_{Hi}) \] (4.84)

for the hot side. \( \dot{Q}_H \) is negative because heat is transferred out of the hot side CV. Thus \( \dot{Q}_H + \dot{Q}_C = 0 \). \( T_{Hi} \) is found as the adiabatic flame temperature of the combustion process. \( C_{pH} \) is the specific heat of the combustion product mixture at that temperature, and it is assumed constant through the heat exchange process. Conserving energy across the combustion process gives

\[ \frac{\dot{Q}}{\dot{n}_{Fuel}} - \frac{\dot{W}}{\dot{n}_{Fuel}} = \overline{h}_P - \overline{h}_R, \] (4.85)

where \( \overline{h}_P \) and \( \overline{h}_R \) are the molar specific enthalpies of the products and reactants, respectively. There is no work output, and the combustion is assumed to be adiabatic. So both terms on the lefthand side of Equation (4.85) are zero, giving

\[ \overline{h}_P = \overline{h}_R. \]

Expanding the molar specific enthalpy terms yields

\[ \sum_e n_e [\overline{h}^o_f + \overline{h} (T_P) - \overline{h} (T_{Ref})]_e = \sum_i n_i [\overline{h}^o_f + \overline{h} (T_R) - \overline{h} (T_{Ref})]_i. \]
\[ \sum_e n_e \overline{h} (T_P) = \sum_i n_i [\overline{h}^o_f + \overline{h} (T_R) - \overline{h} (T_{Ref})]_i + \sum_e n_e [\overline{h} (T_{Ref}) - \overline{h}^o_f]_e \]

Assuming that the reference temperature is equal to the reactant temperature, this simplifies to

\[ \sum_e n_e \overline{h} (T_P) = \sum_i n_i \overline{h}_f + \sum_e n_e [\overline{h} (T_{Ref}) - \overline{h}^o_f]_e \] (4.86)

The values for the molar specific enthalpies and enthalpies of formation are given by Moran and Shapiro [39] in their Tables A-23E and A-25E.
4.4.5 Combined Equation Set

Evaluating the terms on the right hand side of Equation 4.86 yields

\[ \sum e_n e_h (T_P) = 392,858.5 \text{ Btu lb mol}. \]

A manual search is conducted, in which several values of \( T_P \) are substituted in order to evaluate the left hand side of Equation 4.86 and converge on the actual value. Ultimately, the adiabatic flame temperature is determined to be

\[ T_P \approx 3720^\circ R = 3260^\circ F. \]

The effective molar specific heat of the gas mixture is a weighted average of the component molar specific heats

\[ \overline{C}_p \equiv \sum e y_e \overline{C}_p(T_P), \]

where \( y_e = \frac{n_e}{n} \) is the mole fraction and \( n = \sum n_e \) is the total moles of product. The specific heat at constant pressure of any substance is

\( C_p = \left( \frac{dh}{dT} \right)_P. \quad (4.87) \)

For small temperature changes, the molar specific heat can be approximated using Taylor Series expansion as

\[ \overline{C}_p = \frac{\overline{h}(T + \Delta T) - \overline{h}(T - \Delta T)}{2\Delta T}. \quad (4.88) \]

Using \( \Delta T = 20^\circ R \), the molar specific heats in \( \left( \frac{\text{Btu}}{\text{lb mol}^\circ R} \right) \) for the products are

\[
\begin{array}{cccc}
CO_2 & H_2O & N_2 & O_2 \\
\overline{C}_p & 14.525 & 12.325 & 8.625 & 9.05
\end{array}
\]

The effective molar specific heat for the mixture is 4.159 \( \frac{\text{Btu}}{\text{lb mol}^\circ R} \). The effective molar mass of the product mixture is found in a similar manner to the specific heat as

\[ M = \frac{44.01 + 2 \times 18.02 + 9.024 \times 28.01 + 0.4 \times 32.0}{1 + 2 + 9.025 + 0.4} = 27.818 \text{ lb mol}. \]

Thus the mass specific heat of the product mixture, which is the boiler hot side fluid, is

\[ C_{PH} = \frac{\overline{C}_p}{M} = 0.1495 \frac{\text{Btu}}{\text{lb mol}^\circ R}. \]
An additional parameter of interest for the boiler analysis is its efficiency. For the purposes of this thesis, the only loss able to be considered is that associated with the waste heat sent to the exhaust stack. From that standpoint, the appropriate expression is

\[ \eta_{\text{BOILER}} = 1 - \frac{T_{He} - T_{\text{ambient}}}{T_{Hi} - T_{\text{ambient}}} \] (4.89)

A less conservative expression, and one which is more realistic, uses the water inlet temperature as the reference instead of the ambient temperature. The exhaust gas temperature cannot be lower than the liquid water inlet temperature. Thus this new expression, and the one used in this thesis, is

\[ \eta_{\text{BOILER}} = 1 - \frac{T_{He} - T_{Ci}}{T_{Hi} - T_{Ci}}, \]

or

\[ \eta_{\text{BOILER}} = \frac{T_{Hi} - T_{He}}{T_{Hi} - T_{Ci}}. \] (4.90)

### 4.4.6 Effectiveness - NTU Method

The \( \epsilon - \text{NTU} \) analysis is similar to the one carried out for the condenser, with two changes. First, the hot side fluid is a gas with no condensation and the cold side fluid is evaporating steam. So the boiler cold side fluid exhibits three different phase types, much like the condenser hot side. Second, both the inlet and exit conditions are known for the steam side, instead of only the inlet conditions. This second change simplifies the solution process. The basic \( \epsilon - \text{NTU} \) Equations 4.52-4.55 apply to the boiler. The boiler is assumed to behave like a cross-flow heat exchanger with both fluids unmixed. So the relation between \( \epsilon \) and \( \text{NTU} \) from Incropera et al. [36] is

\[ \epsilon = 1 - \exp \left[ \frac{1}{C_r} \left( \text{NTU} \right)^{0.22} \left\{ \exp \left[ -C_r \left( \text{NTU} \right)^{0.78} \right] - 1 \right\} \right]. \] (4.91)

The ratio of specific heats is the same as that given in Equation 4.58 and is repeated here:

\[ C_r = X_{SH}C_{rSH} + X_{MLV}C_{rMLV} + X_{SC}C_{rSC}. \] (4.92)

The weights are found from the following expressions

\[ X_{SC} = \frac{h_{CI} - h_{CI}}{h_{CE} - h_{CI}}. \]
\[ X_{MLV} = \frac{h_{Cg} - h_{Cl}}{h_{Ce} - h_{Ci}}, \]
\[ X_{SH} = \frac{h_{Ce} - h_{Ci}}{h_{Ce} - h_{Ci}}. \quad (4.93) \]

The saturation enthalpies can be found using equations of state
\[ h_{Cg} = h_g [T_{C,sat} (P_{Ci})], \]
\[ h_{Cl} = h_l [T_{C,sat} (P_{Ci})]. \quad (4.94) \]

The region heat capacity ratios are expressed as
\[ C_{rSC} = \frac{C_{minSC}}{C_{maxSC}}, \]
\[ C_{rMLV} = 0.0, \]
\[ C_{rSH} = \frac{C_{minSH}}{C_{maxSH}}. \quad (4.95) \]

The heat capacities are then
\[ C_{minSC} = \text{MIN} (\dot{m}_C C_{pc,SC}, \dot{m}_H C_{ph}), \]
\[ C_{maxSC} = \text{MAX} (\dot{m}_C C_{pc,SC}, \dot{m}_H C_{ph}), \]
\[ C_{minMLV} = \dot{m}_H C_{ph}, \]
\[ C_{minSH} = \text{MIN} (\dot{m}_C C_{pc,SH}, \dot{m}_H C_{ph}), \]
\[ C_{maxSH} = \text{MAX} (\dot{m}_C C_{pc,SH}, \dot{m}_H C_{ph}). \quad (4.96) \]

where
\[ C_{pc,SH} = \frac{C_{pc} + C_{pcg}}{2}, \quad (4.97) \]

and
\[ C_{pc,SC} = \frac{C_{pci} + C_{pci}}{2} \quad (4.98) \]

The weighted maximum theoretical heat transfer rate is
\[
\dot{Q}_{MAX} = X_{SC} C_{minSC} (T_{Hi} - T_{Ci})
+ X_{MLV} C_{minMLV} (T_{Hi} - T_{C,sat})
+ X_{SH} C_{minSH} (T_{Hi} - T_{C,sat}). \quad (4.99)
\]

The expression for \( NTU \) found in Equation 4.67 still applies. It is repeated below:
\[ NTU = U A \cdot \frac{(T_{Hi} - T_{Ci})}{\dot{Q}_{MAX}}. \quad (4.100) \]

An iterative solution is used to find \( \dot{m}_H \), which is then used to find \( \dot{m}_{Fuel} \) from Equation 4.81

The boiler equation set is solved according to Figures A.11 to A.13 in Appendix A.
4.5 Pipe Component and Other Pressure Loss Model

4.5.1 CV Description

This model can be used for any of the pipes throughout the test loop. The following theoretical model could be reused for any test loop configuration so long as the pipe diameter is constant from inlet to exit. It is assumed that the flow is parallel and fully developed. This model can also be used to model equivalent pipe lengths for major and minor losses for any of the other components.

4.5.2 Conservation of Mass

The pipe is a single-input/single-output CV. Therefore application of conservation of mass yields equal inlet and exit mass flow rates:

\[ \dot{m} = \rho_i V_{xi} A_i = \rho_e V_{xe} A_e. \]  

(4.101)

Assuming there is no tangential or radial mixing, the average velocity is equal to the average axial velocity at a control surface:

\[ V_i = V_{xi}, \]

\[ V_e = V_{xe}. \]

4.5.3 Conservation of Momentum

There is a pressure loss associated with flow through a pipe due to friction on the inner wall. This friction generates an axial force on the pipe. The magnitude of this force could be determined using an integral CV approach. Further analysis is unnecessary because the primary purpose of this thesis is to design the thermal-fluids system, and not the structural supports.

4.5.4 Conservation of Energy

Because there is no work or heat extracted from the pipe, the stagnation enthalpy is constant, \( i.e. \)

\[ h_i + \frac{V_i^2}{2g_e} = h_{0,i} = h_{0,e} = h_e + \frac{V_e^2}{2g_e}. \]
So the exit enthalpy is
\[ h_e = h_i + \frac{V_i^2 - V_e^2}{2g_c}. \] (4.102)

For incompressible flows, the inlet and exit enthalpies are equal. However, not all of the system pipe flows can be considered incompressible.

### 4.5.5 Combined Equation Set

The pressure loss due to wall friction is calculated from the Darcy-Weisbach equation. So the exit pressure is
\[ P_e = P_i - f \frac{L \rho V^2}{D}, \] (4.103)

where \( f \) is the Darcy friction factor, \( L \) is the pipe length, and \( D \) is the pipe diameter. \( \rho \) is the mean density,
\[ \rho = \frac{\rho_i + \rho_e}{2}. \] (4.104)

\( V \) is the mean velocity,
\[ V = \frac{V_i + V_e}{2}. \] (4.105)

The Reynolds number is
\[ Re = \frac{\rho V D}{\mu}. \] (4.106)

For laminar flow \((Re < 2000)\), the friction factor is calculated from
\[ f = \frac{64}{Re}. \] (4.107)

For simplicity, Equation [4.107] is used to calculate \( f \) in the laminar and transition flow regimes. For turbulent pipe flow \((Re > 4000)\), the friction factor is calculated using the Colebrook-White equation:
\[ \frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{e/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right), \] (4.108)

where \( e \) is the pipe roughness height. This implicit equation requires either iteration or a 1-D search method to find \( f \). A 1-D search method is used herein. Rearranging Equation [4.101] and setting the inlet and exit areas equal yields
\[ V_e = \frac{\rho_i}{\rho_e} V_i. \] (4.109)
Finally, the exit density is calculated using an equation of state,

\[ \rho_e = EOS(P_e, h_e). \]  \hspace{1cm} (4.110)

Equations 4.102 to 4.110 define the pipe CV. Their solution by iteration is shown in Fig. A.14 of Appendix A.
Chapter 5

Open Source Computer Implementation

5.1 Philosophy of Implementation

The theoretical model from the previous chapter has been implemented in an open source computer program written in the C language. The source code for the model is available at 

http://edge.rit.edu/content/T10100/public/Home.

The main() program which calls all other subprograms is located in LoopRemodel.c. It passes information to all other subprograms as pointers to structures. There are several types of user-defined structures, each serving a specific purpose. An example is the “STATE” structure, which holds thermodynamic state information. The STATE structure has as its members all the possible necessary state properties for any CV within the model. A particular CV may only require the Pressure (P) and Temperature (T) members of the inlet state and might calculate the Pressure (P) and specific enthalpy (h) members of the exit state. At the end of that particular CV, the P,h information would be used in appropriate equations of state to define the entire STATE structure. Similarly the “PROCESS” structure holds all the CV process information. The process information that gets back to an upper level program is selected by the CV subprogram. The list of structure types goes on, but they all serve the same purpose, to pass information between higher level and lower level programs.

Each of the CV models described in Chapter 4 have their own subprogram. Some of them are also parts of other subprograms. The structure of each of the subprograms and the overall test loop program are given in Appendix A. In addition to the source code and header files written for the theoretical implementation, several open source libraries and
tools were used.

5.2 Open Source Tools

5.2.1 Eclipse & GCC

All source code is written in the C language and compiled using the GNU Compiler Collection (http://gcc.gnu.org). The project was compiled and built within the Eclipse IDE (http://www.eclipse.org) using MinGW (http://www.mingw.org).

5.2.2 PLPlot

All plotting is performed using PLplot, a plotting library distributed under the GNU LGPL. PLplot provides both higher-level plotting functions, such as 2D contour plots and surface plots, and lower level functions, such as points, lines, and curves. A combination of these features have been used to create a custom set of plotting functions located in plotting_functions.c. PLplot provides capability for exporting plots to either scaled vector graphic (.svg) or PostScript (.ps) in addition to several on-screen image viewers. More information is available at http://plplot.sourceforge.net/.

5.2.3 freesteam

All equation of state calculations are performed by freesteam, which is distributed under the GNU GPL. Since it is distributed under the GPL, the computer program developed for this thesis must also be distributed under the GPL. This means that it or any derivative programs must be distributed with source code. freesteam utilizes the IAPWS-IF97, which was written by the International Association for the Properties of Water and Steam (IAPWS). This formulation is an international standard, and the one used by ASME Performance Test Codes. More information is available at http://freesteam.sourceforge.net/.

5.2.4 LaTeX

This thesis was written in LaTeX using the MiKTeX (http://miktex.org) distribution and the TeXnicCenter IDE (http://www.texniccenter.org). The thesis is built using a
combination of pdfTeX and latex compilers to enable inclusion of postscript images in a pdf document.

5.3 Validation of Derived Models

5.3.1 Mass

The loop model was run over a 21x21x21 cube of operating conditions for three different turbine rotor configurations. The turbine inlet pressure was held constant at $P_1 = 55psia$, while $T_1$, $\dot{m}$, and $N$ were varied, such that the useful range of turbine operating conditions was exhausted for all turbine configurations. The method of determining this operating range is discussed in Chapter [6]. Over all of the operating points, there was no mass decrement at any point in the system model. This is not surprising, since the steam mass flow rate, $\dot{m}$, is an input to the system model and all the components are modelled as having a single-inlet and a single-exit.

5.3.2 Momentum

The only control volumes for which momentum conservation has been carried out are the turbine rotor and stator. The rotational momentum equations that arise from velocity triangles for these two control volumes are critically linked to the energy equations by the stagnation enthalpy. Therefore, momentum conservation is satisfied when the energy conservation is satisfied for these particular control volumes.

5.3.3 Energy

Turbine

For the turbine, energy conservation must be validated for the turbine as a whole, and for each of the stages. For the entire turbine, the difference between the fluid work absorbed and the shaft work produced should be zero. Therefore a meaningful relative error is

$$error = \frac{\Delta W - (h_{0i} - h_{0e})}{\Delta W} \times 100.0$$  \hspace{1cm} (5.1)
The same can be said for a single turbine stage. Fig. 5.1 shows the relative error for rotor design #1 (as described in Chapter 6) operating with zero losses over a large range of operating conditions. The zero loss condition is not necessary for validation purposes. Fig. 5.2 shows a similar comparison for the fourth stage of that same turbine.

\[ \dot{m}_H (h_{0H_i} - h_{0H_e}) = \left| \dot{Q}_H \right| = \left| \dot{Q}_C \right| = \dot{m}_C (h_{C_e} - h_{C_i}) \]  \hspace{1cm} (5.2)

Condenser

The condenser energy conservation dictates that the magnitude of the energy decrease across the steam side is equal to the magnitude of the increase across the coolant side, thus

\[ \dot{m}_H (h_{0H_i} - h_{0H_e}) = \left| \dot{Q}_H \right| = \left| \dot{Q}_C \right| = \dot{m}_C (h_{C_e} - h_{C_i}) \]  \hspace{1cm} (5.2)
An applicable relative validation error would compare these two energy changes as

\[
error = \frac{\dot{m}_H (h_{0Hi} - h_{0He}) - \dot{m}_C (h_{Ce} - h_{Ci})}{\dot{m}_H (h_{0Hi} - h_{0He})} \times 100.0
\] (5.3)

The variation of relative error for a condenser with \(UA = 50 \frac{Btu}{s \cdot R}\) is shown in Fig. 5.3. The magnitude of this error is attributable to the assumption of constant specific heats for both the steam and coolant side of the condenser.

![Condenser Energy Error](image)

Figure 5.3: Condenser energy validation

**Pump**

Conserving energy across the pump resulted in the power input required being equal to the product of the mass flow rate and the stagnation enthalpy increase. The relative validation error is

\[
error = \frac{\dot{m} (h_{0e} - h_{0i}) - \dot{W}}{\dot{W}} \times 100.0
\] (5.4)

The variation of relative error for the system pumping requirement is shown in Fig. 5.4. The magnitude of the error is negligible.

**Boiler**

The boiler energy validation follows the same premise as that for the condenser, but constant specific heat is assumed on the hot side, and all changes in kinetic energy are neglected. So
the relative validation error is

\[ \text{error} = \frac{\dot{m}_H C_{ph} (T_{Hi} - T_{He}) - \dot{m}_C (h_{Ce} - h_{Ci})}{\dot{m}_H C_{ph} (T_{Hi} - T_{He})} \times 100.0 \]  

(5.5)

The variation of relative error for a boiler with \( UA = \frac{Btu}{\text{s} \, \circ R} \) is shown in Fig. 5.5. The magnitude of the error is negligible.

Pipe

The pipe model assumes there is no heat or work transfer through the CV boundaries and there are no changes in potential energy. So the stagnation enthalpy should be constant
from inlet to exit. The relative validation error can simply compare these two values as

$$error = \frac{h_{0e} - h_{0i}}{h_{0e}} \times 100.0$$  \hspace{0.5cm} (5.6)

Fig. 5.6 shows the variation of relative error for the pipe between the turbine and condenser. The magnitude of the error is negligible.

![Figure 5.6: Pipe energy validation](image)

Cycle

At steady state, the magnitude of the energy input to the steam cycle should be equal to the magnitude of the energy extracted. So the relative validation error compares these two values as

$$error = \frac{E_i - E_e}{E_i} \times 100.0$$  \hspace{0.5cm} (5.7)

or in terms of each of the component energy transfers,

$$error = \frac{\hat{Q}_{\text{boiler}} + \hat{W}_{\text{pump}} - \hat{W}_{\text{turbine}} - \hat{Q}_{\text{condenser}}}{\hat{Q}_{\text{boiler}} + \hat{W}_{\text{pump}}} \times 100.0$$  \hspace{0.5cm} (5.8)

The variation of the relative cycle energy validation error is given in Fig. 5.7. The main contributor to the error appears to be the condenser energy error since they are of the same order of magnitude.
5.4 Verification of Off-design Turbine Performance

In addition to verifying the governing principles of thermodynamics, the performance of each component model must be compared to published or repeatable results. Since the turbine model does not have a loss model built in, it cannot be validated with experimental data. The necessary experimental data also has not been collected, since the test loop which will be used to collect it has not been built. Therefore, the only way to verify the behavior of the turbine model is to analyze the relationship between variables that are independent of the particular loss model. It so happens that the loading and flow coefficients are two such variables. Horlock [10] derived an expression for predicting off-design performance of a turbine stage with constant axial velocity. He stated that the flow coefficient is equal to

\[ \Psi = \Phi \left( \tan \beta_2 S + \tan \beta_3 \right) - 1. \]  

(5.9)

The present stage model does not require constant axial velocity. A plot of \( \Psi \) vs. \( \Phi \) is shown in Fig. 5.8. The colored data reflects the average stage \( \Psi \) and \( \Phi \) of a 4-stage turbine with \( \beta_2 S \approx 76.0^\circ \) and \( \beta_3 \approx 76.0^\circ \). The dotted line gives Horlock’s off-design value for \( \Psi \). The modelled results fit very closely to Horlock’s prediction. The small differences at low and high values of \( \Phi \) are most likely attributed to the accounting for changes in axial velocity in the present theoretical model. The reason the present model fits so closely with the constant axial velocity model is most likely because the axial velocity used to calculate \( \Phi \) in the present model is the average value across the entire stage, \( V_{x,\text{mean}} \). The approach...
5.5 Verification of Loop Model Performance

The present theoretical model for the test loop consists of only the components in an idealized rankine cycle, plus the connecting pipes. Published results for steam cycle models generally include several other components in order to demonstrate their robustness (e.g. [40],[41],[42]). They also use turbine models that require the specification of its inlet and exit pressures. The fundamental reason for creating the current turbine model was so that exit pressure could be predicted in order to select the other loop components accordingly. In order to find an operating point for a specified exit pressure, an optimization would be required. Comparison to specific published test cases presents many difficulties.

Another approach, and the one taken herein, is to compare the computational results for a single operating point to hand calculated results using an elementary thermodynamic analysis. For this process, it is assumed that the input conditions to the computer model are the same as those to the hand calculations. The model input conditions are as follows:

- $P_1 = 55.0$ psia
\( T_1 = 400.0^{\circ}F \)

\( \dot{m} = 5.0 \text{ lb}_m/s \)

\( N = 5000.0 \text{ RPM} \)

The previous section verified the turbine model results for all operating points, so the turbine analysis need not be carried out. The turbine is assumed to have zero losses. The turbine exit conditions may be used as model inputs to the hand calculations. Additionally, solving for the performance of pipe segments requires one and sometimes two levels of iteration, and depends on the performance of the other component models. Therefore, the results for pressure and enthalpy changes across the pipe segments from the computer model will be model inputs to the hand calculation process as well. The condenser hot side exit water is assumed to be subcooled 10\(^\circ\)F. The state results for the computer model are displayed in Table 5.1 where the state numbering is defined in Fig. 4.1. The temperature increase and pressure drop between state 8 and state 1 is associated with the connecting pipe between the boiler and turbine. The large pressure drop indicates that the pipe diameter used is too small at this flow rate.

<table>
<thead>
<tr>
<th>STATE</th>
<th>( P ) (psia)</th>
<th>( T ) ((^\circ)F)</th>
<th>( h ) (Btu/lb(_m))</th>
<th>( s ) (Btu/[lb(_m) (^{\circ})F])</th>
<th>( C_P ) (Btu/[lb(_m) (^{\circ})F])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.00</td>
<td>400.0</td>
<td>1234.438</td>
<td>1.724</td>
<td>0.497</td>
</tr>
<tr>
<td>2</td>
<td>32.98</td>
<td>306.4</td>
<td>1191.795</td>
<td>1.726</td>
<td>0.497</td>
</tr>
<tr>
<td>3</td>
<td>32.98</td>
<td>306.4</td>
<td>1191.795</td>
<td>1.726</td>
<td>0.497</td>
</tr>
<tr>
<td>4</td>
<td>32.98</td>
<td>245.7</td>
<td>214.340</td>
<td>0.361</td>
<td>1.013</td>
</tr>
<tr>
<td>5</td>
<td>32.62</td>
<td>245.7</td>
<td>214.340</td>
<td>0.361</td>
<td>1.013</td>
</tr>
<tr>
<td>6</td>
<td>59.68</td>
<td>245.8</td>
<td>214.443</td>
<td>0.361</td>
<td>1.013</td>
</tr>
<tr>
<td>7</td>
<td>59.67</td>
<td>245.8</td>
<td>214.443</td>
<td>0.361</td>
<td>1.013</td>
</tr>
<tr>
<td>8</td>
<td>59.67</td>
<td>402.9</td>
<td>1235.243</td>
<td>1.716</td>
<td>0.500</td>
</tr>
<tr>
<td>9</td>
<td>30.000000</td>
<td>65.000000</td>
<td>33.163042</td>
<td>0.065109</td>
<td>0.999739</td>
</tr>
<tr>
<td>10</td>
<td>30.000000</td>
<td>130.232727</td>
<td>98.289596</td>
<td>0.182108</td>
<td>0.998499</td>
</tr>
</tbody>
</table>

Table 5.1: State results found by the computer model for the specified test case

The hand calculations will follow the same approach as layed out in Chapter 4, with some modification made for easier table lookups. All steam property lookups were taken
from Table A-2E of Moran and Shapiro [39]. Table 5.2 gives the known state information for the hand calculations. The empty data cells will filled as the hand calculation procedure is carried out. Because all of the pipe pressure drops are assumed to be known and the pressure drops across the condenser and boiler are assumed negligible, all the state pressures are known. Table 5.3 shows the pressure and enthalpy changes across the pipe lengths in

<table>
<thead>
<tr>
<th>STATE</th>
<th>$P$ (psia)</th>
<th>$T$ ($^\circ F$)</th>
<th>$h$ ($Btu/lb_{m}$)</th>
<th>$s$ ($Btu/[lb_{m}^oF]$)</th>
<th>$C_P$ ($Btu/[lb_{m}^oF]$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.00</td>
<td>400.00</td>
<td>1234.438</td>
<td>1.724</td>
<td>0.497</td>
</tr>
<tr>
<td>2</td>
<td>32.98</td>
<td>306.44</td>
<td>1191.795</td>
<td>1.726</td>
<td>0.497</td>
</tr>
<tr>
<td>3</td>
<td>32.98</td>
<td></td>
<td>1191.795</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>32.98</td>
<td></td>
<td></td>
<td></td>
<td>1.013</td>
</tr>
<tr>
<td>5</td>
<td>32.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>59.68</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>59.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>59.67</td>
<td></td>
<td>1235.243</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Table of known state information for the hand calculations

the test loop that are also used as inputs to the hand calculations. The largest pressure drop, by far, is between the boiler and turbine.

<table>
<thead>
<tr>
<th>INLET</th>
<th>EXIT</th>
<th>$P_i - P_e$ (psia)</th>
<th>$h_i - h_e$ (Btu/lb$_m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.29</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>4.68</td>
<td>0.805</td>
</tr>
</tbody>
</table>

Table 5.3: Table of known pipe pressure and enthalpy changes

The condenser subcool is set at $10^\circ F$. So the exit temperature is

$$T_4 = T_{sat}(P_4) - 10.0 = 255.68 - 10.0 = 245.68^\circ F.$$ 

The saturation temperature was found by linear interpolation of the saturation temperatures between 30$psia$ and 35$psia$. Linear interpolation is used for all other table lookups as well. The condenser exit enthalpy is found by assuming constant specific heat in the subcooled
heat transfer region. Thus the change in enthalpy in the subcooled region is

\[ h_f (P_4) - h_4 = C_{P4} (T_{sat} - T_4). \]

The exit enthalpy is then

\[ h_4 = h_f (P_4) - C_{P4} (T_{sat} - T_4) = 224.36 - 1.013703 \times 10.0 = 214.22 \text{ Btu/lb}_m. \]

The temperature and enthalpy do not change in the pipe between the condenser and pump, because the water is an incompressible liquid. Assuming it is also an incompressible liquid at the pump inlet and exit, the specific volumes at those states are

\[ v_5 \approx v_f (P_5) = 0.01704 \text{ ft}^3/\text{lb}_m, \]

and

\[ v_6 \approx v_f (P_6) = 0.01738 \text{ ft}^3/\text{lb}_m, \]

respectively. The mean specific volume is 0.01721 \text{ ft}^3/\text{lb}_m. Assuming a 100\% isentropic efficiency, Moran and Shapiro [39] give the following expression as an approximation of the specific power input to the pump:

\[ \dot{W}_p \dot{m} = v_{mean} (P_5 - P_6). \]

So the specific work for the present case is

\[ \frac{\dot{W}_p}{\dot{m}} = 0.0171 \text{ ft}^3/\text{lb}_m (32.629310 - 59.3680026) \frac{\text{lb}_f}{\text{in}^2} \times 144 \frac{\text{in}^2}{\text{ft}^2} \times \frac{1 \text{ Btu}}{778.17 \text{ft} - \text{lb}_f} = -0.086 \text{ Btu/lb}_m. \]

Since the pump inlet and exit pipe diameters are equal and the water is incompressible, the inlet and exit velocities are equal, and the change in kinetic energy is zero. Therefore the specific power input is equal to the change in static enthalpy across the pump:

\[ \frac{\dot{W}_p}{\dot{m}} = h_5 - h_6. \]

Since the pump is idealized with no losses, the exit enthalpy is

\[ h_6 = h_{6s} = h_5 - \frac{\dot{W}_p}{\dot{m}} = (214.22 + 0.086) \text{ Btu/lb}_m = 214.306 \text{ Btu/lb}_m. \]

The temperature and enthalpy changes across the pipe between the pump and boiler are negligible. The boiler exit state is already known in order to close the cycle. Table 5.4
compares the state properties that were calculated by hand to the corresponding values from the computer model. All percent difference values are normalized by the value resulting from the computer implemented model. Table 5.5 compares the critical process quantities of the two calculation procedures.

<table>
<thead>
<tr>
<th>STATE PROPERTY</th>
<th>Computational Model Result</th>
<th>Hand Calculated Result</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_4$ ($^\circ$F)</td>
<td>245.77</td>
<td>245.68</td>
<td>0.037</td>
</tr>
<tr>
<td>$h_4$ (Btu/lbm)</td>
<td>214.34</td>
<td>214.22</td>
<td>0.056</td>
</tr>
<tr>
<td>$T_5$ ($^\circ$F)</td>
<td>245.79</td>
<td>245.68</td>
<td>0.045</td>
</tr>
<tr>
<td>$h_5$ (Btu/lbm)</td>
<td>214.34</td>
<td>214.22</td>
<td>0.056</td>
</tr>
<tr>
<td>$h_6$ (Btu/lbm)</td>
<td>214.44</td>
<td>214.31</td>
<td>0.061</td>
</tr>
</tbody>
</table>

Table 5.4: Comparison of computational and hand calculated state property values

<table>
<thead>
<tr>
<th>PROCESS QUANTITY</th>
<th>Computational Model Result</th>
<th>Hand Calculated Result</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{Q}_{\text{cond}}$ (MW)</td>
<td>-5.156</td>
<td>-5.156</td>
<td>0.000</td>
</tr>
<tr>
<td>$H_{\text{pump}}$ (ft)</td>
<td>68.856</td>
<td>67.038</td>
<td>2.640</td>
</tr>
<tr>
<td>$Q_{\text{pump}}$ (GPM)</td>
<td>38.075</td>
<td>38.618</td>
<td>1.426</td>
</tr>
<tr>
<td>$\dot{Q}_{\text{boil}}$ (MW)</td>
<td>5.385</td>
<td>5.384</td>
<td>0.019</td>
</tr>
</tbody>
</table>

Table 5.5: Comparison of computational and hand calculated process quantities

The only errors that stand out are for the pump head and flow rate. The hand calculated values assumed that the pump power input is the product of specific volume, the change in pressure, and $\dot{m}$. So changes in internal energy were neglected. The pump model presented in Chapter 4 evaluates the ideal pump exit enthalpy from an equation of state based on the exit pressure and inlet entropy (see Equation 4.73). Such an evaluation is very difficult using lookup tables.

With the exception of the pump process results, the differences between the computer model and the hand calculations, in both state values and process values, are much less than 1%. This shows sufficient accuracy to use the results of the computer implemented model to draw conclusions on test loop design.
Chapter 6

Results

The theoretical model and its computer implementation have been presented for the general case: any steam turbine can be placed in a steam power cycle, which consists of other user-selected components. The user must also select the range of operating conditions to use as model inputs, as well as provide the geometries of each of the components. The turbine geometries presented herein are based on a 4-stage experimental steam turbine housing and a series of bladesets formerly used by Dresser-Rand [12].

There are three rotor designs, and three stator designs. The rotors differ in their blade inlet and exit angles, and the presence of hub endwall slant. All three rotor rows have shroud endwall slant. The blade incidence angles and endwall slant angles are given in Table 6.1. In order to repeat the results given in this chapter, the user would need additional blade height and mean radius values that are proprietary. Also, the blade incidence angles are not identical for every stage, but they are within several degrees. The differences between the three stator designs are related to the axial chord length and endwall profiling. The associated performance differences attributable to these design differences could only be demonstrated with an accurate loss prediction model. Since loss prediction

<table>
<thead>
<tr>
<th>Rotor Design #</th>
<th>$\beta_2R$ (deg)</th>
<th>$\beta_3$ (deg)</th>
<th>Shroud Endwall Slant (deg)</th>
<th>Hub Endwall Slant (deg)</th>
<th>Row Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62</td>
<td>72</td>
<td>12</td>
<td>0</td>
<td>Reaction</td>
</tr>
<tr>
<td>2</td>
<td>62</td>
<td>72</td>
<td>12</td>
<td>12</td>
<td>Reaction</td>
</tr>
<tr>
<td>3</td>
<td>59</td>
<td>66</td>
<td>12</td>
<td>0</td>
<td>Impulse</td>
</tr>
</tbody>
</table>

Table 6.1: Rotor designs modelled [12].
is not incorporated into the present model, only one of the stator designs has been used. Its incidence angles are given in Table 6.2. The blade height and mean radius is omitted again. So there are three complete turbine configurations to model. Because the turbine blade row models are based solely on first principles, there is no loss prediction model. In order to demonstrate the complete possible range of turbine operating conditions, each of the turbine configurations must be modelled for several loss values. Test loop components that meet operational requirements of all turbine configurations at all desired operating conditions and with any value of loss coefficient are sufficient. The turbine model is run for stator and rotor loss coefficients of $\zeta = 0$, $\zeta = 0.1$, and $\zeta = 0.2$. All plots in this Chapter are for rotor design #1 with zero losses. Tables summarizing the results for all turbine configurations and loss coefficients are given in Appendix B.

The turbine has been modeled over the operating conditions given in Table 6.3. This range of conditions was selected because it enabled all three of the turbine configurations to be run over their full envelope. For all three turbines, there are portions of this envelope that do not represent achievable operating conditions. The operating envelope is limited further when the selected loop components are incorporated. The inlet pressure value is 55 psia, because lower pressure values tended to limit the range of achievable non-dimensional turbine variables and higher pressure values tended to increase the boiler heat rate and hence the operating cost for the test loop. The minimum inlet temperature is very close to

<table>
<thead>
<tr>
<th>Stator Design #</th>
<th>$\beta_1$ (deg)</th>
<th>$\beta_{2S}$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>76</td>
</tr>
</tbody>
</table>

Table 6.2: Stator design modelled [12].
the saturation temperature for the selected value of $P_1$, allowing the turbine to operate in the saturated region. Turbine inlet temperatures greater than $500^\circ F$ were not considered, because the maximum temperature limit for either the condenser or pump would be violated at all operating points. Most manufacturers of off-the-shelf U-tube heat exchanger specify a maximum steam temperature of $375^\circ F$. Some pump manufacturers specify a maximum water temperature of $250^\circ F$, while others specify a value of $300^\circ F$. The higher, less stringent value was used herein in order to accommodate more turbine operating points.

The research turbine was specified to run at speeds less than $7600 \text{ RPM}$. The modelled envelope was extended to $8000 \text{ RPM}$ in order to show turbine behavior near the speed limit. The minimum value for $N$ was selected as $2000 \text{ RPM}$, because turbine power output tended to drop off for lower values. If it is desired to operate the turbine at lower velocity ratios, then the limit on $N$ can be reduced further. The limits on $\dot{m}$ came from analyzing the useful turbine operating envelope for each of the three turbine designs. $T_1$, $\dot{m}$, and $N$ were taken at 21 values each, and the pressure only at one, resulting in $21^3 = 9261$ operating points. With the model inputs selected, results are found for all of the specified operating points.

6.1 Turbine

A successful test loop design will enable the turbine to be run over its entire useful range of dimensional and non-dimensional parameters. Two of the critical dimensional values for the assembled turbine are the overall pressure drop across the stages and the torque produced. According to the test turbine specifications, the pressure drop must not be greater than $26.0 \text{ psi}$. Fig. 6.1 shows that the pressure limitation eliminates the high mass flow, low torque operating points, which are synonymous to high mass flow and high speed. The torque produced will weigh heavily in the dynamometer selection. All of the operating points in Fig. 6.1 are within the constraints of the turbine, but remember that losses are not accounted for, and these will increase the pressure drop for a given set of model inputs. Also remember that the torque produced does not account for mechanical losses or gear reduction between the turbine and dynamometer shafts. The “Possible Test Path” in Fig. 6.1 refers to
a potential series of operating points that cover the full non-dimensional turbine operating space. The method used to create the test path is discussed in Section 6.6.

Fig. 6.2 gives the variation of the turbine total-to-static efficiency, $\eta_{ts}$, and the average stage enthalpy-based reaction, $R_h$, vs. the average stage velocity ratio, $\sigma$. $\eta_{tt}$ should be 100% at all operating points for a zero-loss turbine, while $\eta_{ts}$ will vary due to kinetic energy losses at the final stage exit. Substituting the stator exit and rotor inlet metal angles into Equation 4.32 results in a design velocity ratio very close to $\sigma_{des} = 0.5$ as illustrated in Fig. 6.2. A well designed test loop should be able to traverse the velocity ratio across a range that includes the design value, since it approximates the point of maximum efficiency. Testing at other velocity ratios enables off-design performance prediction, which is also crucial to the test loop capability.

Fig. 6.3 compares stage-averaged values for the three non-dimensional parameters used most often in turbine stage design: velocity ratio, work coefficient, and flow coefficient. The average stage velocity ratio is shown to vary between 0.18 and 1.0, which covers the entire range of useful operating conditions for rotor design #1. Theoretically, the velocity ratio could be as low as zero for a turbine that does not rotate, but then it wouldn’t be useful. Since this is the average stage velocity ratio, the $\sigma$ for each stage is either more or less than the value depicted. It will be important when declaring the required operating
Figure 6.2: Turbine total-to-static efficiency vs. average stage $R_h$ and average stage $\sigma$ range, to specify the range of either average stage $\sigma$ or single stage $\sigma$ desired. The profile losses due to friction and separation increase substantially at low stage velocity ratios. This phenomenon is not shown in Fig. 6.2 which omits a realistic loss model. The value of $\sigma$ where performance falls off is difficult to ascertain without such a model or actual experimental data. Additionally, the useful range of $\sigma$ and $\Phi$ depend on the particular turbine design.

The operating range of the turbine depicted in this section includes all real turbine operating points within the specified range of model inputs given in Table 6.3. Many of the candidate operating points are eliminated when the constraints imposed by the operating envelopes of the other loop components are introduced.

### 6.2 Condenser

Other than the connecting pipe, the first loop component to follow the turbine is the condenser. The condenser was constrained to $UA = 25.0 \frac{Btu}{\sigma^2 R}$. Since the value of $U$ will increase as both coolant and steam flow rate increase, constraining the $UA$ value means that each operating point effectively refers to a condenser with a different heat exchange surface area. For heat exchange between condensing steam and water, Incropera [36] estimates $1000 < U < 6000 \frac{W}{m^2 K}$, or $0.0489 < U < 0.2935 \frac{Btu}{\sigma^2 ft^2 \cdot R}$. This corresponds to a surface area
of $85 < A < 511$ ft$^2$ for a $UA = 25.0 \frac{Btu}{\Delta T_R}$. This range of surface areas includes many small to medium off-the-shelf shell-and-tube heat exchangers produced by TACO, Inc. [43] that would be useful for the present application.

Fig. 6.4 shows a standard heat exchanger performance map for the condenser. This type of performance map is often given by manufacturers of air to water fin-and-tube heat
It is shown herein as a means of comparing the performance of the condenser model to an actual heat exchanger. Q/ITD is the ratio between the heat transfer rate and the inlet temperature difference between the steam and coolant. In a performance map for a real heat exchanger, Q/ITD increases as steam rate increases for a constant coolant flow rate. The reason this is not the case for the results in Fig. 6.4 is that the condenser model holds the \( UA \) value constant for all operating points. The \( UA \) value of a real heat exchanger will vary, since the heat transfer coefficient \( U \) increases as steam and coolant flow rates increase. So the heat exchange area of the condenser model changes in order for the \( U \) value to be accurate at all operating points. An additional trend to notice in Fig. 6.4 is an apparent grouping of data points. Each of the groups represents a different coolant exit temperature.

Assuming the coolant inlet temperature is 65 °F, Fig. 6.5 shows the relationship between the condenser heat rate, coolant exit temperature and coolant flow rate. The discrete levels of \( T_{Ce} \) arise as a result of the test loop program structure (defined in Fig. A.1 of Appendix A). There is a minimum flow threshold below which the coolant will boil, a condition which should never occur in the real test loop. The coolant exit temperature can be controlled by changing the coolant flow rate, but it also depends on the condenser heat transfer area. Fig. 6.6 shows the same variables as Fig. 6.5 but for \( UA = 35.0 \text{ Btu/s}^\circ\text{R} \). Comparison of the
Figure 6.6: Condenser heat rate vs. coolant exit temperature and coolant flow rate

two figures shows that higher $UA$ values are useful for achieving higher heat transfer rates at lower coolant flow rates. This benefit comes at the cost of losing lower heat rates, which are associated with lower test loop steam flow rates. A similar plot for a $UA$ value lower than $25.0 \frac{Btu}{s^\circ R}$ would eliminate operating points with high heat rates. Another problem with smaller heat exchange areas is that the necessary coolant flow rate to match the heat transfer rate may cause extreme pressure losses in the coolant loop. There is a sensitive balance between a condenser with too much heat exchange surface area and not enough. The ideal condenser would enable operation of the test loop across a broad range of steam mass flow rates, to be selected by the user as needed. The selection process would be made easier if the condenser $U$ values at some operating points were known. Such information would require additional manufacturer data or a much more detailed condenser model.

It is clear from Figures 6.4 and 6.5 that the coolant exit temperature, which is strongly related to the coolant flow rate, plays a crucial role in determining the condenser performance point. The current condenser model eliminates operating points with coolant exit temperatures above $212^\circ F$ to prevent flashing when drained to atmospheric pressure. The actual coolant exit temperature at the system exit should be much less than $212^\circ F$ in order to avoid a dangerous environmental situation. The potential range of condenser operating points is limited greatly by the maximum coolant temperature specification. Two coolant
Figure 6.7: Two potential coolant system designs

system designs are shown in Fig. 6.7. The first design is that assumed while deriving the condenser model in Section 4.2. The second design proposes to place a bypass pipe parallel to the condenser hot side with an upstream flow-splitting valve and a downstream T-pipe. If sufficient volume is sent through the bypass section, and the condenser inlet pressure is high enough, the maximum temperature limit for the coolant water could be increased substantially.

The condensate must be subcooled in order to avoid cavitation between the condenser and pump. Between those two components, there will be pressure losses associated with the pipe length and a flow measurement section which, by its very design, causes a pressure drop across a throat tap nozzle flowmeter. Fig. 6.8 shows the condenser subcool against the condensate and coolant exit temperatures. The minimum subcool was set to 5 °F in the test loop model. For the range of condensate temperatures shown in Fig. 6.8 this corresponds to

100
the exit pressure being above the vapor pressure by $1.5 - 2.5 \text{ psia}$. A rough approximation of the $NPSH$ available is to multiply this pressure range by 2.4, giving $3.6 - 6.0 \text{ ft}$. This is a very small $NPSH$, especially considering that pressure losses will occur across the measurement section. A more realistic minimum subcool would probably be $10^\circ \text{F}$ or more in order to avoid cavitation at all operating points, but the value will ultimately depend on the $NPSH$ required by the selected pump and the flowmeter design. As the degree of subcool increases, the $NPSH$ available increases, but so does the fuel rate required for the boiler to maintain the same level of superheat. During system operation, the subcool can be reduced by decreasing the coolant flow rate, which also increases the coolant exit temperature. The quantitative dynamic relationships between all of these factors can only be determined by running real experiments or by creating a dynamic model.

### 6.3 Pump

The system head and flow requirement is independent of the pump selected, however an appropriate pump must be selected to cover the required operating range. The pump speed shown in Fig. 6.9 is for an MP Flomax 8 [9] centrifugal pump. The highest dotted black
Figure 6.9: Pump speed vs. system head and flow rate requirement

line is for a pump speed of 3450 RPM, the next lowest for 2850 RPM, and the last for 1750 RPM. The Flomax 8 pump covers all of the modelled loop operating points, with the exception of those that violate the maximum pump inlet temperature of 250°F. Presently, the pressure losses of the boiler and condenser are underestimated since they are assumed to be negligible. When more specific information is known about the pipe lengths on the steam side of both of these components, they can be added to the pipe geometries as extra length. This will probably drive the pressure drop over the maximum limit for the Flomax 8 pump.

6.4 Boiler

The boiler was modeled for a $UA$ value of $5.0 \frac{Btu}{s \cdot R}$. Similar to the condenser, Incropera [36] estimates the $U$ value for heat transfer between air and water as $25 < U < 50 \frac{W}{m^2 \cdot K}$, or $0.001223 < U < 0.002446 \frac{Btu}{s \cdot ft^2 \cdot \circ R}$. This corresponds to a heat transfer surface area of $2044 < A < 4088 \ ft^2$ for a $UA$ value of $5.0 \frac{Btu}{s \cdot R}$. Obviously, this analysis approach grossly oversimplifies the heat transfer process associated with boiling water, but it gives a first guess for the boiler size range.

Coupled with the heat transfer rate range of 3.5 MW to 7.2 MW shown in Fig. 6.10.
the area range enables a very good first guess at the boiler selection. The fuel rate range from that same figure is based on an inlet pressure of 0.25 psig, as specified by most natural gas providers at the meter inlet. Natural gas prices fluctuate year-to-year; however, $10 to $15 per 1000 ft$^3$ is a good estimate for commercial customers according to the Energy Information Administration [44]. Using the data from Fig. 6.10, the widest range of operating costs is $450 to $1440 per hour. This range can be reduced by considering the required range of non-dimensional turbine operating conditions, as will be done in Section 6.6.

### 6.5 Pipe

Four different pipe sections were considered in the model. The lengths and diameters modelled are given in Table 6.4. The length values are estimates based on required measurement sections. The pipe diameters are estimates based on manufacturer specifications for representative test loop components. Any decrease in the pipe diameter will increase the head losses per unit length for that section of pipe. In the present model, the head losses are almost negligible for some sections of pipe, but head losses are inversely proportional to the fifth power of diameter. Halving any pipe diameter would increase head losses by a factor
Table 6.4: Approximate system pipe dimensions

<table>
<thead>
<tr>
<th>Component at Inlet</th>
<th>Component at Exit</th>
<th>L (ft)</th>
<th>D (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbine</td>
<td>Condenser</td>
<td>5.0</td>
<td>12.0</td>
</tr>
<tr>
<td>Condenser</td>
<td>Pump</td>
<td>20.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Pump</td>
<td>Boiler</td>
<td>15.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Boiler</td>
<td>Turbine</td>
<td>20.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

The pipe diameter between the turbine and condenser is equal to the turbine exit flange diameter for the Dresser-Rand [12] research turbine. The head losses in this pipe section are very small, because the diameter is so large. However, it will also be the most expensive pipe, in terms of space requirement and monetary cost, so its length should be given careful consideration.

The pipe diameter between the condenser and pump will ultimately be of variable diameter. After consulting manufacturer catalogs, the condenser exit flange diameter will most likely be between 3.0 and 4.0 inches, while the pump inlet flange diameter will be between 1.0 and 2.0 inches. The maximum value for the pump inlet was used in the present model, because it is equal to the inlet diameter for the MP Flomax 8 [9] pump. Most of the pipe length will probably be equal to the larger condenser outlet diameter, so actual head losses will be less than those estimated with the 2.0 inch diameter. No accounting has been made for the pressure drop associated with the decrease in pipe diameter or the primary system flow measurement section, which will most likely be placed between the condenser and pump. Applying Bernoulli’s equation to a pipe diameter reduction with no losses yields the following equation:

\[
P_i - P_e = 8 \left(1 - \frac{1}{\beta^4}\right) \frac{\dot{m}}{\pi^2 \rho D_i^4 g_c},
\]

(6.1)

where

\[
\beta = \frac{D_e}{D_i}.
\]

(6.2)

For an inlet pressure of 32 psia, and a mass flow rate of 5.5 lbm/s, the pressure drop associated with a reduction in pipe diameter from 4.0 to 2.0 inches is on the order of 0.01 psi. There are some additional losses caused by the pipe fitting that can be accounted
for with equivalent length. Overall, the effect of this pressure drop on the system design is minimal.

The procedure for calculating the pressure drop across the flow meter is very similar. ASME PTC 19.1 gives an equation to relate the upstream and throat pressures of a throat tap nozzle flow meter to the mass flow rate and discharge coefficient. For incompressible flows, this equation is identical to Equation 6.1. Assuming that none of the pressure drop is recovered at the exit of the nozzle, the pressure drop for a flow meter with $\beta = d/D = 0.5$ would be similar to that for the pipe diameter reduction just considered. Therefore the flow meter pressure drop does not significantly impact the pump selection. The condenser to pump pipe length is consistent with ASME Performance Test Code 6 [34], which requires a straight length of 30 pipe diameters for the primary flow measurement section. This length does not include the connecting pipes between the measurement section and the other test loop components.

The pipe between the pump and boiler was set to a 2.0 inch diameter to reflect the outlet size of the MP Flomax 8 pump and the inlet pipe diameter for a series of industrial water tube boilers manufactured by Cleaver-Brooks [13]. Some other pumps that fit the system requirements have outlet diameters as small as 1.0 inches. The diameter for the pipe between the boiler and turbine is an estimate based on dimensions from Cleaver-Brooks, which specifies a steam exit pipe diameter between 4 and 6 inches. For more details on the particular components selected for the system, refer to Chapter 7.

As expected, Fig. 6.11 shows an increase in pressure and energy losses within the pipes as the system mass flow increases. Fig. 6.11 also compares the pipe pressure losses to the turbine pressure drop, which can be used to judge their relative impact on the pump head required. For the current system model and pipe dimensions, the pipes produce $20 - 30\%$ of the total system pressure drop, which is between 11 and 32 psi. The summed pressure losses across the condenser, boiler, a possible superheater, and flow measuring devices will probably be similar to the current total pipe pressure losses. If the desired performance range for the pump is anywhere near its upper speed limit, then a higher head pump will be needed for the system.
6.6 Overall Test Loop

The component result plots are useful for analyzing each component independently, but an effective system design calls for comparison of multiple component model outputs. Because it is one of the most expensive components in terms of capital cost and operational cost, the boiler is early on the list for selection, whereas the other loop components need to fill the remaining operating space. It would also be useful to devise a test path over which to operate the turbine. A logical test path would be for a constant fuel rate, because the boiler has the largest time constant of any of the loop components. A longer system settling time coincides with increased operational cost per steady state operating point. Other test paths should be considered once more information is known about the dynamic behavior of the system, because the final cost per operating point is a function of the system settling time and the total cost per unit time.

Following the constant fuel rate approach, Fig. 6.12 shows a potential test path at a fuel rate of 800 CFM for mass flow rates between $4.7 \leq \dot{m} \leq 5.3 \text{ lb}_m/s$. Other fuel rate values could be selected, but further results will show that changing the value will alter the minimum and maximum achievable average stage velocity ratio, and the maximum required turbine speed. For all subsequent result figures, the points at the extremities of
the “Possible Test Path” are identical to those in Fig. 6.12. The shape of the path on all figures is assumed to be linear.

At the constant 800 CFM fuel rate, Fig. 6.13 shows the pump speed varying between $2550 \leq N_{pump} \leq 3150$ RPM. A second possible test scenario would be to reduce the value of the constant fuel rate if the required minimum velocity ratio is increased. The opposite could be done if the required minimum decreased. The high turbine velocity ratio operating points would be unaffected, so long as the fuel rate is kept below 880 CFM for this particular rotor design and loss scenario. The gains by this method are very small, i.e. $\leq 0.05$ decrease in minimum achievable $\sigma$ for a 25% fuel rate increase from 800 CFM. If required, there are less expensive ways to expand the minimum $\sigma$ envelope limit in independent of, or in conjunction with the fuel rate.

At the constant 800 CFM fuel rate, Fig. 6.14 shows the turbine torque varies between $190 \leq \tau_{turbine} \leq 420$ ft $- lb_f$. The turbine torque is proportional to the required dynamometer back torque. The test path could be changed in much the same as it was discussed for Fig. 6.13, substituting the turbine torque for the pump speed. There is another possible performance envelope alteration that would increase the overall envelope, instead of merely moving within the current modelled envelope. The low turbine velocity ratio limit associated with blue color in Fig. 6.14 occurs at the minimum turbine shaft speed.
limit of 2000 RPM. If the minimum turbine shaft speed is reduced, then the minimum achievable velocity ratio will decrease for the same boiler fuel rate. The maximum required dynamometer back torque will increase. This method will work so long as the turbine speed above minimum allowable value for the coupled turbine-dynamometer system.

Fig. 6.15 shows the variation of the condenser coolant rate relative to the potential test
Figure 6.15: Condenser coolant rate vs. average stage velocity ratio and system mass flow path. All data points with coolant flow rates above 1000 GPM have been ignored, because most off-the-shelf centrifugal pumps are designed for flows less than that value. The flow limitation does not appear to hinder the performance envelope of the system. For the most part, the potential test path crosses operating points with coolant flow rates from 300 to 500 GPM. It appears that the required coolant flow rate has a range of possible values for each combination of velocity ratio and system mass flow rate. For instance, in the region where the system flow rate is 5.0 lbm/s and the velocity ratio is 0.72, the coolant flow rate varies between 500 and 1000 GPM for small changes in the other variables. The reason for this variation is that the condenser could subcool the hot side water to any degree, calling for different coolant flow rates, without affecting the system mass flow rate or the turbine velocity ratio. In other words, coolant flow rate is correlated to steam flow rate and velocity ratio, but it does not have a direct relationship to either variable. Fig. 6.15 is more useful to show that in the desired range of $\sigma$, the condenser coolant flow rate does not need to be above 500 GPM, but it could be if desired, or if additional subcooling is required at the pump inlet due to the NPSH requirement.

Just as the coolant flow rate does not need to vary much along the proposed test path, neither does the condenser heat rate. Fig. 6.16 shows the variation of average stage $\sigma$ with the condenser heat rate and pump speed. The slight change in condenser heat rate along
Figure 6.16: Average stage velocity ratio vs. condenser heat rate and pump speed

the test path is attributable to the varying cycle efficiency. If the boiler heat rate is held
constant, and the turbine power output increases, the condenser does not need to reject as
much heat to meet the subcooling requirement.

Fig. 6.17 shows the variation of turbine exit quality relative to the potential test path. Most of the operating points on the test path are outside the condensation region. Complete
absence of condensation could be guaranteed by increasing the boiler heat rate to 840 CFM. If more condensation is desired, then the fuel rate should be reduced. This would have the effect of reducing the turbine inlet temperature closer to the saturation point at the specified inlet pressure of 55 psia.

6.7 Dynamometer

Until now, the dynamometer component has not been considered. Similar to the other components, the selected dynamometer must be compatible with the turbine operating envelope. An eddy current dynamometer is considered as the primary candidate. Consider the dynamometer performance curve in Fig. 6.18. The red dotted line gives the maximum torque achievable at a particular operating speed. The dynamometer can produce less torque by reducing the resistive eddy current forces produced by the dynamometer. The

![Performance curves for a Dyne Systems Midwest Eddy Current Dynamometer Model MW314HS](image)

Figure 6.18: Performance curves for a Dyne Systems Midwest Eddy Current Dynamometer [11]
green line is the drag torque produced by the cooling water for the dynamometer, which would be in addition to the torque produced by eddy current forces. The blue line indicates that between speeds of 1200 and 8000 RPM, the power absorbed at the maximum torque is constant at 400 hp. So this particular dynamometer could never absorb more than that much power. An appropriate dynamometer for the test loop will enable the turbine to operate over some desired operating range.

A description of dynamometer performance will aid the component selection process. If the device driving the dynamometer produces a larger torque than the dynamometer is able to absorb at a particular speed, then the shaft will accelerate. If the driving torque is above the peak torque of the dynamometer, then the system will keep accelerating until the driving torque at the current speed is less than the maximum absorbable torque at that speed. If that never happens, then the system will accelerate uncontrollably. There should be failsafe mechanisms built into the system that prevent the dynamometer from exceeding it’s maximum speed limit. In order to determine if a particular dynamometer will be useful in the test loop, it’s performance curve is compared to the turbine performance map. Presently, the system is considered without any gearbox.

The turbine dimensional parameters that are most relevant to the dynamometer selection are the shaft speed, torque, and power output. Figures 6.19 and 6.20 give the latter two variables vs. shaft speed, with the color representing the average stage velocity ratio. The torque produced by the turbine is most sensitive to the mass flow rate and shaft speed. The mass flow rate will be controlled by a throttle valve at the turbine inlet. The shaft speed will be controlled by the load torque produced by the dynamometer. The maximum load torque and maximum drive torque follow a similar trend vs. shaft speed. From a dynamometer control standpoint, it would be easiest to operate on the red maximum load torque line. The 300 hp constant power line crosses a large range of velocity ratios in Fig. 6.20. However, operating the dynamometer in this manner would constantly present the danger of overspeeding the system. Also, controlling the remainder of the test loop components to fit that limitation would be very difficult.

Typically, eddy current dynamometer controllers have the capability to bring the system to a constant speed or a constant torque value. The constant speed scheme is the most
likely candidate for the present application, since the actual drive torque range is not known \textit{a-priori}. Under either control scheme, the dynamometer could be run through a series of steady state operating points, with the goal of achieving the full range of turbine velocity ratios. Setting the boiler fuel rate to a constant, as described in Section 6.6, is compatible with this dynamometer control scheme.

### 6.8 Additional Results

Figures B.1 to B.4 in Appendix B show the same results as those in Figures 6.12 to 6.17, with additional combinations of variables taken on the independent axes. The analysis discussed above for rotor design #1 and the zero loss scenario has also been carried out for each of the rotor designs and the three loss scenarios. Equivalent figures for the other rotor designs and loss values are located in Appendix B. Chapter 7 discusses the implications of the computer model results on the test loop design.
<table>
<thead>
<tr>
<th>Velocity Ratio</th>
<th>Power Output (hp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1862</td>
<td></td>
</tr>
<tr>
<td>0.2443</td>
<td></td>
</tr>
<tr>
<td>0.3024</td>
<td></td>
</tr>
<tr>
<td>0.3606</td>
<td></td>
</tr>
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<td>0.4187</td>
<td></td>
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<td>0.8836</td>
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<td>0.9417</td>
<td></td>
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<tr>
<td>0.9998</td>
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</tbody>
</table>

Figure 6.20: Average stage velocity ratio vs. total power output and shaft speed
Chapter 7

Proposed System Design

7.1 Condenser

Table 7.1 contains the condenser specifications found from modelling results and manufacturer specifications. The following descriptions apply to the data in all component

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
<th>Units</th>
<th>Proposed Design</th>
<th>Required by System</th>
<th>Specified by Mfr(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$UA$ Value</td>
<td>25</td>
<td>$\frac{Btu}{s/\degree R}$</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat Exchange Area</td>
<td>85 to 511</td>
<td>$ft^2$</td>
<td>X</td>
<td></td>
<td></td>
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<tr>
<td>Coolant Flow Rate (&quot;Test Path&quot;)</td>
<td>200 to 700</td>
<td>$GPM$</td>
<td>X</td>
<td></td>
<td></td>
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<tr>
<td>Coolant Flow Rate (Extremes)</td>
<td>100 to 5000</td>
<td>$GPM$</td>
<td>X</td>
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<tr>
<td>Heat Transfer Rate (&quot;Test Path&quot;)</td>
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<td>$MW^*$</td>
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<tr>
<td>Heat Transfer Rate (Extremes)</td>
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<td>$MW^*$</td>
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<tr>
<td>Steam Inlet Pipe Diameter</td>
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<td>Condensate Pipe Diameter</td>
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<td></td>
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<td>Coolant Pipe Diameter</td>
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<td>X</td>
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<td>Maximum Temperature</td>
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<td>Maximum Pressure</td>
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<td>Minimum Subcool</td>
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<td>Shell Diameter</td>
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<tr>
<td>Total Length</td>
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<td>$ft$</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight</td>
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<td>X</td>
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</tbody>
</table>

*1W $\approx 3.41 Btu/hr$

Table 7.1: Condenser Requirements
requirement tables in this chapter:

- Items with the “Required by System” designation are the extrema for values required by the system across the full range of modelled operating conditions, so long as the turbine under test is of similar size to those modelled.

- Items with the “Proposed Design” designation are approximations of the actual value extrema. For confirmation of values derived from a “Test Path”, refer to the plots for all rotor configurations and loss coefficients as given in Appendix B.

- Items with the “Specified by Manufacturer(s)” designation were found in product catalogs for one or more manufacturers.

All manufacturer specifications in Table 7.1 were found in U-tube heat exchanger catalogs from Taco Inc. [43], API Heat Transfer Inc. [46], and Diversified Heat Transfer Inc. [47]. All condenser pipe connections are flanged. There are condensers with smaller shell diameters that have NPT connections, but the 12 inch steam inlet required by the turbine automatically limits the condenser to the larger shell diameters for standard off-the-shelf products.

The selection process for the condenser is difficult to complete without more in-depth knowledge of condenser performance. The only criteria by which to limit the condenser selection that shows up in both the theoretical model and in manufacturer catalogs are steam inlet pipe diameter and heat exchanger area. Of the three manufacturers above, Taco Inc. is the only one that provides area information. Even then, the area specified in Table 7.1 is suggested based on approximate $U$ values. The actual $U$ value during operation isn’t definite. Among the three manufacturers, there are 56 different shell size permutations of diameter and length that meet the inlet pipe requirement. All three manufacturers have both 2-pass and 4-pass models for every shell size permutation. The 4-pass models would definitely be more applicable in the lower coolant flow region, as the total flow area would be reduced by half compared to the 2-pass models. However, the quantitative heat transfer characteristics of each condenser would be needed in order to reduce the number of useful models to less than 5 from each manufacturer. There are other criteria that differentiate the manufacturers, but don’t necessarily give preference to one manufacturer over another.
One of the differences between the various manufacturers are the operating condition extrema. The maximum temperature and maximum pressure are given as $375 \, ^\circ F$ and 150 psig, respectively, for the standard models of all manufacturers. Taco Inc. provides the additional option of substituting more expensive construction materials to increase the values to $450 \, ^\circ F$ and 250 psig. The pressure limit is really a concern to the present system design since the highest pressure throughout the whole test loop is at the boiler exit and is between 60 and 70 psia. The $375 \, ^\circ F$ limit was used in the test loop model, and there were sufficient operating points to cover the entire range of velocity ratio. If, for some reason, higher temperature testing is required, then the special Taco Inc. condensers would show some advantage. As of current test loop requirements, they do not.

Another criteria that differs between manufacturers is pipe connection size and type. The coolant pipes will most likely go to an external cooling pond or cooling tower. The long pipes required could cause there to be a large price difference between larger and smaller diameters. The actual coolant system should be able to handle the entire range of “Test Path” coolant flow rates in Table 7.1. Therefore, the selected coolant pump needs to cover that operating range. A regulating valve will most likely be used to control the flow rate along a constant pump speed curve. Pumps that generate the flows required by the coolant system generally have flanged connections. An additional criteria that would eliminate all API Heat Transfer condenser designs (8 total) is that all pipe connections must be flanged. The API Heat Transfer models have threaded coolant pipe connections. Although pipe connections aren’t a crucial criteria, they could be when coupled with the other selection criteria. Ultimately, each of the condenser manufacturers will need to be consulted to get price differences and possibly performance differences between the many candidate models.

7.2 Pump

Much like the theoretical model, the selection process is carried out under the assumption that the pump will have a variable frequency drive. Catalogs for standard off-the-shelf pumps give the head-flow relationship of families of pumps at a constant speed. Sometimes manufacturers provide data for several speeds. In the rare case, such as with MP Pumps
Inc. [9], performance curves are provided for three or more speeds with a single rotor size. All of the manufacturers assume their pumps will be used at a constant speed, which is dependent on the motor size selected and the AC current frequency supplied. In addition to a standard motor, a variable speed pump controller is needed to achieve variable speed operation. Table 7.2 shows the range of operating conditions required by the system performance envelope and the pipe connection size ranges for pumps that cover that envelope.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
<th>Units</th>
<th>Proposed Design</th>
<th>Required by System</th>
<th>Specified by Mfr(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Pipe Diameter</td>
<td>1.0 to 2.0</td>
<td>in.</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Exit Pipe Diameter</td>
<td>1.25 to 2.0</td>
<td>in.</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Temperature</td>
<td>250 to 300</td>
<td>°F</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow Rate</td>
<td>26 to 50</td>
<td>GPM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Head</td>
<td>24 to 78</td>
<td>ft</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NPSHR</td>
<td>5 to 15</td>
<td>ft</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2: Pump Requirements

Although there are many other options on the market, three pumps were considered for the primary test loop pump:

**MP Pumps, Inc. Flomax Pump** [9] The MP Flomax 8 was used in the test loop computer model because the manufacturer provided three constant speed operating curves for its 5.0 inch impeller. This enabled a better estimation of the pump speed. It does not make the pump any more useful for the present application. The Flomax 8 has 2 inch inlet and discharge NPT connections. It is a self-priming pump, available with or without an integrated motor.

**Taco, Inc. 1900 Series In-Line Pump** [48] The Taco 1911 In-Line Pump covers the test loop operating envelope at speeds between 1760 and 3500 RPM. It has inlet and discharge pipe diameters of 1.5 inches. It comes with a removable integrated motor that has a NEMA Standard 56 Frame C Face. There are tapped suction and discharge static pressure measurement ports. The mechanical seals are rated at a maximum temperature of 250°F, with an option for 300°F.

**ITT Goulds G&L Series End Suction Pumps** [49] Two Goulds End Suction Pumps
cover the test loop operating envelope at speeds between 1750 and 3500 $RPM$. Their maximum operating temperature is $250^\circ F$. They are available with ANSI Flanged or NPT pipe connections. The motor is integrated. The two applicable pumps are:

- NPO pump with 1.25 in. inlet and 1.5 in discharge
- ICS/ICS-F pump with 1.25 in. inlet and 1.5 in discharge

There are additional Goulds End Suction pumps that could be applicable to the test loop; however, performance data is only available at a single speed in the product catalog. When more detailed information is necessary, Goulds should be contacted directly.

Other pump manufacturers include Iwaki America Inc., Carver, and others. For a large listing of manufacturers, visit the websites for Glauber Equipment Corporation [50] and PumpCatalog.com [51].

### 7.3 Boiler

Cleaver-Brooks, Inc. is the only manufacturer considered in order to populate the manufacturer specifications of Table 7.3. They provide detailed engineering data and dimensions for all of their boilers. The required system operating range is near, but not at, the low size limit of Cleaver-Brooks Industrial Watertube Boilers [13]. Watertube boilers have combustion gas flow around pipes that carry water, which is heated to generate steam. Cleaver-Brooks lists a superheater as an accessory to its watertube boilers, thus it is assumed that one must be purchased in addition to the boiler. For more precise turbine inlet temperature control, a desuperheater may also need to be purchased.

The particular boiler model selected will depend on the strategy for test loop operation and future use. The extreme heat transfer range in Table 7.3 is very broad. It is possible that it can be covered by a single boiler and desuperheater, but not likely. As was seen in Chapter 6, turbine testing can be performed at a nearly constant heat rate for a given turbine. It is the introduction of additional turbine blade angles that necessitates the variable heat transfer rate. The other loop components (e.g. pump, condenser, and dynamometer)
<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
<th>Units</th>
<th>Proposed</th>
<th>Required</th>
<th>Specified</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U A$ Value</td>
<td>5</td>
<td>$Btu\ \text{h}^{-1}\text{R}^{-1}$</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat Exchange Area</td>
<td>2044 to 4088</td>
<td>$ft^2$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Fuel Flow Rate (“Test Path”)</td>
<td>750 to 900</td>
<td>$CFM$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Fuel Flow Rate (Extremes)</td>
<td>490 to 1257</td>
<td>$CFM$</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Heat Transfer Rate (“Test Path”)</td>
<td>4.9 to 5.8</td>
<td>$MW^*$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Heat Transfer Rate (Extremes)</td>
<td>3.4 to 7.3</td>
<td>$MW^*$</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Feedwater Pipe Diameter</td>
<td>2</td>
<td>$in.$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Steam Exit Pipe Diameter</td>
<td>4 to 6</td>
<td>$in.$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Maximum Temperature</td>
<td>750</td>
<td>$^\circ F$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Maximum Pressure</td>
<td>1000</td>
<td>$psig$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Weight (Dry to Flooded)</td>
<td>45000 to 70000</td>
<td>$lb$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Total Length</td>
<td>173 to 238</td>
<td>$in.$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Total Width</td>
<td>127</td>
<td>$in.$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Total Height</td>
<td>153.25</td>
<td>$in.$</td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

$*1W \simeq 3.41 Btu/hr$

Table 7.3: Boiler Requirements

can be use to traverse the desired non-dimensional turbine operating range. Thus a boiler heat rate range as broad as the “Test Path” values is large enough to cover all turbine designs. The addition of a desuperheater would increase the range further.

The test loop could be operated in a narrow band of mass flow rates so that the boiler heat transfer characteristics could be easily known. The important factor in selecting the boiler will be to ensure it has enough heating capacity to provide the maximum desirable heat transfer rate. If the boiler is oversized, then the desuperheating needs will be much greater and operating costs will be very large. However, the test loop would be more robust for future use with a larger boiler. The expense associated with building a second larger research turbine and with the additional operating costs will most likely outweigh the need for an oversized boiler years down the road. Cleaver-Brooks, Inc. has several boiler models that are applicable to whatever design strategy is reached, and for the range of operating conditions required by the test loop. Table 7.4 lists three of their boilers and their relative design features. If an additional desuperheating unit is required, then the heat transfer areas
Table 7.4: Selected data from the Cleaver-Brooks Industrial Watertube Boiler offering \[13\]

and maximum heat rate will increase. The heat transfer area estimates from the theoretical model are very rough, and account for all heat transfer regimes, including superheat. So the size range of the Cleaver-Brooks boilers is consistent with predictions.

Another manufacturer of watertube boilers is The Babcock and Wilcox Company. Their smallest series of products, the FM Package Boiler, covers the operating range of the turbine test loop. Babcock and Wilcox does not provide information that differentiates each particular model, so they will need to be consulted directly to select the appropriate design.

### 7.4 Dynamometer

One method of selecting the dynamometer would be to say that it should have a maximum load torque curve that is above the maximum turbine drive torque for all speeds. Then, the maximum turbine drive torque is the crucial design specification. However, the dynamometer should not be oversized, because the measurement accuracy of the load torque is defined as a percentage of the full scale, and not of the absolute value of the measurement.

Another selection method would be to ensure the dynamometer can follow the turbine operation over a prescribed test path, such as a constant boiler heat rate path. The dynamometer would then only need to absorb torques less than or equal to the torques on that test path. The ultimate selection method used will should be a combination of the two methods. There should be some open operating space for turbines that produce more torque than the ones modelled in this thesis. Even if the dynamometer torque range is limited, testing can be performed at lower torque values, so long as the entire range of velocity ratio is achieved. Table 7.5 gives the dynamometer selection requirements. The maximum
torque and power values in that table are associated with the zero-loss cases. So the actual maximum values produced will be slightly less than the specified value. Dyne Systems Inc. [11] produces several eddy current dynamometers that would meet the requirements of the system. Their relative utilities depend on the strategy of the final system design. In particular, the following three questions must be answered prior to final dynamometer selection. Should the test loop be able to handle larger turbines in the future? Must the turbine operate near its maximum power value? What is the actual range of required stage velocity ratios and, hence, shaft speeds?

Table 7.5: Dynamometer Requirements

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
<th>Units</th>
<th>Proposed Design</th>
<th>Required by System</th>
<th>Specified by Mfr(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driven Torque (&quot;Test Path&quot;)</td>
<td>140 to 420</td>
<td>ft − lb</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Driven Torque (Extremes)</td>
<td>0 to 850</td>
<td>ft − lb</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Driven Power (&quot;Test Path&quot;)</td>
<td>140 to 350</td>
<td>hp</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Driven Power (Extremes)</td>
<td>0 to 430</td>
<td>hp</td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 8

Conclusions and Recommendations

8.1 Conclusions

The critical review of axial turbine performance prediction methods concluded in a series of conventional empirical loss correlations stemming from Ainley and Mathieson \[5\] and a novel loss correlation presented by Dixon \[3\]. The empirical loss prediction methods used a variety of non-dimensional parameters to generalize the behavior of turbine blade rows. It is the full range of non-dimensional parameters, and not dimensional parameters that must be covered when designing steam turbine experiments.

Discussions with a commercial steam turbine design firm (Dresser-Rand) revealed that empirical loss prediction is heavily integrated in their design and selection process. A new steam turbine test loop is needed in order to develop new loss prediction schemes for more accurate and robust design methods. These discussions also confirmed that the measurement systems required for detailed stage-to-stage performance measurement would be part of a continuing conversation that is not required for successful test loop component selection. An initial review of axial turbine testing methods, specifically relevant codes and standards, introduced the test loop sensor placement for gross steam turbine performance.

Theoretical models were derived from first principles for the turbine, condenser, pump, boiler, and pipe components using control volume analyses. An iterative model was created for the overall test loop that converges on quasi-steady solutions for each component based on a set of thermodynamic inputs to the turbine component, and geometric inputs to all components. All theoretical models were implemented in a new open source simulation environment that carries out the calculation process over a range of up-to three turbine
model inputs. Turbine model input options include inlet pressure, inlet temperature, mass flow rate, and shaft speed. The simulation process was carried out for three different turbine blade configurations and three different values of the blade row enthalpy-loss coefficient in order to demonstrate full coverage of possible turbine operating conditions.

A parametric study was performed to narrow the required operating range of the test loop to a series of turbine test paths. The final operational envelope yielded a set of test loop component requirements for the condenser, pump, boiler, and dynamometer. These requirements were used to recommend off-the-shelf options available from manufacturers of each component type.

### 8.2 Recommendations for Future Work

- The final selection of components can be carried out when more specific customer requirements are known from the steam turbine test loop users.
  - Are there additional stator and rotor blade angle combinations that the test loop should be able to accommodate?
  - Are the turbine flow rates assumed in the present study representative of all future testing needs?
  - Are the turbine inlet pressures and temperatures assumed in the present study representative of future testing needs?
  - Are the turbine shaft speeds assumed in the present study representative of future testing needs?
  - Should the test loop be able to handle turbines with larger flow paths or power outputs?
  - Must the turbine operate near its maximum power value?
  - What is the actual range of required stage velocity ratios?

- Conduct a study of measurement techniques necessary for determination of row-by-row performance. These techniques are not available in measurement codes. Academic literature and industrial expertise must be leveraged.
• Conduct a study of measurement techniques necessary for loss component determination. Once again, academic literature and industrial expertise must be leveraged.
Bibliography


Appendix A

Implementation of Theoretical Models

A.1 Complete Test Loop

Throughout the program flowcharts, there are many different errorcode related blocks. If an errorcode returned to the main program is zero, then the model outputs are written to pre-allocated memory. For the most part, the non-zero error codes are passed back to the user, and the operating point that caused that errorcode is ignored. There are a few exceptions associated with the 1-D search in the condenser subprogram. These types of errorcodes tell the main program to adjust the model inputs, specifically the value of $T_{Ce}$ should decrease. The procedure for this task and for the overall test loop is shown in Fig. A.1. The dashed line of that figure contains the procedure for a single operating point. Each particular operating point is specified by values of $T_1$, $P_1$, $\dot{m}$, and $N$, which are defined in Section 4.1. The ranges of these values evaluated are specified in Chapter 6.

The computer implementation of the theoretical models is subdivided into two CV’s. The first is the Turbine CV. As its name implies, it includes only the turbine component. The second is called the Loop CV. It includes the condenser component, the pump component, the boiler component, and 4 instances of the pipe component. The inlet to the Loop CV is the exit from the Turbine CV, and the exit from the Loop CV is the inlet to the Turbine CV. The thermodynamic inputs to the overall cycle model include the inlet conditions to the turbine, the cycle mass flow rate, the condenser coolant temperature, and the boiler combustion characteristics. All other inputs to the system are either geometric, as is the case for the condenser and boiler UA values, or empirical, as are the cases for the pump curves and the pipe pressure loss correlation. The inlet conditions to the Loop CV for a particular set of thermodynamic model inputs are only known if the Turbine CV model has already been carried out at that input set. The user can select whether the Turbine
CV exit state is found by calculation or by accessing data saved to the hard drive. In either case, the Loop CV model is found in LOOPCV.c and it is assumed that its inlet and exit states are provided. The subprogram structure for the Loop CV is shown in Fig. A.2.

### A.2 Turbine Model

The turbine subprogram calls the Inlet, Stage, and Diffuser subprograms. The Stage subprogram also calls the Stator, Interblade, and Rotor subprograms. This structure is shown in Fig. A.3. As described previously, the turbine model returns an `errorcode`. For most subprograms, the `errorcode` is passed to the next higher level program or subprogram to determine if the subprogram operated as it should. The `errorcode` from the turbine is non-zero when the turbine returns an unachievable operating point, such as when a stage velocity

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ratio is greater than 1 or when the pressure drop is greater than the maximum allowed. Another reason for a non-zero errorcode in any component model is a faulty equation of state calculation due to nonreal state properties. When a non-zero errorcode is returned to the main() program, there are several error handling cases, but for the most part, the particular operating point is ignored.

A.3 Condenser Model

The algorithm used for the condenser subprogram is shown in Fig. A.7. The user has two options for the type of condenser model to run. Type 1 uses the subcooled condenser subprogram, which uses the effectiveness-NTU relationship to determine $\dot{m}_C$ and $T_{Ce}$ when $\Delta T_{subcool}$ is known. The Type 2 subprogram uses a 1-D search method based on Powell’s Method of iteratively solving for the minimum of a 3-point quadratic approximation of $T_{Ce}(\Delta T_{subcool})$. This enables finding of the $\Delta T_{subcool}$ associated with a user-selected value of $T_{Ce}$. In effect, the functionality of the subcooled condenser subprogram is switched to $\Delta T_{subcool}(T_{Ce})$. The upper bound for $\Delta T_{subcool}$ is the value that would result in $T_{He} = T_{Ci}$, which is the theoretical minimum value of $T_{He}$ for a crossflow heat exchanger. The minimum value for $\Delta T_{subcool}$ is the larger of two values. The first is the value that would result if $T_{He} = T_{max,pump}$ so that the maximum pump operating temperature is circumvented. The second value is 5 °F, which is a first guess for sufficient subcooling to guarantee no cavitation between the condenser and pump. The Type 2 model was used to obtain all results for this Thesis.

A.4 Pump Model

The pump subprogram, shown in Fig. A.10 uses the inlet and exit states to determine the volumetric flow rate and head across the pump component. It also calls the pump envelope check subprogram to determine if the inlet temperature violates the maximum allowable value selected by the user. That subprogram also calls the pump curve subprogram to determine the required speed of a user-defined pump, and also to determine whether that speed is within the specified operating range of that pump.
A.5 Boiler Model

The boiler subprogram is described by Figures A.11 to A.13. The equation set is slightly different than that of the Type 1 condenser subprogram, but the procedure is essentially identical.

A.6 Pipe Model

The pipe subprogram structure is described in Fig. A.14.
Figure A.2: Loop CV computer program flow
Figure A.3: Turbine component computer program flow
STATOR CV

Read Inlet State
Read Process Inputs
Read Geometry
Read Assumptions

errorcode = 0

tolerance = 0.00001
Po = 0.0

Initial Guess: \( V_{x2} = 0.25 V_1 \)

\[ error = 1.0, n = 0 \]

\[ error < tolerance? \]

\[ n > 10? \]

\[ tolerance = 1.0 \times tolerance \]

\[ n = 0 \]

\[ V_{x2} = V_{x2} \text{ bar} \]

Write Exit State
Write Process Outputs

errorcode = 8

\[ h_2 < 0.0? \]

\[ \text{NO} \]

\[ \text{YES} \]

\[ V_{o2} = V_{x2} \tan \alpha \]

\[ \text{Equation 4.11} \]

\[ error = 1.0; n = 0; \]

\[ \text{error < tolerance?} \]

\[ \text{NO} \]

\[ \text{YES} \]

\[ \text{errorcode} = 6 \]

\[ \text{NO} \]

\[ \text{YES} \]

\[ \text{errorcode} = 3 \]

\[ \text{errorcode} = 8 \]

\[ \text{Write Exit State} \]

\[ \text{Write Process Outputs} \]

\[ \text{Return errorcode} \]

\[ \text{Equation 4.10} \]

\[ \text{Equation 4.15} \]

\[ \text{Equation 4.16} \]

\[ \text{Equation 4.14} \]

\[ \text{error} = \left( |\frac{P_{2R} - P_o}{P_{2R}}| + |\frac{V_{x2} - V_o}{V_{x2}}| \right) \times 100.0 \]

\[ P_o = P_{2R} \]

Figure A.4: Stator CV computer program flow

INTERBLADE CV

Read Inlet State
Read Process Inputs
Read Geometry
Read Assumptions

errorcode = 0

tolerance = 0.05
Po = 0.0

Initial Guess: \( V_{x2} = V_{x2} \)

Equation 4.11

\[ error = 1.0, n = 0 \]

\[ error < tolerance? \]

\[ n > 10? \]

\[ tolerance = 1.0 \times tolerance \]

\[ n = 0 \]

\[ V_{o2} = V_{x2} \tan \alpha \]

\[ \text{Equation 4.15} \]

\[ \text{Equation 4.16} \]

\[ \text{Equation 4.14} \]

\[ \text{error} = \left( |\frac{P_{2R} - P_o}{P_{2R}}| + |\frac{V_{x2} - V_o}{V_{x2}}| \right) \times 100.0 \]

\[ P_o = P_{2R} \]

Figure A.5: Interblade CV computer program flow
Figure A.6: Rotor CV computer program flow
CONDENSER

Read Inlet State
Read Process Inputs
Read Geometry
Read Settings

P_{He} = P_{Hi}

Read CondenserType

errorcode = \textsc{subcooled,condenser} \implies T_{Ce} (\Delta T_{\text{subcool}})

1-D SEARCH using \textsc{subcooled,condenser}

\begin{align*}
\Delta T_{\text{subcool}} & > \text{MAX} [5.0, (T_{\text{Sat}, \min} - T_{\max, \text{pump}})] \\
\Delta T_{\text{subcool}} & < (T_{\text{Sat}, \min} - T_{\text{Ci}})
\end{align*}

errorcode = (0 \cup -8 \cup -9 \cup -10 \cup -11 \cup -12 \cup -13)

Write All States
Write Process Outputs
Return errorcode

SUBCOOLED_CONDENSER

Read Inlet State
Read Process Inputs
Read Geometry
Read Settings

\begin{align*}
\text{Equation 4.47} \\
\text{Equation 4.51} \\
\text{Equation 4.50} \\
\text{Equation 4.48} \\
\text{Equation 4.49} \\
\text{Equation 4.45}
\end{align*}

errorcode = \textsc{condenser,eps - ntu, model}

(Figure A.8)

Write Exit State
Write Process Outputs
Return errorcode

Figure A.7: Condenser component computer program flow
Figure A.8: Condenser \( \epsilon - NTU \) computer program flow
From EPS–NTU MODEL

error < tolerance?

YES

From EPS–NTU MODEL

NO

n > 10

YES

m_C > m_C?

NO

C_mincb = m_C 

C_mincc = m_C 

SHbool = 0

YES

C_mincb = m_C

C_mincc = m_C

SHbool = 1

NO

m_C > m_C?

YES

C_mincb = m_C

C_mincc = m_C

SCbool = 0

NO

C_mincb = m_C

C_mincc = m_C

SCbool = 1

Equation 4.61

Equation 4.58

Equation 4.66

Equation 4.67

Q_\text{MAX} = \frac{m_C \cdot C_P \cdot (X_{\text{MLV}} (T_{H_{\text{sat}}} - T_C) + X_{\text{SH}} (T_{Hi} - T_C) \ast \text{SH bool} + X_{\text{SC}} (T_{H_{\text{sat}}} - T_C) \ast \text{SC bool})}{C_P}

\text{error} = \frac{m_C \text{error}}{m_C} \ast 100$

Figure A.9: Condenser $\epsilon$ – NTU while loop computer program flow
Figure A.10: Pump component computer program flow

Figure A.11: Boiler component computer program flow
Figure A.12: Boiler $\epsilon$ – NTU computer program flow
Figure A.13: Boiler $\epsilon - NTU$ while loop computer program flow
Figure A.14: Pipe component computer program flow
Appendix B

Extended Results

B.1 Additional Results for Rotor #1 with Zero Losses

Figures B.1 to B.4 show additional comparisons of test loop characteristics for rotor #1 with $\zeta = 0.0$. These figures are merely for illustrative purposes, and are not necessary to specify test loop requirements.

Figure B.1: Average stage velocity ratio vs. turbine torque and pump speed
Figure B.2: Turbine shaft speed vs. boiler fuel rate and turbine torque

Figure B.3: Average stage velocity ratio vs. boiler fuel rate and turbine power output

Figure B.4: Turbine torque vs. average stage velocity ratio and mass flow
B.2 Results for Other Rotor Designs and Loss Coefficients

Figures B.5 to B.12 show the test loop model results for three different rotor designs, each at three different values of the loss coefficient, \( \zeta \). The results in these figures is only meant to supply the data necessary to define the loop component specifications layed out in Chapter 7. Because the present parametric study yields a large amount of data, the results shown are not comprehensive. Results for rotor #1 and \( \zeta = 0.0 \) are given in Chapter 6. Horlock [10] gives several estimates of this range when he compares the results of correlations from Ainley and Mathieson [5, 6] and Soderberg [52] for both design and off-design operation. The range of loss coefficients shown in Table B.1 was intended to cover the entire range of realistic blade-row loss coefficients without considering extreme situations which wouldn’t be of interest to improving performance.

<table>
<thead>
<tr>
<th>Rotor Design #</th>
<th>Loss Coefficient</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>B.5</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>B.6</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>B.7</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>B.8</td>
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<td>0.2</td>
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<tr>
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<td>B.10</td>
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<td>B.11</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>B.12</td>
</tr>
</tbody>
</table>

Table B.1: Table of figures in Section B.2
(a) Turbine torque vs. boiler fuel rate and system flow rate

(b) Average stage velocity ratio vs. boiler fuel rate and pump speed

(c) Condenser coolant rate vs. average stage velocity ratio and system mass flow

(d) Average stage velocity ratio vs. turbine torque and boiler fuel rate

Figure B.5: Results for rotor design #1 and $\zeta = 0.10$
(a) Turbine torque vs. boiler fuel rate and system flow rate
(b) Average stage velocity ratio vs. boiler fuel rate and pump speed
(c) Condenser coolant rate vs. average stage velocity ratio and system mass flow
(d) Average stage velocity ratio vs. turbine torque and boiler fuel rate

Figure B.6: Results for rotor design #1 and $\zeta = 0.20$
## Figure B.7: Results for rotor design #2 and $\zeta = 0.00$

### (a) Turbine torque vs. boiler fuel rate and system flow rate

<table>
<thead>
<tr>
<th>System Flow (lbm/s)</th>
<th>Torque (ft-lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>72.3</td>
</tr>
<tr>
<td>5</td>
<td>144.3</td>
</tr>
<tr>
<td>6</td>
<td>216.3</td>
</tr>
<tr>
<td>7</td>
<td>288.3</td>
</tr>
<tr>
<td>8</td>
<td>360.3</td>
</tr>
<tr>
<td>9</td>
<td>432.3</td>
</tr>
<tr>
<td>10</td>
<td>504.3</td>
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<tr>
<td>11</td>
<td>576.3</td>
</tr>
<tr>
<td>12</td>
<td>648.3</td>
</tr>
<tr>
<td>13</td>
<td>720.3</td>
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<tr>
<td>14</td>
<td>792.3</td>
</tr>
<tr>
<td>15</td>
<td>864.4</td>
</tr>
<tr>
<td>16</td>
<td>936.4</td>
</tr>
<tr>
<td>17</td>
<td>1008.4</td>
</tr>
</tbody>
</table>

### (b) Average stage velocity ratio vs. boiler fuel rate and pump speed

<table>
<thead>
<tr>
<th>Pump Speed (RPM)</th>
<th>Fuel Rate (CFM)</th>
<th>Velocity Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>200</td>
<td>0.1878</td>
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<tr>
<td>800</td>
<td>400</td>
<td>0.2458</td>
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<td>0.3037</td>
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<tr>
<td>1200</td>
<td>800</td>
<td>0.3617</td>
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<tr>
<td>1400</td>
<td>1000</td>
<td>0.4197</td>
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<tr>
<td>1600</td>
<td>1200</td>
<td>0.4777</td>
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<tr>
<td>1800</td>
<td>1400</td>
<td>0.5356</td>
</tr>
<tr>
<td>2000</td>
<td>1600</td>
<td>0.5936</td>
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<tr>
<td>2200</td>
<td>1800</td>
<td>0.6516</td>
</tr>
<tr>
<td>2400</td>
<td>2000</td>
<td>0.7096</td>
</tr>
<tr>
<td>2600</td>
<td>2200</td>
<td>0.7675</td>
</tr>
<tr>
<td>2800</td>
<td>2400</td>
<td>0.8255</td>
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<tr>
<td>3000</td>
<td>2600</td>
<td>0.8835</td>
</tr>
<tr>
<td>3200</td>
<td>2800</td>
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</tr>
<tr>
<td>3400</td>
<td>3000</td>
<td>0.9994</td>
</tr>
</tbody>
</table>

### (c) Condenser coolant rate vs. average stage velocity ratio and system mass flow

<table>
<thead>
<tr>
<th>System Flow (lbm/s)</th>
<th>Coolant Flow (GPM)</th>
</tr>
</thead>
<tbody>
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<tr>
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<tr>
<td>5</td>
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<td>880.6</td>
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<tr>
<td>16</td>
<td>939.9</td>
</tr>
<tr>
<td>17</td>
<td>999.2</td>
</tr>
</tbody>
</table>

### (d) Average stage velocity ratio vs. turbine torque and boiler fuel rate

<table>
<thead>
<tr>
<th>Torque (ft-lbf)</th>
<th>Fuel Rate (CFM)</th>
<th>Velocity Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>200</td>
<td>0.1878</td>
</tr>
<tr>
<td>72.3</td>
<td>400</td>
<td>0.2458</td>
</tr>
<tr>
<td>144.3</td>
<td>600</td>
<td>0.3037</td>
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<tr>
<td>216.3</td>
<td>800</td>
<td>0.3617</td>
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<tr>
<td>288.3</td>
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<td>0.4197</td>
</tr>
<tr>
<td>360.3</td>
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<tr>
<td>1008.4</td>
<td>3000</td>
<td>0.9994</td>
</tr>
</tbody>
</table>

---
(a) Turbine torque vs. boiler fuel rate and system flow rate
(b) Average stage velocity ratio vs. boiler fuel rate and pump speed
(c) Condenser coolant rate vs. average stage velocity ratio and system mass flow
(d) Average stage velocity ratio vs. turbine torque and boiler fuel rate

Figure B.8: Results for rotor design #2 and $\zeta = 0.10$
(a) Turbine torque vs. boiler fuel rate and system flow rate

(b) Average stage velocity ratio vs. boiler fuel rate and pump speed

(c) Condenser coolant rate vs. average stage velocity ratio and system mass flow

(d) Average stage velocity ratio vs. turbine torque and boiler fuel rate

Figure B.9: Results for rotor design #2 and $\zeta = 0.20$
Figure B.10: Results for rotor design #3 and $\zeta = 0.00$
Figure B.11: Results for rotor design #3 and $\zeta = 0.10$.
(a) Turbine torque vs. boiler fuel rate and system flow rate

(b) Average stage velocity ratio vs. boiler fuel rate and pump speed

(c) Condenser coolant rate vs. average stage velocity ratio and system mass flow

(d) Average stage velocity ratio vs. turbine torque and boiler fuel rate

Figure B.12: Results for rotor design #3 and $\zeta = 0.20$